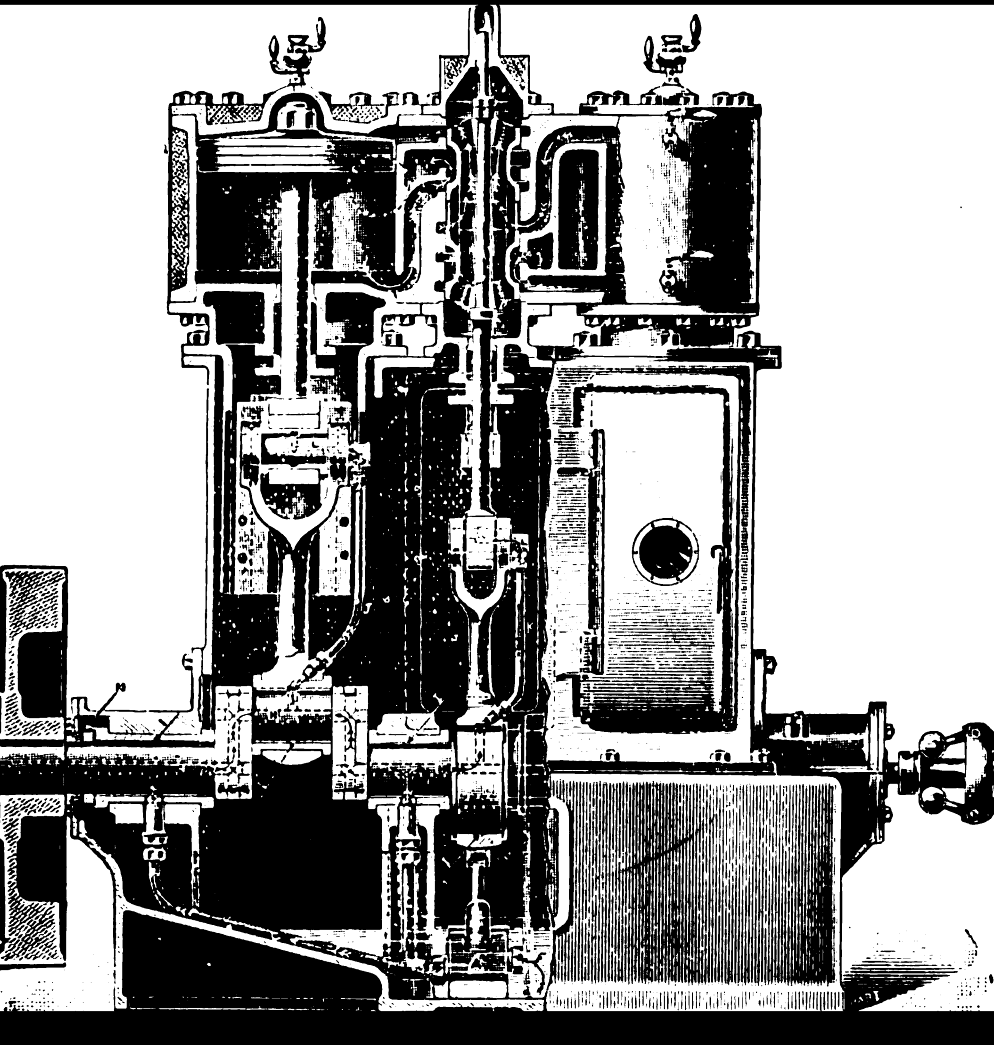

A text-book on applied mechanics

Andrew Jamieson

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A TEXT-BOOK

ON

APPLIED MECHANICS.

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SPECIALLY ARRANGED
FOR THE USE OF SCIENCE AND ART,
CITY AND GUILDS OF LONDON INSTITUTE,
AND OTHER ENGINEERING STUDENTS.

NOTICE.

The QUESTIONS set at the May Examinations of the SCIENCE and ART DEPARTMENT (Advanced and Honours Stages), 1896, are given in an Appendix at the end of this book.

With numerous Diagrams and Examination Questions.

LONDON:
CHARLES GRIFFIN AND COMPANY, LIMITED;
EXETER STREET, STRAND.
1896.

PREFACE.

THIS Text-Book has been written expressly for Second and Third Year Students of Applied Mechanics. It, therefore, forms a suitable companion to the Author's *Text-Book on Steam and Steam Engines.* It also forms a direct continuation of his *Elementary Manual on Applied Mechanics;* for it covers the Advanced Stage of the Science and Art Departments Examinations, and treats on many points demanded by the Honours Section. It will, moreover, be found of considerable use to those who aim at passing the Advanced and Honours Stages of the same Examinations in Machine Construction and Drawing, as well as the Examinations of the City and Guilds of London Institute in Mechanical Engineering. At the same time, the treatment of the subject is sufficiently general to satisfy the wants of other engineering students who do not happen to have these Special Examinations in view.

The book has been divided into six parts:—

I. The Principle of Work and its Applications.
II. Gearing.
III. Motion and Energy.
IV. Strength of Materials.
V. Graphic Statics.
VI. Hydraulics and Hydraulic Machinery.

Parts I. and II. are now issued together as a First Volume. These two parts consist of Nineteen Lectures under the following general headings:—Definitions of Matter and Work—Diagrams of Work—Moments and Couples—The Principle of Work applied to Machines—Friction of Plane Surfaces—Friction of Cylindrical Surfaces and Ships—Work absorbed by Friction in Bearings, &c.—Friction usefully applied by Clutches, Brakes, and Dynamometers—Inclined Plane and Screws—Efficiency of Machines—Wheel Gearing—Friction Gearing—Teeth of Wheels—Cycloidal Teeth—Involute Teeth; Bevel and Mortice Wheels—Friction and Strength of Teeth—Belt, Rope, and Chain Gearing—Velocity-Ratio and Friction of, and Horse-Power Transmitted by, Belt and Rope Gearing—Miscellaneous Gearing.

Great stress has been laid on principles, definitions, and uniformity of notation and symbols. The explanations, illustrations, and examples are such as will enable students to apply leading principles to practical work. In most instances direct reference has been made by footnotes to the latest and best books and to papers read before the leading Engineering Societies at home and abroad.

In every part of the subject a number of examples have been fully worked out, and at the end of each Lecture a series of carefully selected questions has been arranged in the *precise order of*, and relating *solely to*, the subject-matter of the Lecture, so that teachers and students may have a minimum of trouble in finding suitable examples.

The Author has to thank many of his old students

and friends for their kind assistance in connection with the production of this book.

Great care has been taken to avoid errors, but if any should be observed by readers, the Author will be glad to have them pointed out, and to receive any suggestions tending to increase the usefulness of the book.

ANDREW JAMIESON.

THE GLASGOW AND WEST OF SCOTLAND
TECHNICAL COLLEGE,
August, 1896.

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VOLUME II.

APPLIED MECHANICS.

PART I.—THE PRINCIPLE OF WORK AND ITS APPLICATIONS.

LECTURE I.

Applied Mechanics is that branch of applied science which explains the principles upon which machines and structures are made; how they act, and how their strength and efficiency may be tested and calculated.

In this treatise, we shall be chiefly concerned with the application of mechanical laws and principles to the determination of the equilibrium of machines, when acted on by forces; the transmission of power by machines and fluids; the stresses in, and the stability of, structures in general.

Although the student is expected to possess an elementary knowledge of the subject as far as it is treated in the author's *Manual on Applied Mechanics*, yet it is necessary to define, and explain briefly, in their respective places, the more elementary terms which will be used in this book. The student should not content himself with merely learning by rote the definitions herein given, but he should first get a clear understanding of the whole meaning of the things defined, and then endeavour to acquire the facility of defining the terms in his own words.

Definition.—**Matter is anything which can be perceived by our senses, or which can exert, or be acted on by, force.**

What matter is in itself we know not, we only know it by

its properties, its effects on other pieces of matter, and on our senses.

Definition.—**Force is that which produces, or tends to produce, motion or change of motion in the matter upon which it acts.**

So far as we are concerned we shall consider that mechanical force acts on matter either by a "push," "thrust," or "pressure," or by a "pull."

Unit of Force.—Since force is a measurable quantity, we must have a unit of force by which to measure other forces. In this country two units of force are in use, called, respectively, the **Gravitation Unit** and the **Absolute Unit.** The gravitation unit of force is adopted by engineers; and is used in the solution of most Statical problems in Theoretical Mechanics. The absolute unit of force is generally adopted in physical investigations; and, also, for convenience in most Kinetic problems in Theoretical Mechanics.

The distinction between these two units of force will be understood from the following definitions:—

Definition.—**An Absolute Unit of Force may be defined as that force which, acting for unit time on unit mass, imparts to it unit velocity.**

This is the general definition of an absolute unit of force, and by substituting proper units for time, mass, and velocity we get the various absolute units of force for any system in which time, mass, and length are adopted as the fundamental units. An absolute unit of force is, therefore, quite independent of the various values of gravity at different latitudes and of all other variable forces. In other words, it is an independent and invariable unit of force.

If the units of time, mass, and velocity be the second, pound, and foot per second respectively, we then get the following:—

Definition.—**The British Absolute Unit of Force, called the Poundal, is that force which, acting for one second on a mass of one pound, imparts to it a velocity of one foot per second.**

Definition.—**Our Gravitation Unit of Force, called the Pound, is the force required to support a mass of one pound avoirdupois against the attractive force of gravity at Greenwich sea level.**

Hence, the magnitude of a force, in gravitation units, is

numerically equal to the mass in pounds which it is just capable of supporting against gravity.

Since all places on the earth's surface (even when at the same sea level) are not at the same distance from the centre of mass of the earth; since the earth is not of uniform density, and since the effect of centrifugal force due to the earth's rotation, varies with the latitude (being greatest at the equator and zero at the poles), it is evident that the gravitation unit of force will vary with the locality. It is less at places near the equator than at places near the poles. For this reason, then, physicists have adopted the *Absolute* or *Invariable Unit* when dealing with problems in which the results are to be independent of locality and show a high degree of accuracy.

Relation between the Gravitation and Absolute Units of Force.—The symbol g may be defined as the number of feet per second by which the attractive force of gravity would increase, during every second, the velocity of a body falling freely *in vacuo* near the earth's surface. The value of g is about 32·2 at the latitude of London. Clearly, then, the gravitation unit is g times the absolute unit.

Hence, **A force of one pound $= g$ poundals.**

Or, **A force of one poundal $= \frac{1}{g}$ pound.**

DEFINITION.—**Work is said to be done by a force when it overcomes a resistance through a distance along the line of action of the resistance.**

Hence, if a force act upon matter and causes relative motion of its atoms, or relative change of motion between one body and another, then the force is said to do work.

In the mechanical sense of the term, *work* implies two things—(1) that some *effort* has been exerted or a resistance overcome; (2) That something is moved or a *displacement* takes place. Hence the two elements of work are effort (or resistance) and motion (or displacement).*

* The word "*effort*" is a very expressive term, implying the positive or active aspect of force; whereas the word "*resistance*" naturally conveys to one the negative or opposing aspect of force. By Newton's Third Law action and reaction (or effort and resistance) are equal and opposite, hence the terms "effort and action" or "resistance and re-action" are variously used in problems to denote one and the same force, according to the way in which the problem is viewed.

The work done by a force is measured by the product of the *numerical value* of the force and the *numerical value* of the displacement along its line of action.

DEFINITION.—**The British Unit of Work, called the Foot-pound (ft.-lb.), is the work done when a force of one pound acts through a distance of one foot along its line of action.**

DEFINITION.—**The British Absolute Unit of Work, called the Foot-poundal (ft.-pdl.), is the work done when a force of one poundal acts through a distance of one foot along its line of action.**

The student will readily see that the gravitation unit of work is equal to g absolute units. Hence, to convert ft.-lbs. into ft.-pdls., multiply the former by g—*i.e.*, by 32·2 for the latitude of London.

Let P = Force in lbs. (supposed to be constant or uniform).

" L = Displacement of force in ft. (this displacement being along the line of action of the force).

Then, from the above definitions, we get :—

$$\textbf{Work done} = (\mathrm{P} \times \mathrm{L})\ \textbf{ft.-lbs.}$$

The work done by a variable force will be considered in our next Lecture. In any case, if P represents the mean or average force during the displacement L, then P × L is the work done.

EXAMPLE I.—The bore of a pump is 8 inches, and the vertical lift is 54 yards, find the weight of the column lifted. If the stroke of the pump bucket be 9 feet, and the number of strokes 8 per minute, find the work done in one hour.

ANSWER.—Diameter of bucket = 8 ins. = $\frac{2}{3}$ ft. ; vertical lift or head of water = 54 × 3 = 162 ft. ; stroke of bucket = 9 ft.; number of strokes of bucket = 8 × 60 = 480 per hour.

(1) *To find the weight of the column lifted.*

$$\textit{Volume of water lifted} = \textit{Volume of column} = \frac{\pi}{4} d^2 l.$$

$$\text{,, \quad ,,} \quad = \cdot 7854 \times \left(\tfrac{2}{3}\right)^2 \times 162 = 56\cdot 55 \text{ cub. ft.}$$

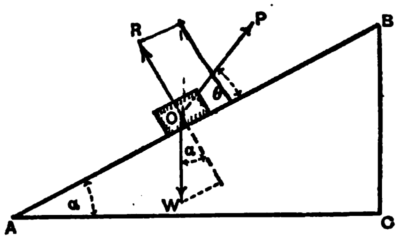
$$\therefore \textbf{ Weight of column lifted} = 56\cdot 55 \times 62\cdot 5 = 3{,}535 \textbf{ lbs.}$$

(2) *To find the work done per hour.*

Work done in one stroke = *Total pressure on bucket* × *stroke.*
" " = 3,535 × 9.
∴ Work done per hour = 3,535 × 9 × 480.
" " = 15,271,200 ft.-lbs.

Work done by a Force Acting Obliquely to the Direction of Motion.—Referring to the previous definition of work, the student will notice that the factor L, in the product P × L, means the displacement of the point of application of the force, P, *along its line of action.* In many cases the line of action of the force is *oblique* to the line of motion, and we now proceed to show how the work done is measured in such cases.

Consider the case of a body being drawn along a *smooth* inclined plane, A B, by an effort, P, whose line of action is inclined at an angle, θ to A B.



WORK DONE BY A FORCE ACTING OBLIQUELY.

Now, from elementary principles we know that P can be resolved into two components at right angles to each other: One (P cos θ) in the direction A B, and the other (P sin θ) at right angles to A B. The point of application, O, of P, moves in a direction parallel to A B, and, hence, by the definition just referred to, the latter component (P sin θ) *does no work.* The only effect of this perpendicular or *normal* component is to diminish the pressure between the body and the plane A B. Hence, the only part of P which is effective in causing motion is the component (P cos θ) parallel to A B.

Let the body be displaced from A to B.

Then, **Work done = P cos θ × A B = P × A B cos θ.**

But, A B cos θ is the length of the *projection* of the displacement, A B, on the line of action of the effort P, or, what is the

same thing, it is the length of the projection of the displacement on a line *parallel* to the line of action of P.*

If we consider the resistance to motion instead of the effort, we get :—

Work done = component of W parallel to AB × displacement, AB.
" " $= W \sin \alpha \times AB$.
" " $= W \times AB \sin \alpha$.
" " $= W \times BC$.

Here, again, B C is the length of the projection of A B on the direction or line of action of the resistance, W.

Hence, we have the following statement, which is often useful :—

The work done by a force is equal to the product of the force into the length of the projection of the displacement on the line of action or direction of the force.

EXAMPLE II.—A body is dragged along a floor by means of a cord which makes a constant angle of 30° with the floor. The tension in the cord is 10 lbs., weight of body 30 lbs. Find (1) the work done in drawing the body 10 feet along the floor; and (2) the pressure between the body and the floor.

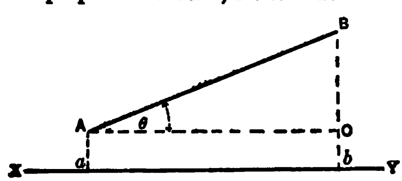
ANSWER.—Here P = 10 lbs.; W = 30 lbs.; $\theta = 30°$; L = 10 ft.

Resolving P into two components at right angles to each other; one in the direction of motion, and the other perpendicular to it, we get :—

Horizontal component $= P \cos \theta$.
Vertical component $= P \sin \theta$.

Hence, (1) **Work Done** $= P \cos \theta \times L$.
" " $= 10 \times \cos 30° \times 10$.
" " $= 100 \times \frac{\sqrt{3}}{2} =$ **86·6 ft.-lbs.**

* Let A B and X Y, be any two lines inclined to each other at an angle, θ. From A and B draw perpendiculars A a, B b to X Y. Then ab is called the



ORTHOGONAL PROJECTION.

orthogonal projection of line A B on line X Y, and clearly $ab = AB \cos \theta$. In the text the term *projection* is to be understood as *orthogonal projection*.

(2) The pressure between the body and the floor is equal to the weight of the body diminished by the vertical or normal component of P.

$\therefore$ Pressure between body and floor $= W - P \sin \theta$.

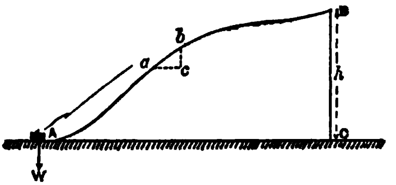
" " $= 30 - 10 \times \sin 30°$.

" " $= 30 - 10 \times \frac{1}{2} = 25$ lbs.

PROPOSITION I.—**The work done in lifting a body is independent of the path taken.**

When a body of weight, W, is lifted through a vertical height, h, the work done is simply W h, and is quite independent of the path described by the body in arriving at its new position.

Suppose the body to be translated from A to B along any route, A a b B. Consider the work done in moving the body from a to b, these two points being taken so near to each other that the part of the curve, a b, lying between them may be regarded as a straight line. Through a draw a c horizontal and meeting a vertical through b at the point c. Then a b c is a small triangle, and since the resistance overcome is simply that of the weight, W, acting vertically downwards, we get:—



WORK DONE IS INDEPENDENT OF THE PATH TAKEN.

$$\textit{Work done from } a \textit{ to } b = W \times b\,c.$$

By dividing the whole path, A B, into a great number of parts such as a b, we get for total displacement, A B :—

$$\textit{Work done} = W \times \Sigma\, b\, c,$$

where $\Sigma\, b\, c$ denotes *the sum of all such small vertical distances like b c.*

But, $\Sigma\, b\, c = B\,C = h$.

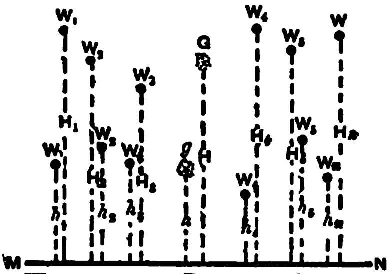
$\therefore$ **Work done** $= W \times h$.*

PROPOSITION II.—**The work done in raising a body or system of bodies is equal to the total weight raised multiplied by the**

* This result might have been deduced at once by assuming the results just previously obtained for the case of the inclined plane, by observing that B C is equal in length to the projection of the displacement, A a b B, on the direction of the resistance, W. This being a more general case than the one cited, we have thought it better to give an independent proof.

vertical height through which the centre of gravity of the body or system of bodies has been raised.

Suppose we have a number of weights, W_1, W_2, W_3, &c., at different heights, h_1, h_2, h_3, &c., respectively, above a given plane, M N. Let the *c.g.* of the system be at g, at a height, h, above M N.



WORK DONE IN RAISING A SYSTEM OF BODIES.

If all the weights be now lifted into different positions, so that the heights above M N are H_1, H_2, H_3, &c., respectively, and their *c.g.* at a height H. Then,

$$\textit{Total work done} = W_1(H_1 - h_1) + W_2(H_2 - h_2) + W_3(H_3 - h_3) + \&c.$$

But, by a property of the *c.g.* we know that

$$W_1 H_1 + W_2 H_2 + W_3 H_3 + \ldots = (W_1 + W_2 + W_3 + \ldots) H.$$

And,

$$W_1 h_1 + W_2 h_2 + W_3 h_3 + \ldots = (W_1 + W_2 + W_3 + \ldots) h.*$$

Subtracting the latter equation from the former, we get :—

$$W_1 (H_1 - h_1) + W_2 (H_2 - h_2) + W_3 (H_3 - h_3) + \ldots$$
$$= (W_1 + W_2 + W_3 + \ldots)(H - h),$$

$$\therefore \textbf{Total work done} = (W_1 + W_2 + W_3 + \ldots)(H - h) = W(H - h).$$

Where, $W = W_1 + W_2 + W_3 +$, &c.

And, $H - h =$ vertical height through which the *c.g.* of the system has been raised.

Although we have taken a system of disconnected weights in proving the above proposition, the student will clearly perceive that the result arrived at is true generally, whatever form the material may have.

The following simple examples will show the application of the two preceding propositions :—

EXAMPLE III.—A uniform beam, 20 ft. long, and weighing 30 cwts., is lying on the ground. Find the work done in raising it into a vertical position by turning it about one end.

* The student will readily see that these results are arrived at by taking the moments of the weights about the plane, M N, and then applying the "*principle of moments.*"

Answer.—The centre of gravity of the beam is 10 ft. from either end, and, during the operation of lifting the beam, this point will describe an arc, which is the quarter of the circumference of a circle whose centre is at the end of the beam in contact with the ground. The vertical height through which the *c.g.* is raised is, therefore, 10 ft.

Hence, by the two preceding propositions,

Work done = whole weight of beam × height through which its *c.g.* is raised.

" " $= (30 \times 112) \times 10.$

" " = **33,600 ft.-lbs.**

Example IV.—A cistern 22 ft. long, 14 ft. broad, and 12 ft. deep, has to be filled with water from a well 7 ft. in diameter. The vertical height of the bottom of the cistern above the free surface of the water in the well is 100 ft. when the operation of filling the cistern is commenced. Water flows into the well at the rate of 462 cubic ft. per hour. Find the work done in filling the cistern, supposing 30 minutes are required for the operation.

Answer.—During the operation of filling, the surface of the water in the well will fall, say x ft., from E F to H K.

The volume of water taken from the well = volume of water E F H K + volume of water run in during the operation.

But, *Volume of water taken from well*

= volume of tank A B C D

$= 22 \times 14 \times 12$ (cub. ft.)

Volume of water represented by E F H K

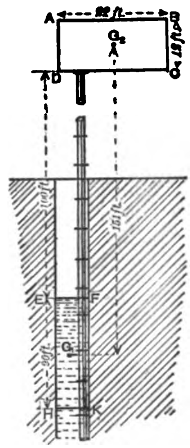
$$= \frac{\pi}{4} d^2 x = \frac{11}{14} \times 7^2 \times x \text{ (cub. ft.)}$$

Volume of water run in in 30 minutes

$$= \frac{462}{2} = 231 \text{ (cub. ft.)}$$

$$\therefore \quad \frac{11}{14} \times 7^2 \times x + 231 = 22 \times 14 \times 12$$

$$\therefore \quad x = \mathbf{90\ ft.}$$



Work done in Filling a Cistern.

Clearly, then, the *c.g.* of the water has been raised from G_1 to G_2, or through a height of $45 + 100 + 6 = \mathbf{151\ ft.}$

$\therefore$ **Work done in filling cistern**

$= (22 \times 14 \times 12 \times 62\tfrac{1}{2}) \times 151 =$ **34,881,000 ft.-lbs.**

LECTURE I.—QUESTIONS.

1. Define the terms *force* and *work done by a force*. What are the units of force and work as adopted by engineers in this country? State the relations between the gravitation and absolute units of force and work, and say why the latter units are so desirable for many scientific purposes.

2. A punching machine is provided with a flywheel and driven by an engine at such a rate that two holes are punched in three minutes. The plate operated on is 1 inch thick, and it is estimated that a mean pressure of 69 tons is exerted through the space of 1 inch. Find the average work done per minute by this machine. *Ans.* 8,586·6 ft.-lbs.

3. A body weighing 100 lbs. is pushed along a horizontal plane by a force of 25 lbs., the direction of which makes an angle of 45° with the plane. Find the work done in moving the body through a distance of 100 feet, and the pressure between the body and the plane. If the direction of the force be reversed, so that it now becomes a pull, find the work done during a displacement of 100 feet, and the pressure between the body and the plane. *Ans.* (1) 1,767 ft.-lbs.; 117·67 lbs. (2) 1,767 ft.-lbs.; 82·33 lbs.

4. Find the work done in turning a cubical block of stone about one of its edges until the diagonals of its end faces are vertical. Length of edge of cube, 4½ ft., *s.g.*, 2·5. *Ans.* 13,270 ft.-lbs.

5. A cistern 22 ft. long, 10 ft. broad, and 8 ft. deep, has to be filled with water from a well 8 ft. in diameter and 40 ft. deep. Supposing no water to flow into the well during the operation of filling the cistern, ascertain how far the surface of the water in the well is depressed, and the work done in filling the cistern when the bottom of the latter is 36 ft. above the free surface of the water in the well at the beginning of the operation. *Ans.* 35 ft.; 6,325,000 ft.-lbs.

LECTURE II.

CONTENTS.—Graphical Representation of Work Done—Diagram of Work for any Varying Force—Case I., When the Force varies directly as the displacement—Examples I., II., III., and IV.—Case II., When the Force varies inversely as the displacement—Boyle's Law—Proposition—Work Done by a Gas Expanding according to Boyle's Law—Example V.—Indicator Diagrams—Rate of Doing Work—Definition of Power or Activity—Definition of Horse-Power—Example VI.—Useful and Lost Work—Definition of Efficiency—Table of Efficiencies—Examples VII. and VIII.—Questions.

Graphical Representation of Work Done.—We have already seen that work is the product of two factors—force and displacement. Now a force can be completely represented by a straight line, and so also can a displacement. Since an area is of two dimensions, it follows at once, that work done can be represented by an area. In our elementary manual on Applied Mechanics, we have shown how to represent by diagrams, the work done for several simple cases. For a uniform force the diagram of work is a rectangle; for a uniformly increasing or uniformly decreasing force the diagram will be triangular or trapezoidal in shape. The shape of the diagram will, however, depend on the manner in which the force varies with the displacement.

A correct diagram of work must fulfil the following conditions:—

(1) *Its area must represent the work done.*

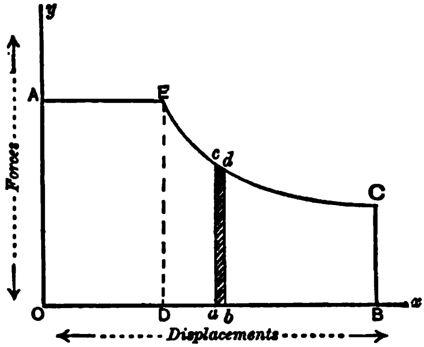
(2) *It must show to the eye the manner in which the force varies in magnitude during the displacement.*

Diagram of Work for any Varying Force.—We shall now show that, if the force during any given displacement be represented in magnitude by the ordinates of the curve, and the displacement by the corresponding abscissæ, the work done will be represented by the area of the figure enclosed between the curve, the initial and final ordinates, and the axis of x.

Let O x, O y be rectangular axes; O x being the axis along which displacements are to be set off, and O y the axis along which forces are plotted.

Suppose the force at the beginning of the motion to be represented by the ordinate, O A, and at the end of the motion by B C, then A E C is called the *curve of resistance.* At any intermediate point, such as a, the force or resistance will be represented by

the ordinate, *a c*. The work done during the displacement, O B, will be represented by the area, O A E C B.



GRAPHICAL REPRESENTATION OF WORK DONE BY A VARYING FORCE.

For, suppose the force to be uniform during the displacement, O D, then :—

$$\left.\begin{matrix}\textit{Work done during}\\ \textit{displacement}\ \mathrm{O\ D}\end{matrix}\right\} = \textit{Area of rectangle}\ \mathrm{O\ A\ E\ D.}^{*}$$

We have now to show that the work done during the displacement, D B, is represented by the area, D E C B.

Take any two ordinates, *a c*, *b d*, indefinitely near to each other. The lengths of these ordinates represent the magnitude of the forces at the points *a* and *b* respectively. Now, since the ordinates are indefinitely near together, the difference in their lengths will be indefinitely small. In that case *a c d b* may be considered a rectangle (its breadth being infinitely small). Hence, the work done during the infinitely small displacement, *a b*, will be represented by the small rectangular strip, *a c d b*. By dividing the displacement, D B, into an infinite number of indefinitely small portions, such as *a b*, and drawing the ordinates at these points, an infinite number of narrow rectangles will thereby be obtained. Hence, it is clear that the work done during the displacement, B D, is represented by the sum of these elementary areas,

i.e., *The Work done during displacement* D B = *Area* D E C B;

∴ *Total work done during displacement* O B = *Area* O A E C B.

* The sign (=) is here used as an abbreviation of the words "*is represented by*," and must not be employed in its usual sense as meaning "*is equal to*." The text will enable the student to attach the proper meaning to the sign used.

Two particular cases of work done by varying forces will now be considered.

Case I.—**When the force varies directly as the displacement.**

When we stretch or compress a piece of any *solid elastic material*—*e.g.*, a helical or spiral spring, or a bar of iron or steel—the resistance offered is directly proportional to the extension or compression produced, when these are small compared with the length of the body. Thus, if a force of 10 lbs. be required to stretch a spiral spring 1 inch, then a force of 30 lbs. will be required to stretch the same spring 3 inches, and so on.

We may state this law thus :—

$$\text{Force} \propto \text{Displacement.}$$

Or,

$$P \propto L$$

$\therefore$

$$P = c\,L\,;\ \text{or}\ \frac{P}{L} = c,$$

where c is some constant quantity depending on the nature of the material.

Hence, if P is the force required to stretch or compress the material by an amount L, and p the force required to stretch or compress the material by an amount l, then

$$P : p = L : l.$$

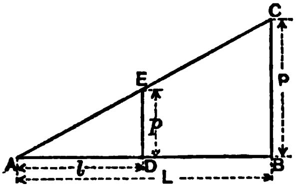


Diagram of Work when the Force varies as the Displacement.

We shall now show that in the diagram of work for this case, the line of resistance is a straight line.

Set out A B to represent the displacement L, and A D to represent l.

Let the ordinate B C represent P. Join A C, and through D draw the ordinate D E. Then D E will represent p.

By similar triangles,

$$DE : BC = AD : AB$$

$$\left.\begin{array}{ll} \textit{i.e.,} & DE : P = l : L \\ \text{But,} & p : P = l : L \end{array}\right\} \therefore DE = p.$$

Again, the areas of the triangles A D E, A B C represent the work done during the displacements A D and A B respectively.

For,

$$\left.\begin{array}{r}\textit{Work done during}\\ \textit{displacement}\ \mathrm{A\,D}\end{array}\right\} = \textit{area of}\ \triangle\, \mathrm{A\,D\,E}. = \tfrac{1}{2}\,\mathrm{D\,E} \times \mathrm{A\,D}.$$

$\therefore$

$$\left.\begin{array}{r}\textit{Work done during}\\ \textit{displacement}\ l\end{array}\right\} = \tfrac{1}{2}\,p\,l.$$

Similarly,

$$\left.\begin{array}{r}\textit{Work done during}\\ \textit{displacement}\ \mathrm{L}\end{array}\right\} = \tfrac{1}{2}\,\mathrm{P\,L}.$$

Also,

$$\left.\begin{array}{r}\textit{Work done during}\\ \textit{displacement}\ \mathrm{D\,B}\end{array}\right\} = \textit{area of trapezoid}\ \mathrm{D\,B\,C\,E}.$$

$$\text{,,} \qquad \text{,,} \qquad = \tfrac{1}{2}\,(\mathrm{B\,C} + \mathrm{D\,E}) \times \mathrm{D\,B}.$$

$$\text{,,} \qquad \text{,,} \qquad = \tfrac{1}{2}\,(\mathrm{P} + p)\,(\mathrm{L} - l).$$

The above results are true whether the force uniformly increases or uniformly decreases. The following examples will render the above principles clear :—

Example I.—Show, by a diagram or otherwise, how the work done in stretching an elastic spring is obtained; 5 ft.-lbs. of work are required to stretch a spiral spring 3 inches; what force in lbs. will be required to stretch the same spring $6\frac{3}{4}$ inches?

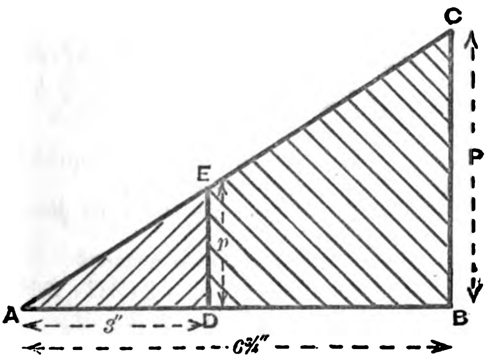


Diagram of Work in Stretching a Spring.

Answer.—Let p, P denote the forces required to stretch the spring 3 inches and $6\frac{3}{4}$ inches respectively.

Then, area of $\triangle$ A D E represents the work done in stretching the spring from A to D.

$\therefore$ $\left.\begin{array}{r}\textit{Work done during}\\ \textit{displacement } AD\end{array}\right\} = \frac{1}{2} DE \times AD,$

$\quad$ " " $= \frac{1}{2} p \times 3 = \frac{3}{2} p$ *inch-lbs.*

But, by the question,

$\left.\begin{array}{r}\textit{Work done during}\\ \textit{displacement } AD\end{array}\right\} = 5$ *ft.-lbs.* $= 60$ *inch-lbs.*

$\therefore$ $\frac{3}{2} p = 60.$

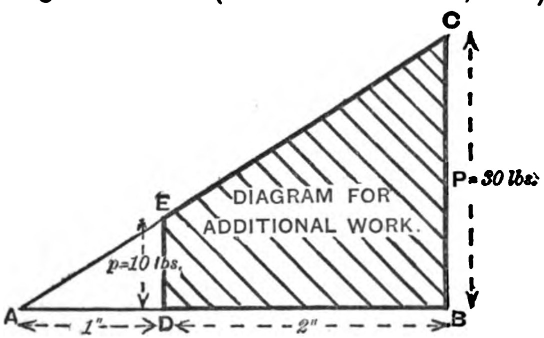
Or, $p = 40$ **lbs.**

Again, $P : p = AB : AD.$

Or, $P : 40 = 6\frac{3}{4} : 3$

$\therefore$ $P = \dfrac{40 \times 6\frac{3}{4}}{3} = \mathbf{90}$ **lbs.**

Example II.—A spiral spring is stretched through 1 inch by a force of 10 lbs. Find the work done in stretching it through an additional length of 2 inches. Draw the diagram of work done, giving dimensions. (Adv. S. and A. Exam., 1890.)



Answer.—Let p and P denote the forces required to stretch the spring 1 inch and 3 inches respectively.

Then, area of $\triangle$, A D E, represents the work done in stretching the spring from A to D.

Also, the work done during the displacement

$DB = \text{area } \triangle ABC - \triangle ADE = \frac{1}{2}(BC + DE) \times BD.$

But, $DE : BC = AD : AB.$

i.e., $10 : P = 1 : 3.$

$\therefore$ $P = \dfrac{10 \times 3}{1} = 30$ lbs.

Substituting this value of P in above equation—

The work done $= \frac{1}{2}(BC + DE) \times DB.$

" " $= \frac{1}{2}(30 + 10) \times 2 = \mathbf{40}$ **inch-lbs.**

Example III.—A chain weighing 2 lbs. per foot passes over a fixed smooth pulley, so that 14 feet hangs over on one side and 6 feet on the other. Show by a diagram the work which will be done in pulling round the wheel until the upper end of the chain is 1 foot above the lower end.

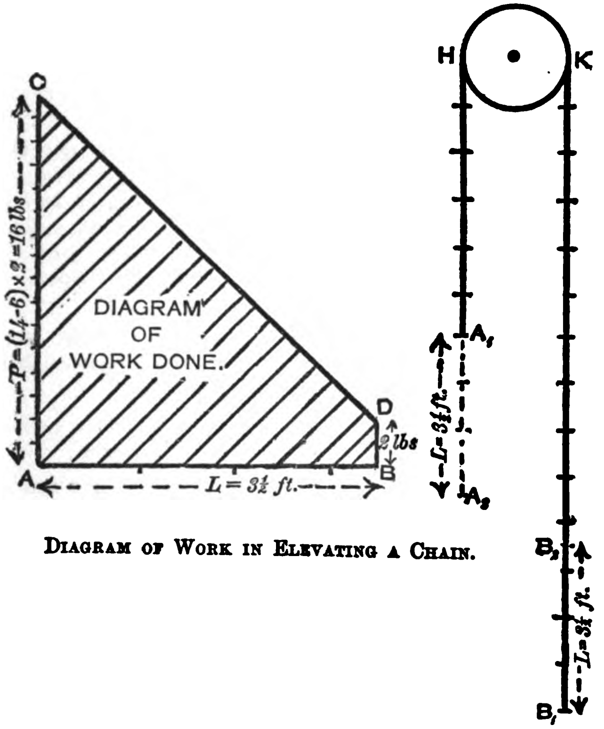


Diagram of Work in Elevating a Chain.

Answer.—Clearly the resistance to be overcome at the beginning of the motion is the weight of the *difference* of the two parts of the chain hanging from the pulley. That is, the initial resistance = weight of a length of (14 − 6) = 8 ft. of chain.

$\therefore$ *Initial resistance* $= P = 8 \times 2 = 16$ *lbs.*

When the upper end of the chain is pulled down so as to be 1 ft. above the lower end, the displacement will be 3½ ft., and the

Final resistance $= p = 1 \times 2 = 2$ *lbs.*

We can now construct the diagram of work. Set off A B to represent the displacement, = $3\frac{1}{2}$ ft. Make A C represent the initial resistance, = 16 lbs., and B D represent the final resistance, = 2 lbs. Join D C, then the trapezoid A B D C is the diagram of work, and its area represents the work done.

$$\therefore \quad \text{Work done} = \text{area A B D C}$$
$$\quad\text{,,} \quad \text{,,} \quad = \tfrac{1}{2}(\text{A C} + \text{B D}) \times \text{A B}$$
$$\quad\text{,,} \quad \text{,,} \quad = \tfrac{1}{2}(16 + 2) \times 3\tfrac{1}{2}$$
$$\quad\text{,,} \quad \text{,,} \quad = 31\cdot5 \text{ ft.-lbs.}$$

Of course, the student will readily see that it is not always necessary to construct a diagram of work before arriving at the answer. All that is necessary to know, is the mean resistance during the displacement. Thus, in the above example, the mean resistance is the arithmetical mean between the initial and final resistances. This, multiplied by the displacement, gives the answer.

EXAMPLE IV.—Four cwts. of material are drawn from a depth of 80 fathoms by a rope weighing 1·15 lbs. per linear foot: how many units of work are expended?

ANSWER.—Here the resistance to be overcome at the beginning of the lift = *whole weight of rope* + *weight of material.*

Whole weight of rope $= (80 \times 6) \times 1\cdot15 = 552$ *lbs.*
Weight of material raised $= 4 \times 112 = 448$ *lbs.*

$\therefore$ *Resistance at beginning of lift* $= 552 + 448 = 1{,}000$ *lbs.*

If we suppose the whole length of rope to be hauled in when the material is brought to the surface, then the resistance to be overcome at end of lift is simply that of the weight of the material to be raised.

$\therefore$ *Resistance at end of lift* $= 448$ *lbs.*

We can now construct the diagram of work. Set off A B to represent the displacement, $= 80 \times 6 = 480$ ft. Set off A C to represent the initial resistance due to weight of rope, = 552 lbs. Make A D represent to the same scale as A C, the resistance due to the weight of the material, = 448 lbs. Join C B, then triangle A B C is the diagram of work for the rope or variable part of the load. Complete the rectangle, A B E D; then A B E D is the diagram of work for the material, or constant part of the load. D C B E is the diagram of work for the whole load.

Hence,

$$\textbf{Work expended during lift} = \textit{Area}\ \mathrm{DCBE},$$
$$\text{,,} \quad \text{,,} \quad = \tfrac{1}{2}(\mathrm{DC} + \mathrm{EB}) \times \mathrm{AB},$$
$$\text{,,} \quad \text{,,} \quad = \tfrac{1}{2}(1{,}000 + 448) \times 480,$$
$$\text{,,} \quad \text{,,} \quad = 347{,}520 \text{ ft.-lbs.}$$

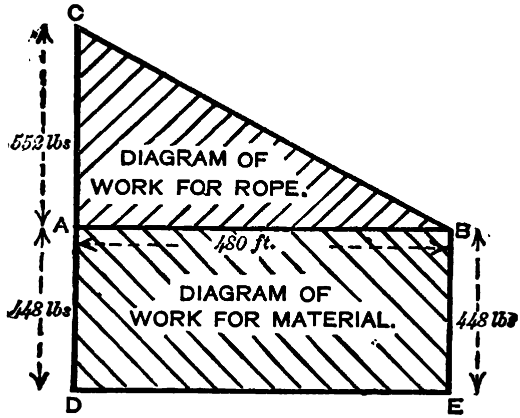


DIAGRAM OF WORK IN ELEVATING A LOAD BY A ROPE OR CHAIN.

We could have arrived at the answer quite simply by finding the work done in lifting the rope and material separately, and then adding together the results. Thus :—

$$\textit{Work done in lifting rope} = \textit{weight of rope} \times \textit{height through which its c.g. is raised}$$
$$\text{,,} \quad \text{,,} \quad = 552 \times \tfrac{1}{2} \times 480 = 132{,}480 \textit{ ft.-lbs.}$$
$$\textit{Work done in lifting material} = 448 \times 480 = 215{,}040 \textit{ ft.-lbs.}$$
$$\therefore \quad \textbf{Total work expended} = 132{,}480 + 215{,}040$$
$$\text{,,} \quad \text{,,} \quad = 347{,}520 \text{ ft.-lbs.}$$

CASE II.—**When the force varies inversely as the displacement.**

We have seen that, when a *solid* elastic material is stretched or compressed within certain limits, the resistance is proportional to the extension or compression. When, however, we compress a gas or allow it to expand, the law expressing the relation between the pressure applied and the expansion or compression produced, is different from that in the case of a solid. By expansion or compression of a gas we mean the increase or decrease produced in its *volume.*

Boyle's Law.—**The pressure of a fixed mass of a perfect gas, at a constant temperature, varies inversely as the volume it occupies.***

Let P = absolute pressure, or elastic force, of gas *per square foot.*

„ V = volume of gas *in cubic feet;*

Then, $$P \propto \frac{1}{V}.$$

Or, $$P\,V = c, \text{ a constant.}$$

The student should carefully note that Boyle's Law is true only for perfect gases, and, also, that the temperature must remain constant throughout the changes of volume. Boyle's Law is very nearly true for dry atmospheric air, and may be applied to most other gases when these are not near their points of liquefaction.

The value of the constant, c, for a given mass, depends on the nature of the gas under consideration; and also, on the constant temperature maintained. Thus, the constant for air at a temperature 32° F. and a mass of one pound is $(14{\cdot}7 \times 144 \times 12{\cdot}34) = 26{,}214$ ft.-lbs., at atmospheric pressure. Where, 12·34 is the volume in cubic feet of 1 lb. of air at 32° F. and 14·7 lbs. the pressure per square inch.

Proposition.—**The work done per unit area on or by a gas during a change of volume is equal to the product of the average pressure per unit area into the change of volume.**

Let P = average pressure of gas in lbs. per sq. ft.,

„ V_1 = initial volume of gas in cub. ft.,

„ V_2 = final „ „ „

Then, $$\textbf{Work done} = P\,(V_2 \sim V_1) \text{ ft.-lbs.}$$

For, suppose we have a cylinder fitted with an air-tight frictionless piston, the area of the latter being A square feet. Let this piston enclose a volume of gas in the cylinder equal to V_1 cubic feet. Now, let the piston move through a distance, L, feet in the cylinder, either by doing work in compressing the gas, or by allowing the gas to do work during its expansion. If the gas

* For an experimental demonstration of this law, and its applications to the steam engine, see the author's works on the "Steam Engine." In all applications of Boyle's Law, *absolute* pressures must be taken. The pressure of the atmosphere may be taken at 14·7, or, roughly, 15 lbs. per square inch absolute.

now occupy a volume, V_2 cubic feet, and the average pressure on the piston during its displacement be P lbs. per square foot, then :—

$$\textit{Work done} = P\,A \times L\ (\textit{ft.-lbs.})$$

But, $$A\,L = \textit{Change of volume of gas.}$$

i.e., $$\text{,,} \quad = V_2 \sim V_1\ (\textit{cub. ft.})$$

$\therefore$ $$\textbf{Work done} = P\,(V_2 \sim V_1)\ \text{ft.-lbs.}$$

This result is true whatever be the size and shape of the vessel containing the gas. When the vessel is of uniform cross sectional area, it may be convenient to consider only the displacement of the piston, the total pressure on the piston being taken as the effort or resistance. Examples of this will be given immediately.

Work done by a Gas Expanding according to Boyle's Law.— We are now in a position to be able to find the work done by or on a gas during a change of volume when the change takes place at constant temperature on a constant mass of gas.

Let p_1 = initial absolute pressure,
,, v_1 = ,, volume,
,, p_2 = final absolute pressure,
,, v_2 = ,, volume.

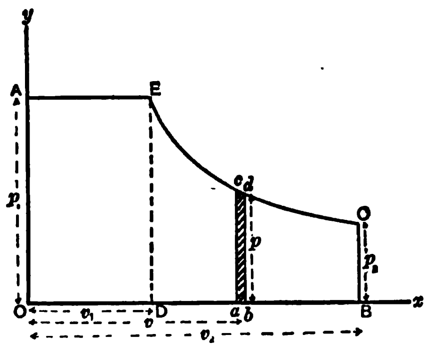


Diagram of Work Illustrating Boyle's Law.

Let O A and O D represent the initial pressure and volume respectively; B C and O B the final pressure and volume.

Consider the work done during the small increase of volume $a\,b$.

$$\text{Let } O\,a = v \text{ and } O\,b = v + \Delta v$$
$$\text{Then } a\,b = \Delta v.$$

Let p = mean pressure during increase of volume Δv.

Then, *Work done from a to b* $= p \, \Delta v$.

$\therefore$ $\left.\begin{array}{r}\textit{Total work done during} \\ \textit{expansion from } \mathrm{D} \textit{ to } \mathrm{B}\end{array}\right\} = \Sigma p \, \Delta v.$

Now, in the limit, when $a\,b$ is taken infinitely small, p will denote the pressure corresponding to the volume v, and Δv, will, according to the notation of the Calculus be denoted by $d\,v$.

$$\therefore \quad \left.\begin{array}{r}\textit{Total work done during} \\ \textit{expansion from } \mathrm{D} \textit{ to } \mathrm{B}\end{array}\right\} = \int_{v_1}^{v_2} p\,d\,v.$$

But, by Boyle's law,

$$p\,v = p_1\,v_1 = p_2\,v_2 = c;$$

$$\therefore \quad p = p_1\,v_1 \times \frac{1}{v}.$$

$$\therefore \left.\begin{array}{l}\textit{Total work done} \\ \textit{during expansion} \\ \textit{from } \mathrm{D} \textit{ to } \mathrm{B}\end{array}\right\} = \int_{v_1}^{v_2} p_1\,v_1 \times \frac{d\,v}{v}$$

$$\text{,, ,,} \quad = p_1\,v_1 \int_{v_1}^{v_2} \frac{d\,v}{v} \text{ (since } p_1\,v_1 = \text{a constant)}$$

$$\text{,, ,,} \quad = \Big[p_1\,v_1 \log_e v \Big]_{v_1}^{v_2}$$

$$\text{,, ,,} \quad = p_1\,v_1\,(\log_e v_2 - \log_e v_1)$$

$$\text{,, ,,} \quad = p_1\,v_1 \log_e \frac{v_2}{v_1}.$$

Similarly,

$$\left.\begin{array}{l}\text{Total work done} \\ \text{during expansion} \\ \text{from D to B}\end{array}\right\} = p_2\,v_2 \log_e \frac{v_2}{v_1}.$$

Where $\log_e \frac{v_2}{v_1}$ is the *Napierian* or *hyperbolic logarithm* of the ratio of the final to the initial volume.* This ratio $\frac{v_2}{v_1}$, is often called the *ratio of expansion*, and is denoted by the letter r. Since the above is true whether the gas be expanded or compressed, we get :—

$$\left.\begin{array}{r}\textbf{Work done during expansion or} \\ \textbf{compression between } v_2 \textbf{ and } v_1\end{array}\right\} = p_1\,v_1 \log_e r.$$

$$\text{Or,} \quad \text{,, ,,} \quad = p_2\,v_2 \log_e r.$$

$$\therefore \quad \text{,, ,,} \quad = c \log_e r.$$

Where c is the constant in the equation $p\,v = c$.

* The curve E$c\,d$C is a *rectangular hyperbola*, the axes Ox, Oy being asymptotes.

The following example will impress the above results more firmly on the mind :—

EXAMPLE V.—Calculate the work done when 10 cub. ft. of air at an initial absolute pressure of 45 lbs. per square inch, is expanded at constant temperature to a volume of 50 cub. ft. Find, also, the final pressure.*

ANSWER.—Here, $P_1 = 45 \times 144 = 6{,}480$ lbs. per sq. ft.

$$V_1 = 10 \text{ cub. ft.}$$

$$V_2 = 50 \quad ,,$$

$$\therefore \quad r = \frac{V_2}{V_1} = \frac{50}{10} = 5.$$

Then,

$$\textit{Work done during expansion} = P_1 V_1 \log_e r$$

$$,, \qquad ,, \qquad = 6{,}480 \times 10 \times \log_e 5 \; (\textit{ft.-lbs.})$$

Referring to a table of hyperbolic logarithms, we find :—

$$\log_e 5 = 1{\cdot}6094.$$

$\therefore$ **Work done during expansion** $= 6{,}480 \times 10 \times 1{\cdot}6094,$

$,, \qquad ,, \qquad =$ **104,289·92 ft.-lbs.**

Next, to find the final pressure.

Here, $\quad p_2 V_2 = p_1 V_1;$

$\therefore \quad p_2 \times 50 = 45 \times 10.$

i.e., $\quad p_2 = \dfrac{45 \times 10}{50} =$ **9 lbs. absolute per sq. in.**

Indicator Diagrams.—As an important application of the diagram of work we may here briefly refer to indicator diagrams obtained from a steam or gas engine. Every engineer knows the importance of obtaining correct diagrams of work done by the steam or gas in the working cylinder of his engine. By an inspection of the cards thus obtained, he is able to detect faults in the working of the engine, which could not be revealed by any other method. For example, from such diagrams he can at once tell whether the valves are properly set; the manner in which the pressure on the piston varies throughout the stroke; the state of the vacuum in the condenser, if it be a steam engine, and a multitude of other facts. He can also calculate the area of the diagram, and thereby deduce the horse-power developed in the cylinder. Lastly, he can

* Expansion at constant temperature is called "*Isothermal*" expansion.

compare this *indicated horse-power* with the *brake horse-power* given out at any point of the machinery during its transmission, and so find the power spent on friction, &c.

Rate of Doing Work.—In the definition and examples of work given in this and the last lecture it will be noticed that no reference was made to the time taken to perform the work. Thus, in Example III., we saw that the work done in raising the 4 cwts. of material and the rope was 347,520 ft.-lbs., and this result is true no matter what time was taken to accomplish it. It did not affect the question of work done whether the material was raised in twenty minutes by the action of men on a windlass, or in one minute by the action of a steam engine. But, if we wish to compare those two agents in respect to the *rate* at which they perform the work, it is clear that this will be in the proportion of 1 : 20. Thus :—

$$\textit{Rate of doing work} \quad . \quad . \quad . \quad . = \frac{\textit{Work done}}{\textit{Time taken}};$$

$$\therefore \qquad \frac{\textit{Rate at which the men work}}{\textit{Rate at which the engine works}} = \frac{1}{20} \Big/ \frac{1}{1} = \frac{1}{20}.$$

Hence, although the *amount* of work done is the same in both cases, yet the *rate* of doing the work is *inversely* as the time taken to do it.

Definition.—**Power and Activity are the terms used to denote the rate of doing work.**

It is evident, that in order to compare the respective powers of two agents, we must have a standard or *unit of power*. The unit of power adopted in this country is the **Horse-Power.** This unit was first introduced by Watt in estimating the power of his engines, and is still the unit adopted by British engineers.

Definition.—**The Unit of Power, called the Horse-power, is the rate of doing work corresponding to 550 ft.-lbs. per second, or 33,000 ft.-lbs. per minute, or 1,980,000 ft.-lbs. per hour.**

Although a horse's power was thus defined by Watt, yet no horse is capable of working at the above rate for any length of time. The actual power of a good horse, working for 10 hours a day, is found to be about 22,000 ft.-lbs. per minute instead of 33,000 ft.-lbs. per minute. The term, however, is still retained by engineers, although it is not now used in its original sense.*

* For historical account of this term, see the author's *Elementary Manual on Steam and Steam Engines*, p. 122.

Let P = Average pressure or effort exerted, in lbs.
„ L = Displacement of P, in feet.
„ T = Time taken to perform a given amount of work.
„ H.P. = Horse-power.

Then, $\text{H.P.} = \dfrac{P \times L}{550 \times T}$, when T is expressed in seconds.

„ $= \dfrac{P \times L}{33{,}000 \times T}$, „ „ minutes.

„ $= \dfrac{P \times L}{1{,}980{,}000 \times T}$, „ „ hours.

EXAMPLE VI.—The two cylinders of a locomotive engine are each 17 inches in diameter. Length of stroke, 24 inches. Mean effective pressure of steam on pistons, 80 lbs. per square inch. Diameter of driving wheels, 6 feet. Speed of engine and train, 30 miles per hour. Find the horse-power exerted by engine.

ANSWER.—(1) *Find the work done per revolution of driving wheels* :—

$$\left.\begin{array}{c}\textit{Total effective pressure}\\ \textit{on each piston}\end{array}\right\} = P = \frac{\pi}{4} d^2 p,$$

„ „ „ $= \dfrac{11}{14} \times 17^2 \times 80 = 18{,}165{\cdot}7$ *lbs.*

Since there are two equal cylinders, and each piston makes two strokes per revolution of the driving wheels, we get :—

$$\left.\begin{array}{c}\textit{Total work done per revolu-}\\ \textit{tion of driving wheels}\end{array}\right\} = 2\,P \times 2\,L,$$

„ „ $= 2 \times 18{,}165{\cdot}7 \times 2 \times 2$;

„ „ $= 145{,}325{\cdot}6$ *ft.-lbs.*

(2) *Find the number of revolutions of driving wheel per hour.*

30 *miles per hour* = $30 \times 5{,}280$ (*ft. per hour*).

$$\left.\begin{array}{c}\text{Circumference of}\\ \text{driving wheels}\end{array}\right\} = \pi D = \frac{22}{7} \times 6 \ (\textit{feet}).$$

$$\therefore \left.\begin{array}{c}\textit{Number of revolutions}\\ \textit{of driving wheels}\end{array}\right\} = \frac{30 \times 5{,}280}{\frac{22}{7} \times 6} = 8{,}400 \ (\textit{per hour}).$$

$$\therefore \left.\begin{array}{c}\textit{Total work done per}\\ \textit{hour}\end{array}\right\} = 145{,}325{\cdot}6 \times 8{,}400 \ (\textit{ft.-lbs.})$$

$$\therefore \quad \text{H.P. Exerted} = \frac{145{,}325{\cdot}6 \times 8{,}400}{1{,}980{,}000} = 616{\cdot}5.$$

Useful and Lost Work.—Up to this point, we have had no occasion to refer to the relation between the *Useful Work* given out by a working agent, and the *Whole Work Expended.* A machine is erected to perform a given amount of work, which is called the **Useful Work,** but during the working of the machine a considerable part of the whole work expended is absorbed in overcoming frictional resistances, &c., and this work is usually spoken of as the **Lost Work.** The sum of the *Useful Work* and the *Lost Work* is equal to the *Total Work Expended;* or,

Total Work Expended = Useful Work + Lost Work.

Definition.—**The Efficiency of a Machine is the ratio of the Useful Work Done to the Total Work expended.**

$$\text{Or,} \qquad \textbf{Efficiency} = \frac{\textbf{Useful Work Done}}{\textbf{Total Work Expended}}.$$

Now, the useful work done is always less than the total work expended, hence the efficiency will always be a number less than unity. What is known as the **Percentage Efficiency** is the efficiency, as found above, multiplied by 100. We shall have examples of the efficiencies of several machines later on; but in the meantime it may be instructive to note the efficiencies of a few of the more common machines.

Table of Efficiencies.

Names of Machines.	Efficiency.	Percentage Efficiency.
Wheel and Compound Axle,	·58	58
Simple Screw Jack,	·25	25
Worm and Worm Wheel,	·3 to ·6	30 to 60
Block and Tackle,	·75	75
Weston's Differential Blocks,	·4	40
Hydraulic Ram,	·6	60
Pumps for Draining Mines,	·66	66
Turbine,	·7 to ·8	70 to 80
Overshot Water-Wheel,	·6 ,, ·8	60 ,, 80
Undershot ,, (Common),	·3 ,, ·4	30 ,, 40
,, ,, (Poncelet's),	·6	60
Breast Wheel,	·5 to ·7	50 to 70
Best Compound Steam Engine,	·8 ,, ·9	80 ,, 90
Gas Engine,	·75 ,, ·8	75 ,, 80

Example VII.—What horse-power is required to lift 3,000 cubic feet of water per hour to a height of 80 feet, supposing $\frac{1}{4}$ of the power to be lost by friction, &c.?

Answer.—*Weight of water raised every minute* $\Big\} = \dfrac{3{,}000 \times 62\cdot5}{60} = $ *3,125 lbs.*

$\therefore$ *Useful work done per minute* $= 3{,}125 \times 80 = $ *250,000 ft.-lbs.*

Let T.H.P. denote the *theoretical* horse-power required, *i.e.*, the power required when all frictional losses are neglected, and A.H.P. the *actual* horse-power required.

Then, $$\text{T.H.P.} = \frac{250{,}000}{33{,}000} = 7\cdot\dot{5}\dot{7}.$$

But, according to the question, $\frac{1}{4}$ of the *actual* power is lost in friction, &c.

$$\therefore \quad \text{A.H.P.} - \tfrac{1}{4}\,\text{A.H.P.} = \text{T.H.P.}$$

$$i.e., \quad \tfrac{3}{4}\,\text{A.H.P.} = \text{T.H.P.}$$

$$\therefore \quad \text{A.H.P.} = \tfrac{4}{3} \times 7\cdot\dot{5}\dot{7} = 10\cdot1.^{*}$$

Example VIII.—If there were 4,000 cubic feet of water in a mine, whose depth is 60 fathoms, when an engine of 70 horse-power began to work the pumps, and the engine continued to work for 5 hours before the mine was cleared of the water, find the number of cubic feet of water which had run into the mine per hour, supposing $\frac{1}{3}$ of the power of the engine to be lost in the transmission.

Answer.—Let x = number of cub. ft. of water run into the mine in one hour.

Then,

Volume of water pumped per hour $\Big\} = x + \dfrac{4{,}000}{5} = (x + 800)$ *cub. ft.*

$\therefore$ *Useful work done per hour* $\Big\} = (x + 800) \times 62\cdot5 \times (60 \times 6)$ *ft.-lbs.*

Now, since $\frac{1}{3}$ of the power of the engine is lost in the transmission, the remaining $\frac{2}{3}$ will be employed in doing the above work.

$\therefore$ *Useful work done by engine per hour* $\Big\} = \frac{2}{3} \times 70 \times 33{,}000 \times 60$ (*ft.-lbs.*)

i.e., $(x + 800) \times 62\cdot5 \times (60 \times 6) = \frac{2}{3} \times 70 \times 33{,}000 \times 60.$

Or, $$x + 800 = \frac{70 \times 88 \times 2}{3} \text{ (cub. ft.)}$$

$$\therefore \quad x = 3{,}306\cdot\dot{6} \text{ cub. ft.}$$

* The method of answering this class of questions is very frequently misunderstood by students. In an example like the above the student is very liable to *increase* the *useful work* by $\frac{1}{4}$ of its amount and then find the H.P. required. The author finds the above method of answering the question appeals more directly to students.

Lecture II.—Questions.

1. A spiral spring is stretched through $\frac{1}{2}$ inch by a force of 10 lbs. Find the work done in stretching it through an additional length of 2 inches. Draw the diagram of work done, giving dimensions. *Ans.* 60 inch-lbs.

2. A chain weighing 3 lbs. per foot passes over a fixed smooth pulley, so that 20 feet hangs on one side, and 10 feet on the other. Find the work done in pulling round the wheel until the upper end of the chain is 6 inches above the lower end. Explain clearly the method of setting out the diagram of work in this case and construct it. *Ans.* 74·8 ft.-lbs.

3. A steel wire rope weighing 9 lbs. per fathom is employed to raise 2 tons of material from a depth of 100 fathoms. Find, by calculation, and by a scale diagram of the work, the work done during the lift, supposing the whole length of rope to be wound on the drum at end of lift. Also find the resistance offered at $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ lift respectively. *Ans.* (1) 2,958,000 ft.-lbs., or 1,320·5 ft.-tons; (2) 5,155 lbs.; (3) 4,930 lbs.; (4) 4,705 lbs.

4. Investigate an expression for the work done when a gas is compressed from a volume v_1, to a volume v_2, the compression being isothermal—*i.e.*, at a constant temperature. Find the work done in compressing 10 cubic feet of air at a pressure of 15 lbs. per square inch absolute till its pressure is 75 lbs. per sq. in. absolute; given $\log_e 5 = 1{\cdot}6094$. *Ans.* 34,763 ft.-lbs.

5. Find the work done in exhausting a chamber of 100 cubic feet capacity to $\frac{1}{10}$ of an atmosphere, atmospheric pressure being taken at 14·7 lbs. per square inch absolute. Hyperbolic expansion being assumed. *Ans.* 141,770·56 ft.-lbs.

6. How is the working power of an agent measured? When is an agent said to work with 1 horse-power? One agent (A) lifts 50 lbs. through 100 feet in 4 minutes; a second agent (B) lifts 2 lbs. through 150 feet in a quarter of a minute; what ratio does A's working power bear to B's? *Ans.* A : B = 25 : 24.

7. The travel of the table of a planing machine cutting both ways is 9 feet, and the resistance to be overcome while cutting is taken at 400 lbs. If the number of double strokes made in one hour be 40, find the horse-power absorbed by the machine. (S. & A. Exam., 1889.) *Ans.* 0·145 H.P.

8. In employing furnace ventilation in a coal mine, there is a furnace at the bottom of a shaft which is estimated to raise 100,000 cubic feet of air at 50° F. through 170 feet in 1 minute. What is the rate at which the furnace does work as estimated in horse-power? *Note.*—A cubic foot of air at 50° F. weighs ·078 lb. (Adv. S. & A. Exam., 1892.) *Ans.* 40·18.

9. A pump is worked directly from the ram of a water-pressure engine, the cylinder of which is 6 inches in diameter, that of the pump being $8\frac{3}{4}$ inches. The head of water in the supply-pipe which gives the pressure is 450 feet, and that in the delivery pipe is 150 feet: find the ratio of work done to total work expended. *Ans.* ·708 : 1.

10. One thousand cubic feet of water has to be raised to a height of 200 feet per minute: the question is, how many horse-power will it be necessary to employ, supposing that one quarter of the power is lost through friction and other causes? *Ans.* 505 H.P.

11. A builder finds that water accumulates in the space for a foundation at the rate of 1,500 cubic feet per hour. This water has to be pumped to a height of 20 feet. The question is, what amount of power will be required

to keep the said foundation dry, supposing that only 0·6 of the power applied is available for useful effect? *Ans.* 1·58 H.P. nearly.

12. Steam enters a cylinder at 80 lbs. per square inch absolute, and is cut off at $\frac{1}{3}$ of the stroke. Diameter of piston, 40 inches, length of stroke, 5 feet. No of revolutions, 50 per minute. Back pressure, 3 lbs. per square inch absolute. Find the horse-power of the engine, assuming the steam to expand hyperbolically, $\log_e 3 = 1·0985$. *Ans.* 1,009 H.P.

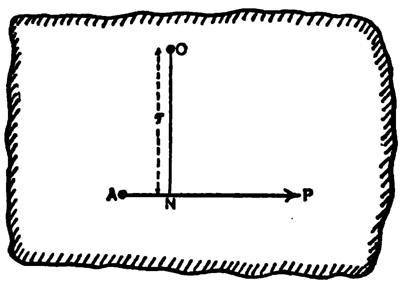
HYPERBOLIC OR NAPIERIAN LOGARITHMS OF RATIOS OF EXPANSION.

No.	Logarithm.	No.	Logarithm.	No.	Logarithm.	No.	Logarithm.
1	0	3·5	1·2527629	6	1·7917595	8·5	2·1400661
1·25	·2231435	3·75	1·3217559	6·25	1·8325814	8·75	2·1690536
1·5	·4054652	4	1·3862943	6·5	1·8718021	9	2·1972245
1·75	·5596157	4·25	1·4469189	6·75	1·9095425	9·25	2·2246236
2	·6931472	4·5	1·5040773	7	1·9459100	9·5	2·2512918
2·25	·8109303	4·75	1·5581446	7·25	1·9810014	9·75	2·2772673
2·5	·9162907	5	1·6094379	7·5	2·0149030	10	2·3025851
2·75	1·0116009	5·25	1·6582280	7·75	2·0476928	12	2·4849065
3	1·0986124	5·5	1·7047481	8	2·0794414	15	2·7080502
3·25	1·1786549	5·75	1·7491998	8·25	2·1102128	18	2·8903847

LECTURE III.

CONTENTS.—Moment of a Force—Definition of the Moment of a Force—Conventional Signs of Moments—Algebraic Sum of Moments—Equilibrium of a Body under the Action of Several Forces—Principle of Moments—Example I.—Couples—Definitions relating to Couples—Propositions I., II., and III.—Example II.—Work Done by Turning Efforts and Couples—Diagram of Work Done by a Couple of Uniform Moment—Work Done by Variable Moments—The Fusee—Correct Form to be given to the Fusee—Questions.

Moment of a Force.—When a body is free to turn about an axis, and is acted on by a force, P, whose line of action is in a plane perpendicular to the axis (but not passing through the same) the effect of P is to rotate the body about that axis.



MOMENT OF A FORCE.

The *measure* of this turning effect depends on two things, viz.—(1) *The magnitude of the force,* and (2) *The perpendicular distance between the axis and the line of action of the force.* Thus, if the axis be perpendicular to the plane of the paper, and O its intersection therewith, then the turning effect of P is measured by the product, P × O N; O N being the length of the perpendicular from O upon the line of action, A P, of the force, P. This product is called the **Moment of the Force, P, with respect to the axis through O.**

When the force acts in a plane perpendicular to the axis, then it is best to define the moment of the force with respect to the *point* O; the point O being the intersection of the axis with the plane of the force. We then get the following definition :—

The Moment of a Force, with respect to a point, is measured by the product of the force into the length of the perpendicular drawn from the given point to the line of action of the force.

From the above it will be seen that a force has no moment about a point in its own line of action.

In general, when we wish to find the moment of a given force with respect to a given *axis* in the body on which the force acts, we have to resolve the given force into two components, viz.—(1) One in a *plane* perpendicular to the axis; (2) The other perpendicular to this plane—*i.e.*, parallel to the axis. The product of the former component into the length of the perpendicular from the axis upon its line of action, gives the required moment. The component parallel to the axis measures the thrust or pull *along* the axis. At the same time the component in the perpendicular plane gives a measure of the transverse pressure at the axis. The proof of these statements will be given immediately.

Conventional Signs of Moments—Algebraic Sum of Moments.—In problems relating to the moments of a number of forces acting on a body which is free to turn about a given axis, it is necessary to distinguish in sign between the moments of those forces which tend to turn the body in one direction about the axis, and those tending to turn the body in the opposite direction. If the moments of the one set of forces be regarded as *positive*, then those of the other set must be regarded as *negative*. Which direction of rotation is to be considered as the positive direction is a matter of little importance, so long as a distinction in sign is made and adhered to throughout the investigation.

By the term "*Algebraic Sum*" is to be understood the sum of the several quantities considered (whether moments or any other quantities differing in sign), each taken with its proper sign attached (+ or −).

Equilibrium of a Body under the Action of several Turning Forces.—The tendency of a force to turn a body about a given point depends only on the product of the two factors (1) effort and (2) its perpendicular distance from the point. It therefore follows that if any number of forces act in the same plane on a body and tend to turn it about a given point, the result will be the same (so far as the turning effect is concerned) as that of a single force acting in the same plane, and having a moment equal to the sum of the several moments. If some of the forces tend to turn the body in one direction and the others in the opposite direction; and, further, if the sum of the moments of the one set be equal to the sum of the moments of the other set, so that the *algebraical* sum of the moments is zero, it follows that the body will have no tendency to turn in the one direction more than in the other. In other words, the body will be in equilibrium so far as rotation is concerned.*

* The proofs of these statements are given in books on Theoretical Mechanics.

Principle of Moments.—If any number of forces, acting in the same plane, keep a body in equilibrium, then the sum of the moments of the forces tending to turn the body about any axis in one direction, is equal to the sum of the moments of the forces tending to turn the body about the same axis in the opposite direction.

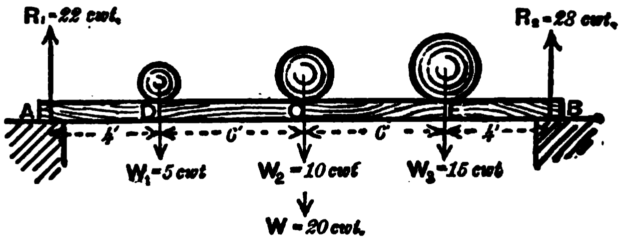
Conversely.—If the sum of the moments of the forces in the one direction is equal to the sum of the moments in the opposite direction, the body will be kept in equilibrium.

The *Principle of Moments* is sometimes stated in the following brief but useful form :—

When a body is kept in equilibrium by any number of co-planer forces, the algebraical sum of the moments of all the forces about any point in their plane is zero.

Conversely.—If the algebraical sum of the moments about any point in their plane is zero, the forces are in equilibrium.

Example I.—A uniform beam weighing 1 ton rests on supports at its ends, 20 ft. apart. Weights of 5, 10, and 15 cwts. rest on the beam at distances of 6 ft. apart, the weight of 5 cwts. being 4 ft. from one of the supports. Find the reactions at the points of support.



To Illustrate Example on Moments.

Answer.—According to the *Principle of Moments* just stated, we may take moments about any point in the plane of the forces, in order to find R_1 and R_2 the reactions at the points of support. The student, however, will find it advantageous to take moments about *one* of the points of support; for then, the moment of the reaction at that point will vanish, and he will thus have an equation containing only one unknown quantity—viz., the other reaction.

Suppose we take moments about the point B, then we get:—

$$R_1 \times AB = W_1 \times DB + (W_2 + W)\,CB + W_3 \times EB.$$

Substituting values, we get:—

$$R_1 \times 20 = 5 \times 16 + (10 + 20) \times 10 + 15 \times 4 = 440 \text{ (cwt.-ft.)},$$

$$\therefore \qquad R_1 = \frac{440}{20} = 22 \text{ cwts.}$$

Now, we can either take moments about A, and find R_2 in the same way as we have found R_1; or, we may make use of our knowledge of parallel forces (since the above system is one of parallel forces) and get R_2. The latter method is the simpler. Adopting this method, we get:—

$$R_1 + R_2 = W_1 + W_2 + W + W_3$$

$$\therefore \qquad R_2 = 5 + 10 + 20 + 15 - 22 \text{ (cwts.)}$$

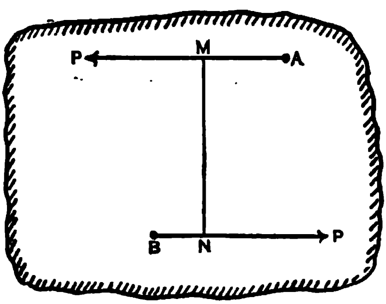
Or, $\qquad R_2 = 28$ cwts.

Couples.—We shall now show that all questions relating to turning forces are really questions involving couples compounded with single forces.

DEFINITION.—**A Couple is a system of two equal and oppositely directed parallel forces, whose lines of action do not coincide.**

DEFINITION.—The **Arm of a couple is the perpendicular distance between the two equal forces.**

DEFINITION.—**The Moment of a couple is the product of one of the equal forces into the arm.**



MOMENT OF A COUPLE.

Thus, if a body be acted on by two equal and opposite parallel forces, P, P, whose points of application are A and B respectively, then these forces constitute a **Couple.** If M N be drawn $\perp$ to A P and B P, then the length of this perpendicular is called the **Arm of the Couple, and the Moment of the Couple** = P × M N.

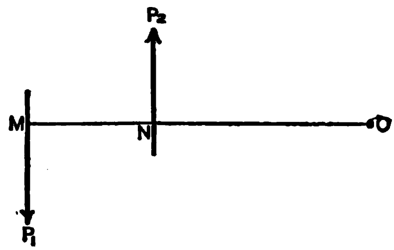
From an inspection of the figure it will be seen that the effect of a couple acting on a body is to produce rotation. A couple

has no effect in producing translation of the body on which it acts.

We shall now prove the following important *Propositions* regarding couples.

PROPOSITION I.—**The Algebraic Sum of the Moments of the two forces of a couple about any point in their plane is constant**; or in other words,

The Moment of a Couple about any point in its plane is constant.



MOMENT OF A COUPLE ABOUT A POINT.

Let P_1, P_2 be the equal forces constituting the couple and O be *any* point in the plane of the couple.

From O, drop the perpendicular O N M on the lines of action of P_1 and P_2.

Then, Moment of P_1 about O $= P_1 \times OM$.

And, " P_2 " $= - P_2 \times ON$.

∴ **Moment of Couple about O** $= P_1 \times OM - P_2 \times ON$.

i.e., " " $= P_1 \times \mathbf{M}N$.

But $P_1 \times MN$ is clearly a constant quantity. It is, in fact, what we have already defined as the *Moment of the Couple.* Hence, we see that the moment of a couple about any point in its plane, is independent of the position of that point with respect to the couple.

Remembering, then, that a couple has no translatory effect on the body on which it acts, and that its rotatory effect is measured by its moment, we at once obtain the following corollaries from the above Proposition :—

(1) **A Couple may be considered as acting anywhere in its own plane.**

(2) **A Couple may be replaced by another of equal moment and sign and acting in the same plane.**

(3) **The Resultant of two or more Couples acting in the same plane, is a couple whose moment is equal to the algebraic sum of the moments of the component couples.***

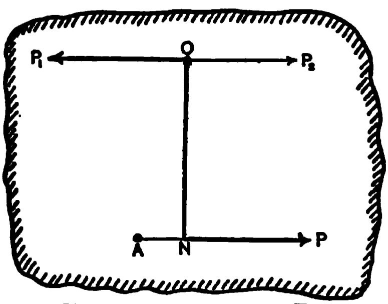
* Independent proofs of these propositions are usually given in books on Theoretical Mechanics.

Proposition II.—**A force acting on a rigid body can always be replaced by an equal force acting at any given point together with a couple.**

Let P be a force acting at the point, A, in a rigid body, and let O be the given point.

At O, introduce two opposite forces, P_1 and P_2, each equal to P, and having their line of action, $P_1 O P_2$, parallel to A P.

Then, obviously, the introduction of these two equal and opposite forces at O will not affect the action of P at A. We have now a system of three forces acting on the body, which is equivalent to the single force, P, at A. But, clearly, two forces of this system—viz., P and P_1—constitute a *couple*, the moment of which is P × O N. The action of this couple is simply to produce rotation of the body. The remaining force, P_2, is that part of the system which produces or tends to produce translation. The magnitude and direction of P_2 are always equal and parallel, respectively, to those of the original force, P.



A Force Replaced by a Force and a Couple.

If O represents the intersection of the plane of the forces with an axis round which the body is free to turn, then the moment = P × O N, and the transverse pressure on the axis is P_2 = P.

Proposition III.—**A force and a couple acting in the same plane are equivalent to or, may be replaced by, a single force in that plane.**

This is the converse of *Proposition II.*, and might have been assumed here without proof; but we prefer giving a proof since it exhibits a method or process of reasoning useful for other purposes.

Let a force, P, and a couple whose moment is Q × q, act in the same plane on a rigid body.

Replace this couple by another of equal moment and similar in sense, and having its forces each equal to P, the given force. [See Cors. (1) and (2), Prop. I.]

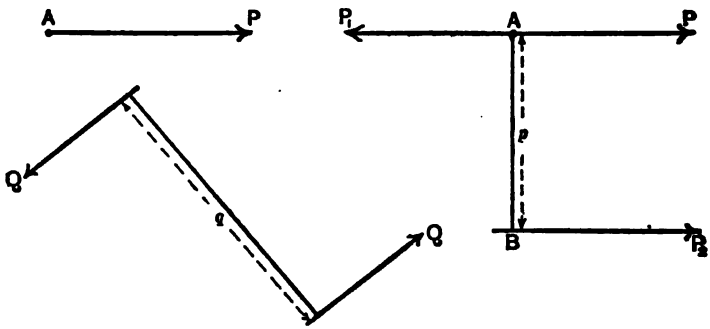
Let the arm of this new couple be p. Then we must have:—

$$P \times p = Q \times q.$$

Or,
$$p = \frac{Q \times q}{P}.$$

Now, rotate this new couple in its plane into such a position that one of its forces acts along the same line as the given force P at A, but having its direction opposite to that of P. [See Cor. (1) Prop. I., and right hand fig. below]. We have then a system of three equal forces, two of which—viz., P, P_1 at A—neutralise each other, and then we are left with the single force P_2, acting along a line B P_2 parallel to A P, and at a distance p from it, such that $p = q\,Q\,/\,P$.

Several important applications of the preceding propositions will be met with throughout the present treatise.



A FORCE AND A COUPLE REPLACED BY A FORCE.

EXAMPLE II.—A uniform platform, A C, turning about a hinge at A, is kept in a horizontal position by means of a chain, C H, fixed to a hook, H, in the wall vertically over A. A barrel weighing 6 cwts. is placed on the platform at B. Determine the tension in the chain, and the magnitude and direction of the reaction at the hinge, A; given weight of platform = 2 cwts., A C = 6 feet, A B = 5 feet, and A H = 8 feet.

ANSWER.—(1) *To find* T *the tension in the chain,* C H.

From A drop the perpendicular A N on C H. Take moments about the hinge, A, so as to eliminate the reaction at that point. Then, by the *Principle of Moments*, we get :—

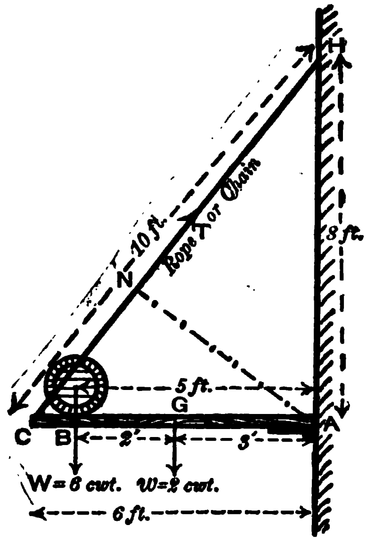
$$T \times AN = W \times AB + w \times AG$$

$$,, \quad ,, \quad = 6 \times 5 + 2 \times 3 = 36 \text{ cwt.-ft.}$$

$$\therefore \qquad T = \frac{36}{AN} \text{ cwt.} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

We have now to determine the length of A N in feet.

The easiest way to find this, is to express the area of the right angled triangle, ACH, in two ways, and then equate these.



To Illustrate the Tension in the Chain and the Reaction at the Platform Hinge.

Thus, $\quad \text{Area } \triangle ACH = \frac{1}{2}(CH \times AN).$

Also, $\quad$ „ „ $= \frac{1}{2}(AC \times AH).$

$\therefore \quad \frac{1}{2}(CH \times AN) = \frac{1}{2}(AC \times AH).$

i.e., $\quad AN = \dfrac{AC \times AH}{CH}.$

Or, $\quad$ „ $= \dfrac{6 \times 8}{CH} = \dfrac{48}{CH}$ ft., . . (2)

But, $\quad CH = \sqrt{AC^2 + AH^2}.$

Or, $\quad$ „ $= \sqrt{6^2 + 8^2} = 10$ ft.

From eqn. (2) $\quad AN = \dfrac{48}{10} = 4 \cdot 8$ ft.

And, „ (1) $\quad T = \dfrac{36}{4 \cdot 8} = 7 \cdot 5$ cwts.

(2) *To find the reaction at the hinge,* A.

Resolve the tension, T, in the chain, C H, into two components, viz., one along C A and the other perpendicular to C A.

Let T_h = horizontal component of T

„ T_v = vertical „ „

Then, $T_h = T \cos \angle HCA$

$\therefore$ „ $= 7{\cdot}5 \times \frac{6}{10} = 4{\cdot}5$ cwts.

Also, $T_v = T \sin \angle HCA$

$\therefore$ „ $= 7{\cdot}5 \times \frac{8}{10} = 6$ cwts.

Now, let R denote the reaction of the hinge at A, and let R_h, R_v represent the horizontal and vertical components of R. Then since the only horizontal forces acting on the platform are R_h and T_h, these must be equal and act in opposite directions.

$\therefore$ $R_h = T_h = 4{\cdot}5$ cwts.

Again, R_v, T_v, W, and w constitute a system of parallel forces in equilibrium.

$\therefore$ $R_v + T_v = W + w$

$\therefore$ $R_v = 6 + 2 - 6 = 2$ cwts.

But, $R^2 = R_h^2 + R_v^2$

i.e., $R^2 = 4{\cdot}5^2 + 2^2 = 24{\cdot}25$

$\therefore$ $R = \sqrt{24{\cdot}25} = 4{\cdot}92$ cwts.

(3) *To find the direction of the reaction,* R.

Since R_h acts from A to C, and R_v acts vertically upwards, it at once follows that the direction of R lies along some line between A C and A H.

Let r denote the length of the perpendicular from C upon the line of action of R. Then, taking moments about C, we get by the *Principle of Moments*:—

$$R \times r = W \times CB + w \times CG$$

Or, $4{\cdot}92 \times r = 6 \times 1 + 2 \times 3 = 12$ cwt.-ft.

$\therefore$ $r = \frac{12}{4{\cdot}92} = 2{\cdot}43$ ft.

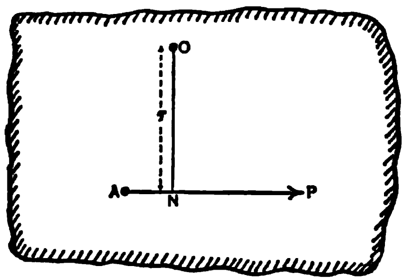
To set out the line of action of R we may proceed thus:— With centre, C, and radius, $r = 2{\cdot}43$ ft. draw an arc of a circle above line A C. From A draw a tangent to this arc. Then the direction of the reaction is along this tangent.

Note.—When the student has studied the Lectures on the Graphical Methods of determining the stresses in structures he should return to this problem, and determine graphically the tension in the chain, C H, and the magnitude and direction of the reaction of the hinge at A.

Work Done by Turning Efforts and Couples.—We are now in a position, to be able to find the work done by turning forces and couples, and to construct the diagrams of work done by such efforts.

Case I.—*Work Done by Uniform Moments.*—Let O be a point in the plane of the turning or twisting effort, P, round which the body is rotating.

Then, $\quad$ Moment of P about $O = M = P \times ON = Pr$.



Uniform Moment.

Let the body make n complete turns.

Then, $\quad$ Displacement of $P = 2\pi r n$

$\therefore$ *Work done by* $P = P \times 2\pi r n = Pr \times 2\pi n = M \times 2\pi n$.

But $2\pi n$ is the *circular measure* of the angle turned through by the effort, P.

Let θ denote this angle—*i.e.*, let $\theta = 2\pi n$,

$$\text{Then,}\quad \left.\begin{array}{r}\text{Work done by moment, M, dur-}\\ \text{ing angular displacement, } \theta\end{array}\right\} = M\theta, \quad . \quad . \quad . \quad (1)$$

Now we have already seen that every turning effort may be regarded as equivalent to a couple of equal moment to the effort and a force equal and parallel to the effort acting at the point O.

For the present we are only concerned with the turning effect of the effort or couple; hence, if a body is rotated by a couple of moment, M, through an angular displacement, θ:—

Then, **Work done by couple = M θ**, ... (2)

Where, $\theta = 2\pi n$.

Diagram of Work Done by a Couple of Uniform Moment.—From equations (1) and (2) it will be seen, that the quantity, θ, or $2\pi n$, has the same relation to the equation for the work done by a couple, that L had in the previous expressions (P × L) for the work done by a force; only, that here θ represents an *angular* displacement while in the previous case L represented a *linear* displacement. And, just as we can construct a diagram of work for linear displacements, so also, can we construct a similar diagram of work for angular displacements.

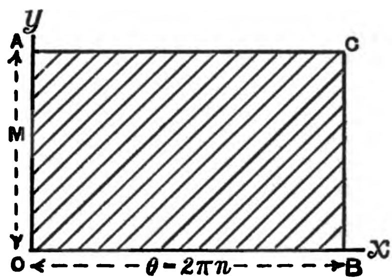


DIAGRAM OF WORK FOR A UNIFORM MOMENT.

Hence, draw two rectangular axes, $o\,x$, $o\,y$. Set off O A to represent the turning moment, M, and O B to represent the angular displacement, θ.

Then, if the moment of the couple be uniform, the area of the rectangle O A C B represents the work done.

i.e., *Work Done* = Area O A C B = M θ.

CASE II.—*Work Done by a Couple of Variable Moment.*—If we wind up a flat spring (such as the main spring of a watch or clock) or twist a helical spring or a wire or shaft by an effort in a plane perpendicular to its axis, the twisting moment required is proportional to the angle of twist within certain limits. This law may be stated briefly, thus:—

$$M \propto \theta.$$

We can prove, as in Lecture I., that the diagram of work for such cases as the above will be a triangle or a trapezoid according as the spring or shaft is in a neutral or initial state of stress when we begin to further twist or untwist it.

Thus, let the material be in an unstressed condition to begin with; and let M_2 be the twisting moment corresponding to the

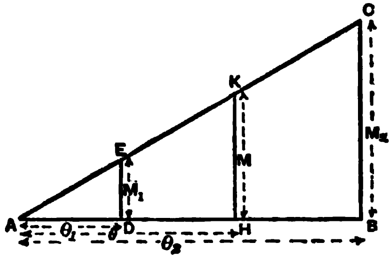


DIAGRAM OF WORK FOR A VARIABLE MOMENT.

angle of twist, θ_2. Then, for any other angle of twist, θ, we get the corresponding twisting moment, M, from the proportion

$$M : M_2 = \theta : \theta_2.$$

Hence, if A B, A H, and B C represent θ_2, θ, and M_2 respectively, we see that M will be represented by the ordinate H K. For, obviously,

$$HK : BC = AH : AB = \theta : \theta_2.$$

$\therefore$ H K represents the twisting moment for angle of twist, θ, to the same scale that B C represent M_2.

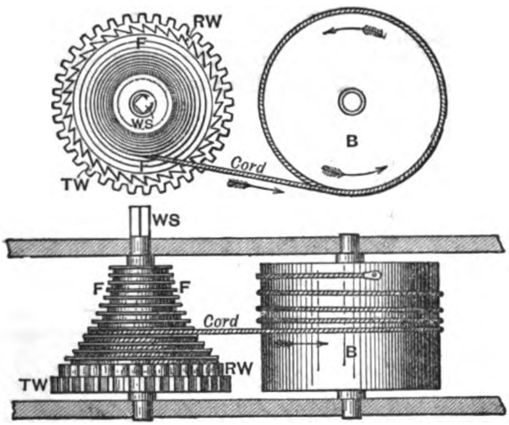
Hence, $\left.\begin{array}{r}\textit{Work done in twisting}\\ \textit{material through angle, } \theta\end{array}\right\} = \textit{Area } \triangle \text{ ABC} = \frac{1}{2} M \theta.$

If the initial and final angles of twist be θ_1 and θ_2 respectively, then

$$\left.\begin{array}{c}\textit{Work done in twisting material}\\ \textit{through angle } (\theta_2 - \theta_1)\end{array}\right\} = \textit{Area } \text{DBCE}$$
$$\text{,,} \qquad \text{,,} \qquad = \tfrac{1}{2}(M_2 + M_1)(\theta_2 - \theta_1).$$

The Fusee.—As an illustration of the manner in which the variable twisting moment of a coiled spring may be compensated, and thus secure a uniform turning effort, we may instance the case of the *fusee* as adopted in many watches, clocks, and chronometers. In such cases, the driving of the works at a constant rate is the object aimed at, and this naturally requires a constant turning effort in the wheelwork, this effort being just sufficient to overcome the frictional and other resistances offered by the mechanism. Now, one of the most compact and con-

venient pieces of mechanism into which mechanical energy can be stored is that of a coiled spring, and since the very nature of the spring is such, that its moment decreases as it uncoils, we must employ some compensating device between this variable driving force and the constant resistance. The fusee does this in a most accurate and complete manner. Looking at the accompanying figures and index to parts, we see that the barrel, B, which con-



The Fusee for a Clock or Watch.

Index to Parts.

B	represents	Barrel.	TW	represents	Toothed wheel.
F	,,	Fusce.	WS	,,	Winding square.
RW	,,	Ratchet wheel.			

tains the watch or clock spring, is of uniform diameter, and that between the outside of this barrel and the fusee, or spirally grooved cone, there passes a cord or chain. When the winding key is applied to the winding square, WS, and turned in the proper direction, a tension is applied to the cord, and it is wound upon the spiral cone, thus coiling up the spring inside the barrel, B; for the outer end of this spring is fixed to the periphery of the barrel, and the inner end to its spindle or axle. When the spring is fully wound up it exerts the greatest force, but it acts at the least leverage, since the cord is on the groove of least diameter. When the spring is almost uncoiled it acts at the greatest leverage, for then the cord is on the groove of largest diameter. Consequently, the radii of the grooves

of this cone are made to increase in proportion as the force applied to the cord decreases, in order that there shall be a constant turning effort on the works of the clock or watch.

Correct Form to be given to the Fusee Curve.—We shall now show that the true form of the fusee curve is that of a rectangular hyperbola for equalising the effect of a spring of uniform elasticity, and when neglecting the other connections.

Let A B C D be the diagram of work for the spring inside the barrel, B. Then, from what has been said above, A B C D will be a trapezoid.

Let B C represent P, the force which the spring (inside the barrel) exerts on the cord or chain when it is fully wound up.

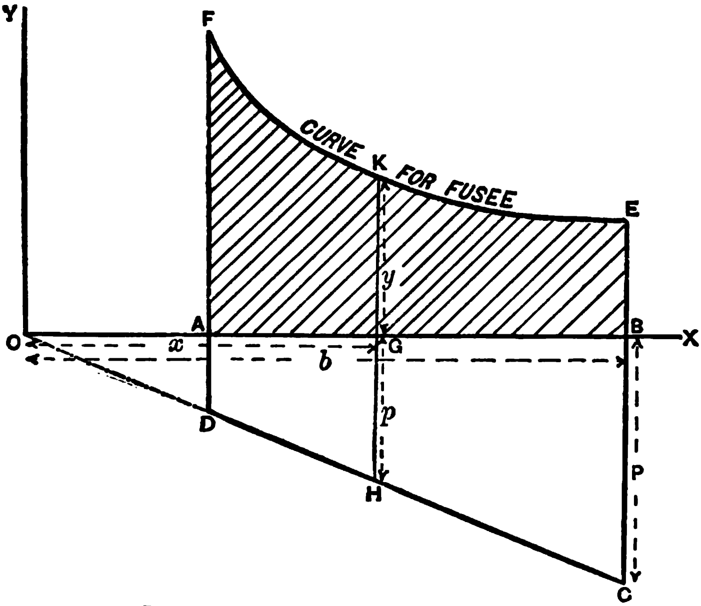


Diagram of Work, &c., for the Fusee.

Similarly, let G H represent p, the tension in the cord or chain at any other instant.

Let the ordinates E B, K G, and F A, represent the several radii at which the cord acts on the fusee. Then E K F will be the curve required to be given to the fusee. Thus, B E represents the radius at which the tension, P, in the cord acts when the spring

inside B is completely wound up. Similarly, G K represents the radius of the fusee corresponding to the tension, p, in the cord.

Produce C D till it meets B A at O. Take O as the origin. Let O G = x, G K = y, and O B = b.

Then, if we have a *constant twisting moment* acting on the fusee spindle :—

$$p \times y = \text{a constant } (m) \quad . \quad . \quad . \quad (1)$$

But,
$$p : \mathrm{P} = x : b$$

$\therefore$
$$p = \frac{\mathrm{P}}{b} x = n x$$

Where
$$n = \frac{\mathrm{P}}{b} = \text{a constant.}$$

Substituting this value for p in equation (1) we get :—

$$n x y = m$$

Or,
$$x y = \frac{m}{n} = \text{a constant.}$$

But this is the equation for a rectangular hyperbola referred to the axes O X, O Y which are its asymptotes. We have met with this curve before when treating of the expansion or compression of gases according to Boyle's law.

In practice, the fusee is made as nearly as possible to this shape. Then the fusee and spring are connected, as shown by the figures on page 43, and tested by fixing an L-shaped lever (with an adjustable weight on the long arm of this adjusting rod) to the winding square, W S, and finding whether the tension in the cord or chain (as due to the spring enclosed in B) is balanced in every position by the same turning effort on the lever.

Should the turning moment of the combined spring and fusee be thus found to be greater when the spring is fully wound up than when it is nearly run down, the initial tension of the spring is too great. To lessen this, the ratchet wheel, R W, is eased back a tooth or two, the cord readjusted, and the above experiment repeated until the nearest approach is arrived at to a uniform turning effort on the works of the timepiece.

LECTURE III.—QUESTIONS.

1. Define the moment of a force with respect to a point. When is a moment reckoned positive and when negative? Draw an equilateral triangle, A B C, and suppose each side to be 4 feet long. A force of 8 units acts from A to B, and a force of 10 units from C to A. (*a*) Find the moment of each force with respect to the middle point of B C; (*b*) Find a point with respect to which the forces have equal moments of opposite signs. *Ans.* (*a*) $18\sqrt{3}$; (*b*) Any point on the resultant.

2. State the principle of moments and hence show that the moments of two forces about any point in their resultant are equal and opposite. A rod is supported horizontally on two points, A and B, 12 feet apart. Between A and B points C and D are taken such that A C = B D = 3 feet. A weight of 120 lbs. is hung at C, and a weight of 240 lbs. at D. Take a point O midway between A and B and find with respect to O the algebraic sum of the moments of the forces acting on the rod on one side of O. (You may neglect the weight of the rod.) *Ans.* 540 ft.-lbs.

3. In a blowing engine of the overhead beam construction the area of the steam piston is 2,712 square inches, and the mean pressure of the steam is 30 lbs., while the area of the piston of the blowing cylinder is 16,272 square inches. The leverage of the working beam is as 15 on the steam side to 20 on the opposite side; what is the pressure of the air as it leaves the blowing cylinder? *Ans.* 3·75 lbs. per square inch.

4. A safety valve, 3 inches in diameter, is held down by a lever and weight. The distance from the fulcrum to the pin of the valve is 6 inches. Weight of valve 5 lbs. Weight of lever 15 lbs. Distance from fulcrum to centre of gravity of lever 16 inches. Find where a weight of 60 lbs. must be placed on the lever so that the steam may blow off at a pressure of 56 lbs. per square inch. *Ans.* 35·1 inches from fulcrum.

5. Define a couple, its arm, and its moment. Show that two couples, whose moments are equal and of opposite signs, are in equilibrium when they act in the same plane on a rigid body. If forces act from A to B, B to C, and C to A, along the sides of a triangle, A B C, and are proportional to the sides along which they respectively act, show that they are equivalent to a couple.

6. Show that a force acting at a given point A, may be replaced by an equal parallel force acting at any other point B, and a couple whose moment equals moment of original force about B.

7. Find the resultant of a force and a couple acting in the same plane. Draw a square, A B C D, and its diagonal, A C. Two forces of 10 lbs. each act from A to B and from C to D respectively, forming a couple. A third force of 15 lbs. acts from C to A. Find their resultant, and show in a diagram exactly how it acts. *Ans.* R = 15 lbs.

8. State the principle of moments, and apply it to the solution of the following question:—A B, A C are sheer poles secured to a base plate in the ground at B and C, and held in position by a wire guy or tension rope, A E, attached to the ground at E, D is the middle point of the line joining B and C, and B C is perpendicular to E D. Given A B = A C = 25 feet; B C = 14 feet; D E = 40 feet; A E = 55 feet. Find tension in the guy rope when a weight of 20 tons is suspended from A. *Ans.* 13·55 tons.

9. Explain, with a sketch, the use of a fusee in equalising the variable force of a spring coiled within the barrel of a watch. Find the theoretical form to be given to the curve of the fusee.

10. A safety valve, $3\frac{1}{2}$ inches in diameter, is held down by a lever and spring. The arrangement has to be so constructed that each pound of additional pressure per square inch on the valve will be registered as such on the spring at the end of the lever. Neglecting the weights of the lever and valve, you are to determine the relative distances of spring and valve from the fulcrum of the lever. After the valve has been set, determine the additional pressure per square inch which will be necessary to lift the valve $\frac{1}{10}$ inch, the spring requiring a force of 10 lbs. to extend it 1 inch. You may neglect the weights of the lever, valve, and spring. Sketch the arrangement. *Ans.* 9·625 : 1; 4·82 lbs.

LECTURE IV.

BEFORE taking up the subject of simple machines we shall give a brief statement of another important principle in Mechanics known as the "principle of work."

Principle of Work.—If a body or system of bodies be in equilibrium under the action of any number of forces, and receive a small displacement, the algebraical sum of the work done by all the forces is zero.

Conversely.—If the work done be zero, the forces are in equilibrium.

We may verify the truth of the principle of work by assuming the principle of moments, or the principle of the parallelogram of forces, &c.; or, conversely; having assumed the principle of work we can verify the truth of the principle of moments, or the principle of the parallelogram of forces. After all, the principle of work is only a particular case of the more general principle called the **Principle of the Conservation of Energy,** which is now universally accepted by all scientists, and may be stated thus:—

Principle of the Conservation of Energy.—The total energy of any material system is a quantity which can neither be increased nor diminished by any action between the parts of the system, though it may be transformed into any of the forms of which energy is susceptible. (*Clerk Maxwell.*)

DEFINITION.—**Energy confers upon a body possessing it the ability to do work.**

The principle of the conservation of energy, therefore, asserts that there can be no increase or decrease in the energy of any system without an equivalent loss or gain of energy in some other system. If in any isolated system there be an increase in one form of energy this can only happen at the expense of some of the other forms of energy in the system.

4

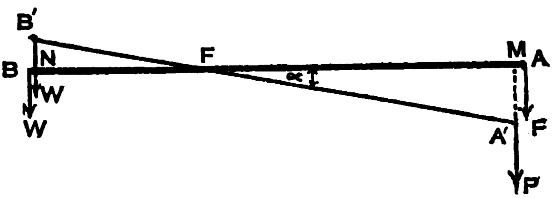
Hence, the total energy in the Universe is a constant quantity.

We may change energy of one form into that of another, but we can never change the total amount.

Useful and Lost Work in Machinery.—In the case of machines, it is true, that the useful work is much less than the work put into the machine. A part of the total energy exerted is rendered unavailable for useful work, this part being employed in overcoming the friction at the rubbing surfaces; by setting up vibrations in the machinery, &c.; and these reappear in the forms of heat and sound energy, &c. The energy thus mis-spent is a direct loss to the engineer, and he has to contrive means for its reduction; although, he can never hope to entirely eliminate it.

Since the student is still expected to give a demonstration or verification of the *Principle of Work;* when, say, the truth of the *Principle of Moments* is assumed, we herewith give such a demonstration in its usual form as it appears in most works on this subject.

PROPOSITION.—**To verify the truth of the Principle of Work by assuming the truth of the Principle of Moments.**



PRINCIPLE OF WORK AND OF MOMENTS.

Let A F B be a rigid lever capable of turning about a fulcrum at F. Let forces P and W act at the extremities A and B respectively. Let the three forces, P, W, and the reaction at F, be a system of forces in equilibrium.

Then, by the *Principle of Moments*, we have:—

$$P \times AF = W \times BF$$

Or, $$P : W = BF : AF \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Now, conceive the system to receive a small displacement, the forces being still in equilibrium. For this displacement it is best to conceive the lever tilted through a very small angle α, round the fulcrum, F; its new position being A′ F B′.

Then,

$$\textit{Work done by } P = P \times A'M$$

$$\text{,, } \quad \text{,, } \quad W = -(W \times B'N)$$

$$\therefore \quad \textit{Total work done} = P \times A'M - W \times B'N \quad . \quad . \quad (2)$$

But, from the similar Δ^s, N F B′ and M F A′, we get:—

$$B'N : A'M = B'F : A'F$$

$$\text{,, } \quad \text{,, } \quad = BF : AF$$

$\therefore$ From eqn. (1), $B'N : A'M = P : W$

$$\therefore \quad W \times B'N = P \times A'M$$

Hence, from eqn. (2), we get:—

$$\textbf{Total work done} = P \times A'M - P \times A'M = 0.$$

This verifies the principle as stated above.

The student should now prove in a similar manner the converse statement, and also, verify the truth of the *Principle of Moments* by assuming the *Principle of Work.*

Principle of Work Applied to Machines.—When applied to machines the *Principle of Work* takes the form :—

Total work expended = Useful work done + Work lost in the machine.

Or, **Work put in = Work got out + Lost work.**

If we denote these three quantities by W_T, W_U, and W_L respectively, we can write the above equation thus :—

$$W_T = W_U + W_L \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (I)$$

Definition.—**The ratio which the useful work done bears to the total work expended is called the efficiency of the machine.**

$$\textit{i.e.,} \qquad \textbf{Efficiency} = \frac{\textbf{Useful work done}}{\textbf{Total work expended}} = \frac{W_U}{W_T} \quad . \quad . \quad (II)$$

The efficiency of an actual machine is always a proper fraction. The efficiency could only be unity in the case of a perfect machine, or where we assume the entire absence of frictional and other losses. In such theoretical cases we state the *Principle of Work* in the following form :—

Total work expended = Useful work done.

Object of a Machine.—The object of a machine is to enable us to perform work of various kinds, either by our muscular exertions or by utilising the forces of nature.

We may define a machine either from a *statical* or from a

kinematical point of view. Regarded **Statically,** ***it is an instrument for changing the magnitude, direction, or place of application of a given force.*** **Kinematically,** ***it is an instrument for changing the direction or the velocity of a given motion, or both direction and velocity.***

Combining these two statements, we get the following :—

Definition.—**A machine is an instrument, or combination of movable parts, constructed for the purpose of transmitting and modifying, in various ways, force or motion, or both force and motion.**

Or, a machine may be defined to be a combination of resistant bodies whose relative motions are completely constrained, and by means of which the natural energies at our disposal may be transformed into any special form of work. (*Prof. A. B. W. Kennedy.*)

Simple or Elementary Machines.—All machines, however complicated, are merely combinations of two or more of the following mechanisms :—

1. The Lever and Fulcrum.
2. The Pulley.
3. The Wheel and Axle.
4. The Inclined Plane.
5. The Wedge.
6. The Screw.

In reality, there are only *two* elementary mechanisms distinct in principle—viz., the **Lever** and the **Inclined Plane.** The Pulley and the Wheel and Axle are but modifications of the Lever; whilst the Wedge and the Screw are but particular cases of the Inclined Plane.*

Force Ratio—Velocity Ratio—Mechanical Advantage.—In considering any machine it is desirable to know the ratio which the applied force or effort bears to the resistance or load overcome. This is termed the **Force Ratio.** Also, the ratio of the velocities of the points of application of the effort and resistance. This is termed the **Velocity Ratio.** In this treatise we shall denote the applied force or effort by P or Q according as frictional resistance is neglected or taken into account; W being the resistance or load in both cases.

Then, P = *Theoretical* force required to overcome resistance, W.
And, Q = *Actual* „ „ „ W.

$$\left.\begin{array}{ll}\text{Also,} \quad \textbf{Theoretical Force Ratio} & = \dfrac{P}{W} \\ \text{And,} \quad \textbf{Actual Force Ratio} & = \dfrac{Q}{W}\end{array}\right\} \quad \ldots\ldots \quad \text{(III)}$$

* For descriptions, &c., of these simple mechanisms, see the author's *Elementary Manual of Applied Mechanics.*

Let V = Velocity of the point of application of the effort P or Q.
„ v = „ „ „ „ resistance, W.

Then, $$\text{Velocity Ratio} = \frac{\text{Velocity of P or Q}}{\text{Velocity of W}} = \frac{V}{v} \quad \ldots \quad \text{(IV)}$$

Let x = Displacement of the point of application of P or Q.
„ y = „ „ „ W.

Then, in a given time or period of motion of the machine, it is clear that:—

$$\frac{\text{Velocity of P or Q}}{\text{Velocity of W}} = \frac{\text{Displacement of P or Q in a given time}}{\text{Displacement of W in the same time}}.$$

$$\left.\begin{array}{ll} \text{Or,} & \dfrac{V}{v} = \dfrac{x}{y} \quad \ldots\ldots\ldots \\ \textit{i.e.,} & \text{Velocity Ratio} = \dfrac{x}{y} = \dfrac{\text{P or Q's displacement}}{\text{W's displacement}} \end{array}\right\} \quad \ldots \quad \text{(V)}$$

The reciprocal of the force ratio is usually spoken of as the *Mechanical Advantage* of the machine.* Hence:—

$$\text{Theoretical advantage} = \frac{\text{Resistance overcome}}{\text{Theoretical force required}} = \frac{W}{P} \qquad \text{(VI)}$$

$$\text{Actual advantage} \ldots = \frac{\text{Resistance overcome}}{\text{Actual force required}} \ldots = \frac{W}{Q} \qquad \text{(VII)}$$

Relations between the Advantage, Velocity Ratio, and Efficiency of a Machine.—Neglecting friction and applying the "*Principle of Work*" to any machine, we get:—

$$W \times \text{its displacement} = P \times \text{its displacement}.$$

Or, $$W \times y = P \times x.$$

$$\left.\begin{array}{ll} \therefore & \dfrac{W}{P} = \dfrac{x}{y} \\ \text{Or, from Equation (V),} & \dfrac{W}{P} = \dfrac{V}{v} \end{array}\right\} \quad \ldots\ldots\ldots \quad \text{(VIII)}$$

i.e., **Theoretical Advantage = Velocity Ratio.**

* In some treatises on Applied Mechanics the *force ratio* and *Mechanical Advantage* are synonymous terms, but, since in many problems it is desirable to know what ratio P or Q bears to W, we have chosen the former term (force ratio) to denote either of these ratios, retaining the term "*advantage*" in its original sense, to denote the *reciprocal* of the *force ratio* or the ratio of the load overcome to the force required to overcome it.

Again, in any machine we get:—

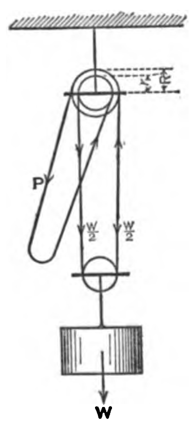
$$\text{Efficiency} = \frac{\text{Useful work done}}{\text{Total work expended}} = \frac{W\,y}{Q\,x}$$

$$\text{,,} \quad = \frac{W}{Q} \times \frac{P}{W} \text{ from Equation (VIII).}$$

$$\left.\begin{aligned} \text{,,} \quad &= \frac{P}{Q} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \\ \textit{i.e.,} \quad \text{Efficiency} &= \frac{\text{Theoretical force to overcome W}}{\text{Actual force to overcome W}} \end{aligned}\right\} \; . \; . \; \text{(IX)}$$

Example I.—Determine the relation between P and W in Weston's differential pulley block—(1) by the Principle of Moments, (2) by the Principle of Work. The radii of the pulleys are $4\frac{1}{2}$ inches and $4\frac{1}{4}$ inches. Taking the efficiency of the machine at 40 per cent., find the effort required on the hauling chain in order to raise a weight of $\frac{1}{4}$ ton. What is the actual advantage in this machine?

Weston's Differential Pulley Block.



Skeleton Figure of Weston's Differential Pulley Block.

Answer.—Let R and r denote the radii of the larger and smaller pulleys respectively. Then:—

(1) Since the weight, W, is supported by two parts of the chain, it is clear that the tension in each part is W/2. Considering the upper or differential pulley we see that it is acted on by three forces at the circumferences, viz.: —The tensions in the two parts of the chain supporting W, and the pull, P, along the hauling part of the chain.

Taking moments round the centre of the pulley pin, and applying the Principle of Moments, we get :—

$$P \times R + \frac{W}{2} \times r = \frac{W}{2} \times R.$$

Or, $$P \times R = \frac{W}{2}(R - r),$$

$\therefore$ $$\frac{P}{W} = \frac{R - r}{2R}.$$

(2) Suppose the system to receive such a displacement that the differential pulley makes one complete turn, W being raised during the operation. Then :—

The displacement of $P = x = 2\pi R$.

One part of the chain supporting W is overhauled by an amount $= 2\pi R$, while the other part is let out by a length $= 2\pi r$. The weight, W, will, therefore, be raised by an amount equal to the *algebraical* mean of these two displacements of the supporting chain.

Or,

Displacement of $W = y = \frac{1}{2}(2\pi R - 2\pi r) = \pi(R - r)$.

Hence, by the *Principle of Work*, we get :—

$$Px = Wy; \text{ or, } \frac{P}{W} = \frac{y}{x}.$$

i.e., $$\frac{P}{W} = \frac{\pi(R - r)}{2\pi R} = \frac{R - r}{2R}.$$

This is, however, the same result as before.

$$\text{Efficiency} = \frac{\text{Useful work done}}{\text{Total work expended}} = \frac{Wy}{Qx} = \frac{W}{Q} \times \frac{R - r}{2R}.$$

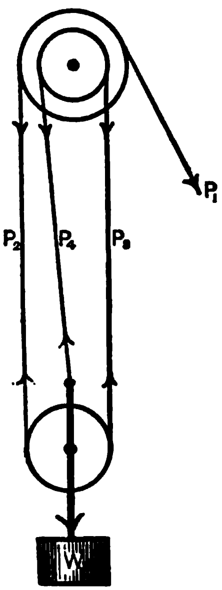
In the example, the efficiency = 40 per cent. = $\cdot 4$; W = 560 lbs.; $R = 4\frac{1}{2}''$; $r = 4\frac{1}{4}''$.

Hence, $$\cdot 4 = \frac{560}{Q} \times \frac{4\frac{1}{2} - 4\frac{1}{4}}{2 \times 4\frac{1}{2}} = \frac{560}{Q} \times \frac{1}{36},$$

$\therefore$ $$Q = \frac{560}{\cdot 4 \times 36} = 38\cdot\dot{8} \text{ lbs.}$$

$$\text{Actual advantage} = \frac{W}{Q} = \frac{560}{38\cdot\dot{8}} = \frac{14\cdot 4}{1}.$$

Example II.—It is found by trial that when P is on the point of lifting Q by means of a single fixed pulley, $P = (1 + m) Q$, where m is a fraction depending on the friction of the parts of the machine. If three such pulleys are combined into "a block and tackle," find the effort requisite to raise a given weight by means of it. If m equals 0·2 so that an effort of 60 lbs. would just raise a weight of 50 lbs. in a single fixed pulley, find the number of ft.-lbs. of work done against friction, when a weight of 1,000 lbs. is raised 20 ft. by means of a block and tackle of three such pulleys. (S. and A. Adv. Theor. Mechs. Exam. 1883.)



Question on the Pulley Block.

Answer.—(1) Let the figure represent the block and tackle consisting of three pulleys or sheaves. (In this figure we have drawn the pulleys of different sizes for the purpose of showing clearly the various ropes and exhibiting the forces).

Let P_1 be the effort required to raise a given weight, W, by means of this system of blocks. Let P_2, P_3, P_4, denote the tensions in the three parts of the rope as shown.

Then, from conditions stated in question, we get:—

$$P_1 = (1 + m) P_2 \quad . \quad . \quad . \quad (1)$$
$$P_2 = (1 + m) P_3 \quad . \quad . \quad . \quad (2)$$
$$P_3 = (1 + m) P_4 \quad . \quad . \quad . \quad (3)$$

Adding together the corresponding sides of equations (1), (2), and (3), we get:—

$$P_1 + P_2 + P_3 = (1 + m) (P_2 + P_3 + P_4).$$
$$\text{,, \quad ,,} \quad = P_2 + P_3 + P_4 + m (P_2 + P_3 + P_4).$$

i.e., $\quad P_1 = P_4 + m (P_2 + P_3 + P_4).$

But, $\quad P_2 + P_3 + P_4 = W.$

$\therefore \quad P_1 = P_4 + m\,W \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$

Multiplying together equations (1), (2), and (3), we get:—

$$P_1 P_2 P_3 = (1 + m)^3 P_2 P_3 P_4.$$

$\therefore \quad P_1 = (1 + m)^3 P_4.$

Or, $\quad P_4 = \dfrac{P_1}{(1 + m)^3}$

Substituting this value for P_4 in equation (4), we get:—

$$P_1 = \frac{P_1}{(1+m)^3} + m\,W.$$

$$\therefore \qquad P_1\{(1+m)^3 - 1\} = m(1+m)^3\,W.$$

Or, $$(m^3 + 3m^2 + 3m)P_1 = m(1+m)^3\,W.$$

i.e., $$\left.\begin{aligned}(m^2 + 3m + 3)\,P_1 &= (1+m)^3\,W.\\ \text{Or,}\qquad \frac{P_1}{W} &= \frac{(1+m)^3}{m^2+3m+3}\end{aligned}\right\} \quad . \quad . \quad . \quad (5)$$

(2) In the example given, $m = 0{\cdot}2$, and we may find the effort, P_1, required to raise a weight, $W = 1{,}000$ lbs.

From equation (5), we have :—

$$P_1 = \frac{(1{\cdot}2)^3}{0{\cdot}2^2 + 3 \times 0{\cdot}2 + 3} \times 1{,}000 \text{ lbs.}$$

$$,, = \frac{1{\cdot}728 \times 1{,}000}{3{\cdot}64} = \mathbf{474{\cdot}72\ lbs.}$$

When W rises 20 feet, then clearly P_1 will be displaced $3 \times 20 = 60$ feet.

$\therefore$ *Work done by* $P_1 = 474{\cdot}72 \times 60 = 28{,}483{\cdot}2$ ft.-lbs.

And, *Work done on* $W = 1{,}000 \times 20 = 20{,}000$ ft.-lbs.

$\therefore$ **Work done against friction** $= 28{,}483{\cdot}2 - 20{,}000 =$ **8,483·2 ft.-lbs.**

We might also find the efficiency of this machine.

$$\textit{Efficiency} = \frac{W\,y}{P_1\,x} = \frac{m^2 + 3m + 3}{(1+m)^3} \times \frac{1}{3}$$

Where x = displacement of P_1, and y = corresponding displacement of W, and $y : x = 1 : 3$, there being three parts of rope supporting W.

$$\therefore \qquad \textbf{Efficiency} = \frac{3{\cdot}64}{3 \times 1{\cdot}728} = \mathbf{{\cdot}7021} = \mathbf{70{\cdot}21\ per\ cent.}$$

Hence, 29·79 per cent. of total work expended is lost in friction.

LECTURE IV.—QUESTIONS.

1. State the principle of work, and apply it to show that a balanced lever whose arms are 2 and 3 will remain in equilibrium when weights which are as 3 and 2 are suspended at its ends.

2. Apply the principles of moments and of work in determining the relation between P and W in the wheel and compound axle. A weight of 20 lbs. draws up W lbs. by means of a wheel and compound axle. The diameter of the wheel is 5 feet, and the diameters of the parts of the compound axle are 9 and 11 inches respectively; find W. *Ans.* 1,200 lbs.

3. A compound axle consists of 2 parts, the diameters being 10 and 12 inches respectively, and a rope is coiled round them in opposite directions so as to form a loop, upon which hangs a pulley loaded to 48 lbs. Considering the parts of the rope to be vertical, find the force which, acting at a leverage of 4 feet upon the axle, will just balance the weight. Sketch the arrangement. *Ans.* ½ lb.

4. In a compound wheel and axle, where the weight hangs on a single movable pulley, the diameters of the two portions of the axles are 3 and 2 inches respectively, and the lever handle which rotates the axle is 12 inches in length. If a force of 10 lbs. be applied to the end of the lever handle, what weight can be raised? *Ans.* 480 lbs.

5. Define the terms force ratio and velocity ratio as applied to machines. What must be the difference in the diameters of a compound wheel and axle so that the velocity of P may be 100 times that of W, the length of the handle being 2½ feet? (S. and A. Adv. Exam., 1887.) *Ans.* 1·2 inches.

6. In a compound wheel and axle, let the diameter of the large axle be 6 inches, and that of the smaller axle 4 inches, and the length of the handle 20 inches; find the ratio of the velocity of the handle to that of the weight raised. *Ans.* 40 : 1.

7. Define the terms, force ratio, velocity ratio, theoretical and actual advantages and efficiency of a machine. A tackle consists of two blocks, each weighing 10 lbs. The lower or movable block has two sheaves, and the upper or fixed one has three sheaves. It is found that a force of 56 lbs. is required to raise a weight of 200 lbs. suspended from the hook of the lower block. Find (1) the theoretical advantage, (2) the actual advantage, (3) the efficiency of the machine, (4) the percentage efficiency. If W rises 6 feet, what length of rope must be hauled in? *Ans.* (1) 4·76 : 1; (2) 3·57 : 1; (3) ·71; (4) 71; 30 feet.

8. Describe Weston's differential pulley. If the weight is to be raised at the rate of 5 feet per minute, and the diameters of the pulleys of the compound sheave are 7 and 8 inches respectively, at what rate must the chain be hauled? (S. and A. Adv. Exam. 1888.) *Ans.* 80 feet per minute.

9. State and explain the principle of the conservation of energy and show that the principle of work is only a particular case of this general principle.

10. State the principle of work and apply it to determine the relation between P and W in Weston's differential pulley block. In such a block the radii of the pulleys are 5 inches and 4½ inches respectively. Taking the efficiency of the machine at 50 per cent.; what force must be applied to the hauling chain in order to raise a weight of 1 ton? What is the actual advantage in this machine? *Ans.* 224 lbs.; 10 : 1.

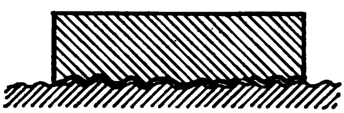
11. Explain the methods which you would adopt to find the mechanical advantage and efficiency of any machine, such as the ordinary block and tackle, or a Weston's differential block. Having found the theoretical pull (P) and the actual pull (Q) required to raise a given weight (W), what would be the efficiency of the machine? Give reasons for your answer. *Ans.* Efficiency = P/Q.

LECTURE V.

CONTENTS.—Definition of Friction—Limiting Friction—Definition of Coefficient of Friction—Static and Kinetic Friction—Ordinary Laws of Friction for Plane Surfaces—Morin's Experiments—General Results of Recent Experiments on the Friction of Plane Surfaces—Simple Methods for Finding the Coefficients of Friction and Angles of Repose—Definition of Angle of Repose—Limiting Angle of Resistance and its Definition—Examples I. and II.—The best Angle of Propulsion or Traction—Example III.—Questions.

DEFINITION.—**Friction is the term used to denote the resistance to motion which is experienced when one body is made to slide over the surface of another.**

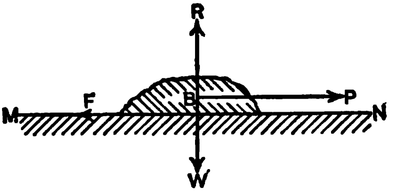
The true cause of friction is the roughness of the surfaces in contact. The surfaces of all bodies are more or less rough, and, when examined by means of a microscope, they are found to be covered with minute projections, which are smaller the smoother the surface. When one surface rests upon another, the projections of the one appear to fit into corresponding hollows in the other. Hence, to move the one surface relatively to another a certain force must be exerted either in separating (*i.e.*, lifting) the surfaces sufficiently to clear these projections; or, in breaking off some and clearing others. By interposing a lubricant, such as oil or grease, between the surfaces, the friction may be greatly diminished. In such cases, the surfaces do not appear to be in actual contact, but are separated by a thin film of the lubricant, over which they slide. The amount by which the friction is thus diminished depends on the nature and quantity of the lubricant between the rubbing surfaces.



MAGNIFIED SECTION THROUGH TWO ROUGH SURFACES IN CONTACT.

Limiting Friction.—Friction is thus a *tangential* resistance offered to the motion of one body over the surface of another. Thus, if the body, B, is made to slide along the surface, M N, by the force, P, the frictional resistance, F, always acts along the common tangent to the two surfaces in contact. Whilst B is

just beginning to move, the resistance, F, increases from nothing to a certain limit, so that any further increase of P causes the body to slide. The greatest amount of friction thus called into play is usually spoken of as the **Limiting Friction.** It depends for its magnitude on the reaction, R, between the surfaces in contact (due to the weight of the body, W) and the nature of those surfaces. The limiting friction, F, is *measured* by the least force, P, which just causes sliding to take place in a horizontal plane.



LIMITING FRICTION.

DEFINITION.—**The Coefficient of Friction (μ) is the ratio of the Limiting Friction, F, to the Normal Reaction, R, between the surfaces in contact.**

i.e., $$\mu = \frac{F}{R}; \text{ or, } F = \mu R.$$

Static and Kinetic Friction.—It has been proved experimentally that the "limiting friction" between surfaces at rest relatively to each other, is slightly different in magnitude from that between the same surfaces when in motion. The former has been called **Static Friction** or **Friction of Rest,** whilst the latter is called **Kinetic Friction** or **Friction of Motion.**

Ordinary Laws of Friction for Plane Surfaces.—In 1785, Coulomb, a French officer, published the results of a series of experiments carried out by him on the friction of plane surfaces. These results he embodied in the following statements, usually called the ordinary laws of friction :—

LAW I.—**The friction between two bodies is directly proportional to the normal pressure between them.**

LAW II.—**The friction is independent of the areas of the surfaces in contact.**

LAW III.—**Kinetic friction is less than static friction, and is independent of velocity.***

It will at once be seen, that these three laws may be comprised in the single statement that, **The coefficient of friction depends only on the nature of the surfaces in contact.**

* For experimental methods of verifying these laws see the Author's *Elementary Manual on Applied Mechanics.*

Morin's Experiments.—Coulomb's experiments were not considered sufficiently extensive to thoroughly establish the truth of the above so-called laws. The whole subject has, however, been reinvestigated by several persons, notably by General Morin during the years 1831-34. The results of Morin's experiments were, for a long time, regarded as conclusively establishing the above laws. This was no doubt true within the limits of the pressures and the velocities he employed; but, in some recent experiments, which have been carried out with much greater care and wider variations, both in pressures and velocities, the laws of Coulomb were found to be erroneous. The coefficients of friction, instead of being independent of pressure and velocity, are shown to vary considerably with the pressure, velocity, and temperature.

General Results of Recent Experiments on Friction of Plane Surfaces.—(1) With *dry* surfaces the coefficient of friction *increases* with the intensity of the pressure. The highest pressure employed by Morin was little more than 100 lbs. per square inch, and it is just about this pressure that deviation from Coulomb's law appears to begin. This increase in the friction with high pressures is probably due to abrasion of the surfaces; but, when the same surfaces were well lubricated the reverse took place.

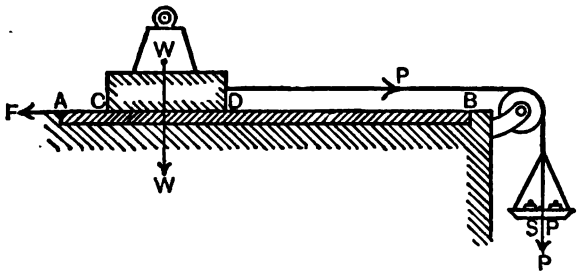
(2) The lowest pressure employed by Morin was about $\frac{3}{4}$ lb. per square inch, but in recent experiments with pressures lower than this, the coefficient of friction was found to *increase* as the pressure *decreased.*

(3) With *high* velocities the coefficient *diminishes* as the velocity *increases.* These results are only true with velocities greater than those employed by Morin.* With all velocities under 10 feet per second it has recently been found that the coefficient of friction is quite independent of the speed.

Simple Methods for Finding the Coefficients of Friction and Angles of Repose.—(1) Take two pieces of the materials to be tested. Let one of these, A B, be shaped like a lath and laid on a table, while the other, C D, is made to slide on its upper surface as shown by the figure. The bodies are pressed together by a weight, W, which may be varied at pleasure. The reaction, R, between the two surfaces will be W + weight of block, C D. Now load the scale pan, S P, with small shot until the block, C D, moves freely along A B. The motion may require to be aided by a little tapping on the table, since static friction is greater than kinetic friction. The experiment should be re-

* 10 feet per second was the highest velocity in Morin's experiments.

peated two or three times, taking care that P, the force causing motion, is not more than is necessary to just keep C D moving

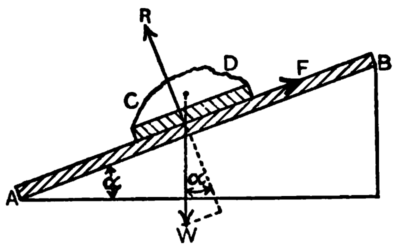


FINDING THE COEFFICIENT OF FRICTION.

at a *uniform* rate. Next remove S P and weigh it carefully. Let this weight be P units.*

Then, The coefficient of friction $= \mu = \frac{F}{R} = \frac{P}{R}$.

(2) Incline the plane A B gradually until C D just begins to slide downwards, the table being gently tapped to overcome the



FINDING ANGLE OF REPOSE.

static friction. Let α be the angle which the plane A B now makes with the horizontal, then :—

The coefficient of friction $= \mu = \tan \alpha$.

For, resolve W along A B and at right angles to A B. Then:—

The component along A B $= F = W \sin \alpha$.

And the component at right angles to A B, $\Big\} = R = W \cos \alpha$.

$$\therefore \qquad \mu = \frac{F}{R} = \frac{W \sin \alpha}{W \cos \alpha} = \tan \alpha.$$

* If P be in pounds, ounces, grammes, or grains, R must also be in pounds, ounces, grammes, or grains.

The angle α is called the "*angle of repose*," or "*angle of friction*" for the materials A B and C D. Hence:—

DEFINITION.—**The angle of repose is the greatest angle at which a plane may be inclined to the horizon before a body placed on it will begin to slide.**

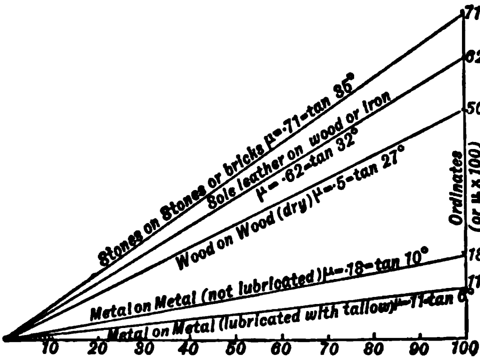
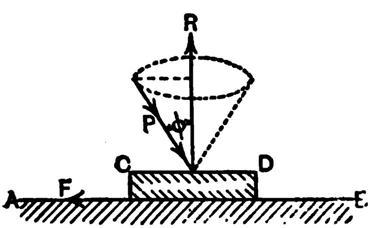


DIAGRAM OF ANGLES OF REPOSE.

The above diagram exhibits the "angles of repose" for several of the more common materials, together with the values of their coefficients of friction.

Limiting Angle of Resistance.—If we attempt to push the block, C D, along A B by means of a sharp pointed rod inclined to the vertical as shown; then, it will be found that, however great the pressure exerted, no relative motion will take place unless the rod be inclined to the vertical at an angle at least equal to the angle of repose.



LIMITING ANGLE OF RESISTANCE.

Thus, let φ be the angle which the direction of the force, P, makes with the normal reaction, R, when sliding is just about to take place. Resolve P parallel and perpendicular to A B. Then, clearly, the limiting friction, F, between A B and C D is equal to the component of P parallel to A B—*i.e.*, $F = P \sin \varphi$. Also, the perpendicular pressure between the surfaces (neglecting the weight of C D) is $R = P \cos \varphi$.

$$\therefore \qquad \mu = \frac{F}{R} = \frac{P \sin \varphi}{P \cos \varphi} = \tan \varphi.$$

But we have just seen that $\mu = \tan \alpha$.

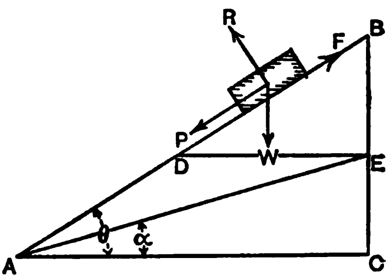
$$\therefore \qquad \mu = \tan \varphi = \tan \alpha. \quad \text{Or, } \varphi = \alpha.$$

Hence, when a body is made to slide along the surface of another, the direction of the total reaction makes an angle with their common normal (at least) equal to the angle of repose. This angle is called the *limiting angle of resistance* and may be thus defined.

Definition.—**The limiting angle of resistance is the greatest angle which the total reaction between two surfaces can make with the normal before sliding takes place.**

(3) Another method for finding the average coefficient of friction is the following:—

Take a plane, A B, made of one substance and inclined at any angle, θ (greater than the angle of repose). Allow a block of the other substance to slide along a given length, B A, of the inclined plane, and note its velocity when it reaches the point, A. Next calculate the *vertical* height, B E, corresponding to the length, B D, of the plane through which the body would require to slide in order to acquire the *same* velocity as before, *if there was no friction.*



Finding the Coefficient of Friction.

To get this height, B E, let v be the actual velocity of the body at A. Then, neglecting friction, this velocity would be acquired when the body reached the point D, such that:—

$$v^2 = 2g \times BE, \therefore BE = \frac{v^2}{2g}*$$

Set off this distance along B C. Join A E.

Then :—**Average coeff. of frict. from B to A** $= \dfrac{EC}{AC} = \tan \alpha$.

Let B C represent the *total* force, P, impelling the body down, B A, against friction and generating the velocity, v, then

* See Contents for Lecture on Motion.

B E will represent the force which goes to generate the velocity alone, since in the second case the final velocity is the same as in the first and, by hypothesis, no frictional resistances are overcome. Hence, the force which overcomes the friction alone, will be represented by the difference between B C and B E—*i.e.*, by E C.

Now, we know that:—

$$P : R = BC : AC.$$

Hence,
$$F : P : R = EC : BC : AC.$$

$$\therefore \qquad \mu = \frac{F}{R} = \frac{EC}{AC}.$$

The chief difficulty in making an experiment of this kind would be in finding the velocity, v, at A. It is much easier to find the time taken to move from B to A. Suppose this time to be found. Let it be, t, seconds. Then assuming the body to be uniformly accelerated, we get $s = \frac{1}{2} v\, t$, or $AB = \frac{1}{2} v\, t$.*

$$\therefore \qquad v = \frac{2\,AB}{t}.$$

Example I.—Let the plane, A B, be 10 feet long and inclined to the horizon at an angle of 30°. Suppose the time taken to slide from B to A to be $1\frac{1}{2}$ seconds. Then the velocity at the foot of the plane is $v = \dfrac{2 \times 10}{1{\cdot}5} = \dfrac{40}{3}$ feet per second.

$$\text{The height } BE = \frac{v^2}{2g} = \frac{\frac{40}{3} \times \frac{40}{3}}{2 \times 32{\cdot}2} = 2{\cdot}76 \text{ feet.}$$

Then, since angle $BAC = 30°$, $\therefore BC = \frac{1}{2} AB = 5$ feet.

$$\therefore \qquad CE = 5 - 2{\cdot}76 = 2{\cdot}24 \text{ feet.}$$

Also,
$$AC = \sqrt{AB^2 - BC^2} = \sqrt{75} = 8{\cdot}66.$$

$$\therefore \qquad \mu = \tan \angle EAC = \frac{EC}{AC} = \frac{2{\cdot}24}{8{\cdot}66} = {\cdot}2586$$

and this corresponds to an angle of friction of 15° nearly.

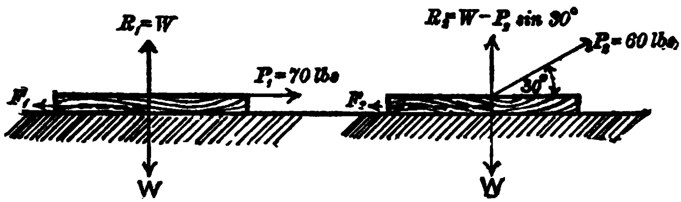
This method of obtaining the coefficient of friction, although both interesting and instructive, is, however, so complicated and attended with so many difficulties that reliable results could not be obtained in a class-room or ordinary laboratory.

Example II.—A plank of oak lies on a floor with a rope

* See Lecture on Motion, &c., in this treatise.

attached to it. When the rope is pulled horizontally with a force of 70 lbs. it just moves, but when pulled at an angle of 30° to the floor a force of 60 lbs. moves it. What is the weight of the plank and the coefficient of friction between it and the floor?

ANSWER.—Let W denote the weight of the plank in lbs.,
" μ " coefficient of friction between the floor and the plank.
" F_1, F_2 " frictions in the two cases.



EXAMPLE ON COEFFICIENT OF FRICTION.

In the first case, when P is parallel to the floor, we get $R_1 = W$, and $P_1 = F_1 = \mu R_1$,

$$\therefore \qquad \mu W = 70 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

In the second case, when P is inclined at an angle of 30° to the floor, the reaction between the floor and the plank will be less than W by the vertical component of P_2.

$$\therefore \qquad R_2 = W - P_2 \sin 30° = W - 60 \times \tfrac{1}{2} = (W - 30) \text{ lbs.}$$

$$\therefore \qquad F_2 = \mu R_2 = \mu (W - 30) \text{ lbs.}$$

$$\text{But} \qquad F_2 = P_2 \cos 30° = 30\sqrt{3} \text{ lbs.}$$

$$\therefore \mu(W - 30) = 30\sqrt{3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

We have now obtained two equations, (1) and (2), containing the two unknown quantities, W and μ. By solving these equations these quantities can be found.

Divide (2) by (1), then

$$\frac{W - 30}{W} = \frac{30\sqrt{3}}{70} = \frac{3\sqrt{3}}{7}.$$

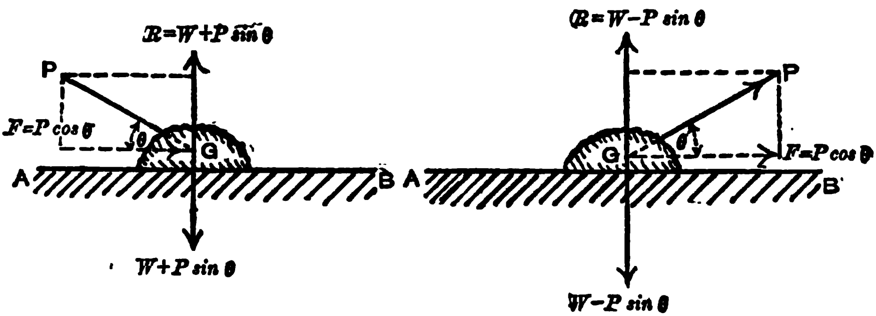
$$\therefore \qquad 7W - 210 = 3\sqrt{3}, W.$$

$$\therefore \qquad W = \frac{210}{1{\cdot}8} = 116{\cdot}\dot{6} \text{ lbs. nearly.}$$

From equation (1), $\mu = \dfrac{70}{W} = {\cdot}6$ nearly.

From this example it appears that the pull, P (which is sometimes called the *traction*), *decreases* as its direction becomes more inclined to the line of motion. Thus, when P is parallel to the floor, the pull required is 70 lbs., but when inclined at an angle of 30° it is only 60 lbs. It is also clear that P does not continually decrease as the *angle of traction* increases. For, when P makes an angle of about 90° with the line of motion, the tendency of P is to lift W and not to move the body along the floor at all. Hence, there must be some definite angle for which the pull, P, will have its minimum value.

The Best Angle of Propulsion or Traction.—We shall now show that the best angle of propulsion or traction for given materials is equal to their angle of repose.



THE BEST ANGLE OF PROPULSION OR TRACTION IS THE ANGLE OF REPOSE.

Let P make an angle, θ, with the direction of motion. Let α be the angle of repose for the two materials. Then $\mu = \tan \alpha$.

Normal pressure between the bodies $= R = W \pm P \sin \theta$.*

Resistance to motion . . . . $= F = \mu R = \mu (W \pm P \sin \theta)$.

Force causing motion . . . . $= P \cos \theta$.

$$\therefore \quad P \cos \theta = \mu (W \pm P \sin \theta)$$

$$\therefore \quad P (\cos \theta \mp \mu \sin \theta) = \mu W.$$

* The sign is + when P is pushing the body, as shown by the left-hand figure; and − when P is pulling the body, as shown by the right-hand figure. Hence, throughout the following investigation the *upper* sign will refer to the left-hand figure and the *lower* sign to the right-hand figure.

Or, $\quad P = \dfrac{\mu W}{\cos\theta \mp \mu \sin\theta}$. $\qquad$ But, $\mu = \dfrac{\sin\alpha}{\cos\alpha}$.

Hence, $P = \dfrac{W \sin\alpha}{\cos\theta\cos\alpha \mp \sin\theta\sin\alpha}$, $\therefore P = \dfrac{W\sin\alpha}{\cos(\theta \pm \alpha)}$ (1)

Hence, P will have its ***minimum*** value, for a given load, W, when the fraction $\dfrac{\sin\alpha}{\cos(\theta \pm \alpha)}$ is a *minimum*. Now, the denominator of this fraction is the only quantity which can be made to vary, since α, and therefore also, $\sin\alpha$ is a constant quantity for the same materials. Consequently, the fraction will be a *minimum* when its denominator is a *maximum*—*i.e.*, when $\cos(\theta \pm \alpha)$ is a *maximum*. But the maximum value of a cosine is *unity*, and this occurs when the angle is 0.

$\therefore$ When $\cos(\theta \pm \alpha) = 1$, $\theta \pm \alpha = 0$, or $\theta = \mp \alpha$.*

Hence, the least push or pull required to move a load, W, along a horizontal plane is, by equation (1),

$$P = W \sin\alpha$$

and the direction of the push or pull makes an angle α, equal to the angle of repose, with the horizontal plane.

Example III.—A body weighing 200 lbs. is drawn along a horizontal plane, by a rope making an angle of 30° to the plane. Find the force necessary to move the body, supposing the coefficient of friction to be ·5. Find, also, the least force which would just suffice.

Answer.—(1) Referring to the previous right-hand figure, we get:—

$$P\cos 30° = F = \mu R = \mu(W - P\sin 30°)$$

$$\therefore \quad P \times \frac{\sqrt{3}}{2} = \cdot 5\,(200 - P \times \tfrac{1}{2})$$

$$\therefore \quad \cdot 866\,P = 100 - \cdot 25\,P$$

$$\therefore \quad P = \frac{100}{1\cdot 116} = 89\cdot 6 \text{ lbs.}$$

* The upper or (−) sign, here refers to the case wherein the body is being *pushed*, while the lower or (+) sign refers to the case wherein the body is being *pulled*. In the first case, we see that P will be a *minimum* when $\theta = -\alpha$,—*i.e.*, when the force, P, is directed from *below upwards* and inclined to the horizon at an angle, α, equal to the "angle of repose." Similarly, the *pull*, P (right-hand figure), will be a *minimum* when $\theta = +\alpha$, —*i.e.*, when P is directed upwards and inclined to the horizon at an angle, α, equal to the "angle of repose."

(2) The least force necessary to move the body is, according to the above results,

$$P = W \sin \alpha.$$

Now, $\tan \alpha = \mu,$

$\therefore$ $$\frac{\sin^2 \alpha}{\cos^2 \alpha} = \mu^2.$$

Or, $\sin^2 \alpha = \mu^2 (1 - \sin^2 \alpha),$

$\therefore$ $$\sin \alpha = \frac{\mu}{\sqrt{1 + \mu^2}} = \frac{\cdot 5}{\sqrt{1 \cdot 25}} = \cdot 447$$

$\therefore$ $$P = 200 \times \cdot 447 = 89 \cdot 4 \text{ lbs.}$$

In this case the direction of P makes an angle of $26\frac{1}{2}°$ nearly with the horizontal plane.

The student should now prove that the same holds true when a body is pulled up an inclined plane by a force, P, which makes an angle, θ, with the incline—viz., that the force will be least when $\theta = \alpha$, the angle of repose.

LECTURE V.—QUESTIONS.

1. What is friction? State the ordinary laws of friction, and explain by aid of sketches and concise descriptions how they may be proved experimentally. What is meant by the "coefficient of friction;" "angle of repose;" "angle of friction;" and "limiting angle of resistance?"

2. Define the coefficient of friction and the angle of friction. A weight of 500 lbs. is placed on a table, and is just made to slide by a horizontal pull of 155 lbs. Find the coefficient of friction, and the number of degrees in the angle of friction, by drawing it to scale. (S. & A. Adv. Theor. Mechs. Exam., 1886.) *Ans.* 0·31.

3. The saddle of a lathe weighs 5 cwts., and it is moved along the bed of the lathe by a rack and pinion arrangement. What force, applied at the end of a handle 10 inches in length, will be just capable of moving the saddle, supposing the pinion to have 12 teeth of $1\frac{1}{4}$-inch pitch, and the coefficient of friction between the saddle and lathe-bed to be 0·1, other friction being neglected? Sketch the arrangement. (S. & A. Adv. Exam., 1891.) *Ans.* 13·36 lbs.

4. A body weighing 50 lbs. is pulled along a rough horizontal plane by a force whose line of action makes an angle of 45° with the plane. If the coefficient of friction between the body and the plane be 0·2, find the magnitude of the pull and the pressure between the body and the plane. *Ans.* 11·78 lbs.; $41 \cdot \dot{6}$ lbs.

5. A body is resting on a rough horizontal plane, and is acted on by a force whose line of action is inclined 45° to the plane. The force is gradually increased until the body is just about to move; find the ratio of the force exerted, to the weight of the body, the coefficient of friction being 0·25. *Ans.* $\sqrt{2} : 5$, or $1 : 3{\cdot}5$.

6. A body lying on a rough table can just be moved by a horizontal pull of 20 lbs.; but, when pulled at an angle of 30° to the horizon, the force required is found to be only 18 lbs. Will you explain the reason for this difference, and find the weight of the body and the coefficient of friction between it and the table? *Ans.* 41 lbs.; ·49.

7. A body weighing 100 lbs. is drawn along a horizontal plane by a rope, making an angle of 20° with the plane. Find the force required, supposing the coefficient of friction to be 0·15. Find, also, the least force which would just pull the body along the plane, and the angle which its direction would make with the plane. *Ans.* 15 lbs.; 14·75 lbs.; $8\frac{1}{2}$°.

8. A body of known weight is placed on a rough horizontal plane and pulled in a certain direction. Find (1) the force of the pull which will just make the body slide, and (2) what must be the direction of the pull that it may be the least that will make the body slide? Suppose that P is the least pull as above determined, and that the body when pushed by a force, P_1 (acting along the same line as that in which P acted), is on the point of sliding, show that $P_1 (1 - \mu^2) = P (1 + \mu^2)$, μ being the coefficient of friction. (S. & A. Adv. Theor. Mechs. Exam., 1887.)

9. A force of 15 lbs. per ton of load, is required to maintain the motion of a train on a level line. Determine the coefficient of friction between the driving-wheels and rails when an engine of 30 tons weight can just keep in motion a train of 350 tons (including weight of engine). Give a diagram illustrating the method employed by you in arriving at the answer. *Ans.* 0·078.

LECTURE VI.

CONTENTS.—Friction of Cylindrical Surfaces—General Morin's Experiments—Hirn's Experiments—Prof. Thurston's Experiments—Prof. Fleeming Jenkin's Experiments—Beauchamp Tower's Experiments on Journals—Practical Examples of Lubricating Journals—Experiments on Collar Friction—Friction of a Pivot Bearing—Results of the Experiments—Friction of Railway Brakes—Friction between Water and Bodies Moving through it—Frictional Resistance of a Ship Propelled through Sea Water—Examples—Questions.

Friction of Cylindrical Surfaces.—In the preceding lecture we only dealt with the friction of plane surfaces. It is equally necessary, however, that the engineer should study the friction of cylindrical surfaces. With this object in view we shall give a brief summary of the results of the experiments carried out by the principal authorities on this subject prior to the year 1883, and then state the conclusions arrived at by the "Research Committee on Friction," appointed by the "Institution of Mechanical Engineers," which now constitute the standard and most reliable experiments on this subject.

General Morin's Experiments.—Morin also made experiments on the friction of axles, and he arrived at the same general conclusions as were explained in Lecture V. for plane surfaces; the only difference being, in the values of the coefficients of friction. The diameters of his journals reached a maximum of 4 inches, but the speeds never exceeded a sliding velocity of more than 30 feet per minute, and the pressures 160 lbs. per square inch of the nominal bearing surface. By nominal bearing surface is meant, the projected area of the journal on a diametrical plane—*i.e.*, on a plane containing the axis of the journal. Thus, let d denote the diameter and l the length of the journal in inches, then:—

Nominal bearing surface $= d\,l$ square inches.

If R be the total reaction or load in pounds, and p the intensity of pressure, or the pressure in pounds per square inch on the journal, then:—

$$R = p\,d\,l; \text{ or } p = \frac{R}{d\,l}.^{*}$$

* If the arc of the circle embraced by the brass bush, upon which the shaft actually bears, be indicated by the chord, d', subtended by it, then:—

$$R = p\,d'\,l, \text{ or } p = \frac{R}{d'\,l}.$$

The values of μ in the equation $F = \mu R$, as given by Morin, are:—for dry journals ·18 to ·25, for those greased and wet with water, ·14 to ·19; intermittently lubricated, ·07 to ·12; and for continuous lubrication, ·03 to ·05. *The friction was found to be independent of the velocity and proportional to the load.* The coefficient of friction thus depending *only* on the nature of the bearing surfaces. In more recent experiments with cases approaching those which occur in actual practice, it has been shown that the values of μ are much smaller than those given by Morin, and, further, that the friction is entirely dependent on the more or less thorough lubrication of the bearing.

Hirn's Experiments.*—In 1885, M. Hirn published the results of a long series of experiments, chiefly on lubricated journals. These results show, that instead of the coefficient of friction being a constant quantity for the same materials, it is more nearly proportional to the square root of the velocity of rubbing, v, and inversely proportional to the square root of the intensity of pressure, p.

Or, $$\mu = c\sqrt{\frac{v}{p}}$$

Where c is a constant quantity found by experiment. Hence, we see that the friction in those experiments varied directly as the square root of the load, area, and velocity.

$$\text{For, } F = \mu R = c\sqrt{\frac{v}{p}} \times R = c\sqrt{\frac{v}{\frac{R}{dl}}} \times R = c\sqrt{R\,d\,l\,v}.$$

For ordinary conditions of working the friction thus appears to have varied as the square root of the velocity. The friction diminished as the temperature increased,† and the best results were obtained after the lubricant had been working for some time between the surfaces.

* In 1880, C. J. H. Woodbury, of the Institute of Technology, Boston, Mass., U.S.A., read a paper before the American Society of Mechanical Engineers (see vol. i., p. 74, *et seq.*) on "Measurement of the Friction of Lubricating Oils." He states that his experiments proved that the coefficient of friction varies in an inverse ratio with the pressure for high speed lightly-loaded spindles. Further, that the coefficient of friction at 150° F. was about 75 per cent. less than at 75° F.; and, therefore, mill owners should keep their machinery warm in winter.

† See a paper by M. G. Adolphus Hirn, read before the Société Industrielle de Mulhouse, June 28, 1884, where water is used to control the temperature of the bearing surfaces of oil testing machines; also, The 1886 Cantor Lectures on "Friction," by Prof. Hele Shaw.

Prof. Thurston's Experiments.*—Professor R. H. Thurston, of U.S.A., has carried out a number of experiments to determine the effect of changes, not only in velocity, but also in pressure and temperature, upon the frictional resistance of lubricated bearings. His conclusions are, that the coefficient at first decreases, but after a certain point increases with the velocity; the point of change varying with the pressure and temperature. Very few details, however, are given of the way in which these experiments were carried out, and, consequently, we cannot here enlarge further upon them.

Prof. Fleeming Jenkin's Experiments.†—A number of experiments was carried out by Prof. Jenkin in connection with the difference between Static Friction or the Friction of Rest and Dynamic Friction or the Friction of Motion. He experimented at extremely low velocities, and showed, that in certain cases, the coefficient of friction decreases gradually as the velocity increases, between speeds of ·012 and ·6 foot per minute, thus indicating the probability of a *continuous* rather than a sudden change in the value of the coefficient of friction between the conditions of rest and motion.

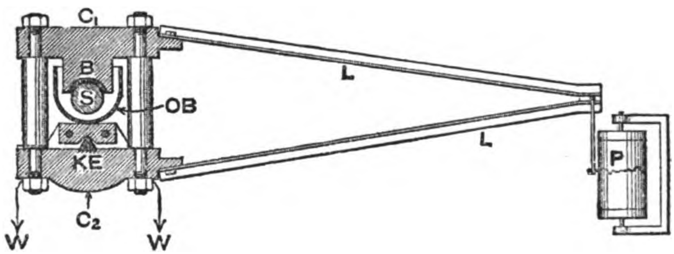
Beauchamp Tower's Experiments‡—(1) *Description of Machine.*—In experimenting on the friction of lubricated bearings, and on the value of different lubricants, one of the difficulties which is first met with is the want of a method of applying the lubricant, which can be relied upon as sufficiently uniform in its action. All the common methods of lubrication are so irregular in their action that the friction of a bearing often varies considerably. This variation, though small enough to be of no practical importance, and to pass unnoticed, in the working of an ordinary machine, would be large enough utterly to destroy the value of a set of experiments, say, on the relative values of various lubricants; for it would be impossible to know whether an observed variation was due to a difference in the quality of the oil, or in its rate of application. The first problem, there-

* "Friction and Lubrication," p. 185. "American Association for the Advancement of Science," Aug. 1878, p. 61. "The Theory of the Finance of Lubrication and on the Valuation of Lubricants by Consumers," "Friction and Lost Work in Machinery," N.Y., 1885, and on "The Real Value of Lubricants," Jany. 5th, 1885, see *Trans. Am. Soc. Mech. Engs.*, vol. xiii. Also see the 1891 vol. for "Special Experiments with Lubricants," by B. J. E. Denton of Hobokin, N.J., U.S.A. He deals with the lubrication of steam cylinders and of journals subjected to heavy pressures.

† *Proceedings of the Royal Society*, 1877, p. 93.

‡ By the kindness of the Council of the Institution of Mechanical Engineers, London, the author is permitted to make the following extracts from their Proceedings and Report on Friction Experiments, Nov. 1883.

fore, which presented itself, in the present experiments, was to devise a method of lubrication such as would be perfectly uniform in its action, and would form an easily reproducible standard with which to compare other methods. These conditions were best fulfilled by making the bearing run immersed in a bath of oil. By this method the bearing is always supplied with as much oil as it can possibly take; so that it represents the most perfect lubrication possible, and is a good standard with which to compare other methods. It is at all times perfectly uniform in its action. It is very easily defined and reproduced; and it also has the advantage that the temperature of the bearing can be easily regulated by gas jets under the bath. Experiment showed that the bath need not be full; the results obtained were the same when it was so nearly empty that the bottom of the journal only just touched the oil.

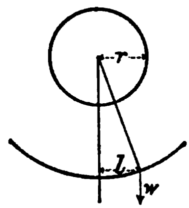


BEAUCHAMP TOWER'S APPARATUS FOR TESTING THE FRICTION OF JOURNALS.

The above figure represents the arrangements for conducting the experiments. The shaft, S, was of steel, 4 inches diameter and 6 inches long, with its axis horizontal and driven by a belt acting on a pulley keyed to its outer end.

A gun-metal brass, B, embracing somewhat less than half the circumference of the journal, rested on its upper side. The exact arc of contact of this brass was varied in the different experiments. Resting on this brass was a cast-iron cap, C_1, from which was hung by two bolts a cast-iron cross-bar, C_2, carrying a knife-edge, K E. The exact distance of this knife-edge below the centre of the journal was 5 inches. On this knife-edge was suspended the cradle which carried the weights, W, W, applied to the bearing. The cap, bolts, and cross-bar were put together in such a manner as to form a rigid frame, connecting the brass with the knife-edge. If there had been no friction between the brass and the journal, the weight would have caused the knife-edge to hang perpendicularly below the

axis of the journal. Friction, however, caused the journal to tend to carry the brass and the frame to which it was attached, round with it, until the line through the centre of journal and the knife-edge, made such an angle with the perpendicular, that the weight multiplied by the distance from the knife-edge to that perpendicular, offered an opposing moment just equal to the moment of friction.



INDICATING COEFFICIENT OF FRICTION.

Suppose r = radius of the journal.

„ l = distance of the knife-edge from the perpendicular.

„ w = the weight.

And,

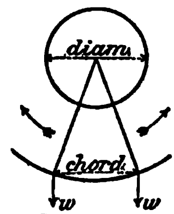
$(l \times w)$ = the moment of friction.

$$\text{The friction at the surface of the journal,} = \frac{\text{the moment}}{r} = \frac{w \times l}{r}.$$

$$\text{Hence, the coefficient of friction,} = \frac{\text{Friction at surface of journal}}{w} = \frac{l}{r}.$$

So that the coefficient of friction is indicated by l in terms of r, no matter what the weight is. As an example, suppose l was equal to r, the coefficient of friction would obviously be 1; or if l was $\frac{1}{10}$ of r, then the coefficient of friction would be $\frac{1}{10}$.

In order to avoid the difficulty of determining accurately when the knife-edge was perpendicularly under the centre of the journal (a knowledge which was necessary in order to obtain a measurement of l, and which was very difficult to obtain owing to the considerable friction between the brass and the journal when at rest), each experiment was tried with the journal revolving in both directions, and the sum of the values of l on both sides was measured; and then the coefficient of friction was indicated by the chord of the whole angle, included between the two lines of inclination caused by the friction, with the rotation in the two directions, the chord being expressed in terms of the diameter of the journal (see figure). Each result was thus a mean of two experiments, one with the axle running in one direction, and the other with it running in the other direction. In order to read the value of the co-



INDICATING COEFFICIENT OF FRICTION.

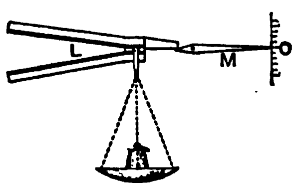
efficients thus obtained, a light horizontal lever, L, was attached to the frame connecting the brass to the knife-edge. It was $62\frac{1}{2}$ inches long, or twelve and a-half times the distance between the centre of the journal and the knife-edge; so that, at the end of the lever, the chord indicating the coefficient of friction was magnified twelve and a-half times. As a chord of 4 inches at the knife-edge represents a coefficient of 1, a chord of 50 inches at the end of the lever also represents a coefficient of 1, while 5 inches represents a coefficient of $\frac{1}{10}$, $\frac{1}{2}$-inch of $\frac{1}{100}$, and $\frac{1}{20}$-inch of $\frac{1}{1000}$. The position of the end of the lever during each experiment was recorded by a tracing point, attached to the end of the lever, and marking on metallic paper carried upon a revolving vertical cylinder, P. The distance between the two lines obtained by running the axle both ways, when measured on the above scale, indicated the value of the coefficient.

(2) *Method of Experimenting.*—Early in the experiments it was found, that immediately after the motion of the shaft was reversed, the friction was greater than it was when the shaft had been running in the same direction some time. This increase of friction, due to reversal, varied considerably. It was greatest with a new brass, and diminished as the brass became worn, so as to fit the journal more perfectly. Its greatest observed amount was at starting and was about twice the normal friction, and it gradually diminished until the normal friction was reached after about ten minutes continuous running. This increase of friction was accompanied by a strong tendency to heat and seize, even under a moderate load. In the case of one brass, which had worked for a considerable time without accident, and had consequently become worn so as to fit the journal very accurately, this tendency to increase of friction after reversal almost entirely disappeared; and it could be reversed under a full load without appreciable increase of friction or a tendency to heat or seize. The phenomenon must be due to the surface fibres of the metal, which have been for some time stroked in one direction, meeting point to point and interlocking when the motion is reversed. The very perfectly fitting brass was probably entirely separated from the journal by a film of oil; and there being no metallic contact the phenomenon did not show itself. In consequence of the above facts, it was found necessary to proceed with the experiments in the following order. A complete series of experiments, with a gradually increasing load, was taken with the journal running in one direction; the load was then diminished by the same steps as it had been increased, and the experiments thus repeated, the shaft still running in the same direction, until the load had thus

been reduced to 100 lbs. per square inch, which was the load due to the unweighted cradle. The direction of motion was then reversed, and the shaft run for half an hour, so as to get it thoroughly used to going the other way; after this the load could be increased and the experiments taken without any difficulty. The experiments, as before, were taken at each step whilst both increasing and decreasing the load; so that each recorded result is really the mean of four experiments, which have in many instances been taken several hours apart.

This method of obtaining a direct indication of the coefficient of friction, by the angular displacement of the frame connecting the brass and knife-edge, would undoubtedly have been the best had the coefficient of friction been nearly as constant as it has hitherto been supposed to be. But as shown by the results, the coefficient of friction was found, instead of being constant, to vary nearly inversely as the load, and also to be much smaller in quantity than was expected; the consequence was, that with high loads the height of the diagram was very small. In the cases where with the greatest loads, a coefficient of only $\frac{1}{1000}$ was observed, the distance between the two lines was only $\frac{1}{20}$ inch.

Owing to these experiments showing that the moment of friction was much more nearly constant than the coefficient, it was resolved to alter the method of observation, and to measure the moment directly, instead of the coefficient. For this purpose the paper cylinder was removed, and a small lever, M (see accompanying figure), was connected to the main indicating lever in such a manner that the motion of the end of the main lever was magnified five times at the end of the small lever. The end of the small lever was pointed; and when the machine was working, this point was brought exactly opposite a fixed mark by putting weights into a scale-pan on the end of the main lever.



SECOND ARRANGEMENT OF INDEX.

The main lever was so overbalanced that under all circumstances some weight was required to be added to the scale-pan, in order to bring the end of the small lever to the mark, even when, in addition to the friction being greatest, the direction of motion of the journal tended most to depress it. The method of running in both directions, and loading and unloading, was followed as before. The weights in the scale-pan being noted, the moment of friction was given by half the difference between the weights in the scale-pan, when running in one direction and in the other.

The following table is selected from those recorded in the *Proceedings of the Institution of Mechanical Engineers* as an example of the results obtained :—

BATH OF MINERAL OIL. TEMPERATURE 90° F. 4-INCH JOURNAL, 6 INCHES LONG. CHORD OF ARC OF CONTACT OF BRASS = 3·92 INCHES.

Nominal Load Lbs. per Sq. In.	COEFFICIENTS OF FRICTION, for speeds as below.						
	100 rev. 105 ft. per min.	150 rev. 157 ft. per min.	200 rev. 209 ft. per min.	250 rev. 262 ft. per min.	300 rev. 314 ft. per min.	350 rev. 366 ft. per min.	400 rev. 419 ft. per min.
Lbs.							
625	...	·0013	·00139	·00147	·00157	·00165	...
520	...	·00123	·00139	·0015	·00161	·0017	·00178
415	...	·00123	·00143	·0016	·00176	·0019	·002
310	...	·00142	·0016	·00184	·00207	·00225	·00241
205	·00178	·00205	·00235	·00269	·00298	·00328	·0035
100	·00334	·00415	·00494	·00557	·0062	·00676	·0073

The above coefficients × the nominal load = nominal frictional resistance per square inch of bearing.

Nominal Load Lbs. per Sq. In.	NOMINAL FRICTION RESISTANCE per square inch of bearing.						
	100 rev. 105 ft. per min.	150 rev. 157 ft. per min.	200 rev. 209 ft. per min.	250 rev. 262 ft. per min.	300 rev. 314 ft. per min.	350 rev. 366 ft. per min.	400 rev. 419 ft. per min.
Lbs.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.
625	...	·81	·865	·92	·98	1·03	...
520	...	·64	·72	·782	·84	·886	·924
415	...	·51	·594	·664	·73	·785	·83
310	...	·44	·494	·57	·64	·695	·745
205	·364	·419	·48	·55	·61	·67	·716
100	·334	·415	·494	·557	·62	·676	·73

N.B.—The bearing carried the 625 lbs. per sq. in. running both ways, but seized on the weight being increased.

The nominal load per sq. in. is the total load divided by 4 × 6.

The actual load per sq. in. is the total load divided by 3·92 × 6.

These quantities were obtained by a direct load on the lever.

This was a thinner sample of mineral oil than that used in the previous experiments; it was fluid at 50°, while the oil previously used could only be described as grease at 50°. This will account for these experiments showing less friction than the former, except with the highest load, at which, the thin oil being overloaded and on the point of seizing, the friction is greater than with the thick oil.

Experiment showed that the friction varied considerably with temperature. All the oil-bath experiments were therefore taken at a nearly uniform temperature of 90°; the variation above or below this temperature was never allowed to be more than 1½°.

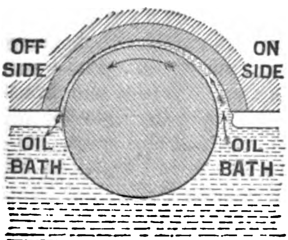
(3) *Results of Experiments.*—The results of the experiments are recorded in Tables I. to IX. in the *Proceedings of the Institution of Mechanical Engineers.* The general results of the oil-bath experiments may be described as follows:—*The absolute friction* (that is the actual tangential force per sq. in. of bearing, required to resist the tendency of the brass to go round with the journal) *is nearly a constant under all loads, within ordinary working limits.* Most certainly it does not increase in direct proportion to the load, as it should do according to the ordinary theory of solid friction. The ordinary theory of solid friction is, that it varies in direct proportion to the load; that it is independent of the extent of surface; and that it tends to diminish with an increase of velocity beyond a certain limit. The theory of liquid friction, on the other hand, is, that it is independent of the pressure per unit of surface, is directly dependent on the extent of surface, and increases as the square of the velocity. The results of these experiments seem to show that the friction of a perfectly lubricated journal follows the laws of liquid friction much more closely than those of solid friction. They show that under these circumstances the friction is nearly independent of the pressure per sq. in., and that it increases with the velocity, though at a rate not nearly so rapid as the square of the velocity.

The experiments on friction at different temperatures indicate a very great diminution in the friction as the temperature rises. Thus, in the case of lard oil, taking a speed of 450 revolutions per minute, the coefficient of friction at a temperature of 120° is only one-third of what it was at a temperature of 60°.

A very interesting discovery was made when the oil-bath experiments were on the point of completion. The experiments being carried on were those on mineral oil; and the bearing having seized with 625 lbs. per sq. in., the brass was taken out and examined, and the experiment repeated. While the brass was out, the opportunity was taken to drill a ½-in. hole for an ordinary lubricator through the cast-iron cap and the brass. On the machine being put together again and started with the oil in the bath, oil was observed to rise in the hole which had been drilled for the lubricator. The oil flowing over the top of the cap made a mess, and an attempt was made to plug up the hole, first with a cork and then with a wooden plug. When the machine was started the plug was slowly forced out by the oil in a way which showed that it was acted on by a considerable pressure. A pressure-gauge was screwed into the hole, and on

the machine being started the pressure, as indicated by the gauge, gradually rose to above 200 lbs. per sq. in. The gauge was only graduated up to 200 lbs., and the pointer went beyond the highest graduation. The mean load on the horizontal section of the journal was only 100 lbs. per sq. in. This experiment showed conclusively that the brass was actually floating on a film of oil, subject to a pressure due to the load. The pressure in the middle of the brass was thus more than double the mean pressure. No doubt if there had been a number of pressure-gauges connected to various parts of the brass, they would have shown that the pressure was highest in the middle, and diminished to nothing towards the edges of the brass.*

* Another set of experiments was afterwards made by Mr. Beauchamp Tower in order to investigate this point more thoroughly. The results formed the second report on Friction presented to the Institution of Mechanical Engineers in January, 1885. This report confirms the above statement. Small holes were bored in the brass bush, and a different one of these having been connected during each test with a Bourdon pressure gauge, and the bearing having then been immersed in an oil bath, the exact oil pressures at nine different points on the bearing were measured. The pressure was found to be greatest a little to the off side of the centre line of the bearing—*i.e.*, to that side towards which the shaft turned, gradually falling to zero at each edge. It was also found to be greatest in the middle of the length of the bearing. The total upward force of these recorded pressures was found to be within a few pounds of the actual total load on the bearing, thus showing that the load was wholly supported by the film of oil which existed between the shaft and its brass bearing. Or, to quote Mr. Tower's own words, "It was possible to make the brass on a journal work so nicely that there was absolutely no metallic contact between the brass journal and the brass, the whole of the weight being borne by the oil. It seemed to him that the important practical inference was, that it was actually possible so to lubricate a bearing, that not only would metallic friction be altogether done away with, and thereby the amount of power lost by friction be reduced, but metallic wear and tear would also be done away with. He would not say that such a result was actually possible in practice now; but it was a reasonable one to aim at in mechanism. By giving a sufficiently profuse lubrication, and by having the brasses so arranged, that there should be a uniform pressure all over their surface, it was possible to have wear and tear between metal and oil, instead of between metal and metal."



JOURNAL AND OIL BATH.

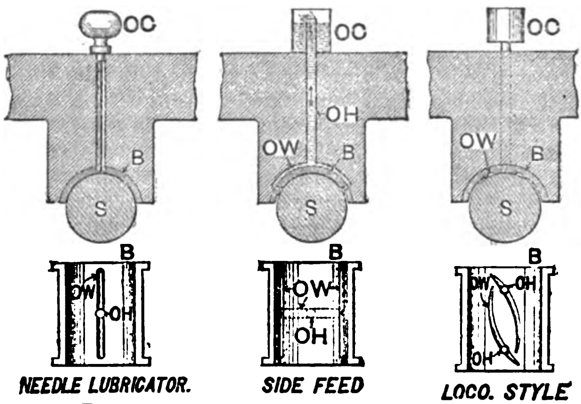
It is now generally recognised that the oil acts as a lubricant by merely furnishing molecules rolling in between the two surfaces, but unless these molecules can be got in, there is no possibility of the diminution of friction. The accompanying figure shows, in an exaggerated manner, what happens in the case of a well-lubricated journal when the brass is bored to a slightly larger radius than the shaft and the oil circulates freely, as shown by the small arrows.

The experiments with ordinary lubrication were begun with a needle lubricator, the hole from which penetrated to the centre of the brass. A groove in the middle of the brass, and parallel to the axis of the journal, extended nearly to the ends of the bearing for distributing the oil (see the first of the following three figures). It was found, that with this arrangement, the bearing would not run cool when loaded with only 100 lbs. per sq. in.; and that not a drop of oil would go down even when the needle-lubricator was removed and the hole filled completely with oil, thus giving a head of 7 inches of oil to force it into the brass. It appeared as though the hole and groove, being in the centre of pressure of the brass, allowed the supporting oil-film to escape. This view was confirmed by the following experiment:—The oil-hole being filled up to the top, the weight was eased off the journal for an instant. This allowed the oil to sink down in the hole and lubricate the journal; but immediately the load was again allowed to press on the journal the oil rose in the hole to its former level, and the journal became dry, thus showing that this arrangement of hole and groove, instead of being a means of lubricating the journal, was a most effectual one for collecting and removing all oil from it. It should be mentioned that care was taken to chamfer the edges of the groove, so as to prevent any scraping action between them and the journal.

As the centre of the brass was obviously the wrong place to introduce the oil, it was resolved to try to introduce it at the sides. Accordingly the centre hole and groove were filled up, and two grooves were made. These grooves were parallel to the axis of the journal, extending nearly to the ends of the brass, and were placed at equal distances on either side of the centre; they formed boundaries to an arc of contact, the chord of which was $3\frac{1}{4}$ inches (see the second of the following three figures). With this arrangement of groove the lubrication appeared to be satisfactory, the oil going down into the journal and the bearing running cool. The bearing nevertheless seized with an actual load of only 380 lbs. per square inch.

The arrangement of grooves was then altered to that usual in locomotive axle-boxes (see the third of the above three figures). The oil was introduced through two holes, one near each end of the brass, and each connected to a curved groove; the two curved grooves nearly enclosing an oval-shaped space in the centre of the brass. At the same time the arc of contact was reduced till its chord was only $2\frac{1}{4}$ inches. *This brass refused to take its oil or run cool.* It would sometimes run for a short time with an actual load of 178 lbs. per square inch, but rapidly

heated on the slightest increase of the load. The brass having been a good deal cut about by altering and filling up grooves, it was considered desirable to have a new brass, and one was accordingly obtained. *The grooves being made exactly the same as in the last experiment with the old one, this brass seized with an actual load of only about 200 lbs. per square inch.* The oil-box was completely cut away so as to allow a freer current of air round the bearing, and the lubricator pipes were soldered into the brass. The wicks were taken out of the lubricators and the



THREE METHODS OF LUBRICATING BEARINGS.

INDEX TO PARTS.

O C	represents	Oil cups.	O W	represents	Oil ways.
B	,,	Brass bearings.	S	,,	Shaft.
O H	,,	Oil holes.			

lubricators filled full of oil, by which means oil was supplied to the brass under a full head of 9 inches; and yet the oil refused to go down, and the underside of the journal felt perfectly dry to the hand, and speedily heated with a load of only 200 lbs. per square inch.

The fact that this arrangement of grooves, which is found to answer in the axles of railway vehicles, was found to be perfectly useless in this apparatus, can only be accounted for by the fact, that a railway axle has a continual end play while running, which prevents the brass from becoming the perfect oil-tight fit which it became in this apparatus. The attempts to make this arrangement of lubrication answer were not abandoned until after repeated trials. It now became clear that there was no use in trying to introduce the oil directly to the part of the

brass against which the pressure acted, and that the only way to proceed was to oil the lower side of the journal, and trust to the oil being carried round by the journal to the seat of the pressure.

The grooves and holes in the brass were accordingly filled up, and an oily pad, contained in a tin box full of rape oil, was placed under the journal, so that the journal rubbed against it in turning. The pad was only supplied with oil by capillary attraction from the oil in the box, and the supply of oil to the journal was thus very small; the oiliness in fact was only just perceptible to the touch, but it was evenly and uniformly distributed over the whole journal. The bearing fairly carried 551 lbs. per square inch, and three observations were obtained with 582 lbs., but the bearing was on the point of seizing and did seize after running a few minutes with this load. It will be observed that in this instance, the bearing seized with very nearly the same load as it did in the oil-bath experiment with rape oil.

These experiments with the oily pad show a nearer approach to the ordinarily received laws of solid friction than any of the others. The coefficient is approximately constant, and may be stated as about $\frac{1}{100}$ on an average. There does not in this case appear to be any well-defined variation of friction with variations of speed, according to any regular law.

The results of the experiments with rape oil, fed by a syphon lubricator to side grooves, follow nearly the same law as the results obtained from the oil-bath experiments, as far as the approximate constancy of the moment of friction is concerned; but the amount of the friction is about four times the amount in the oil-bath.

The oil-bath probably represents the most perfect lubrication possible, and the limit beyond which friction cannot be reduced by lubrication; and the experiments show that with speeds of from 100 to 200 feet per minute, by properly proportioning the bearing-surface to the load, it is possible to reduce the coefficient of friction as low as $\frac{1}{1000}$. A coefficient of $\frac{1}{500}$ is easily attainable, and probably is frequently attained in ordinary engine-bearings, in which the direction of the force is rapidly alternating and the oil given an opportunity to get between the surfaces, while the duration of the force in one direction is not sufficient to allow time for the oil-film to be squeezed out. The extent to which the friction depends on the quantity of the lubrication is shown in a remarkable manner by the following table, which proves that the lubrication can be so diminished that the friction is seven times greater than it was in the oil bath, and yet that the bearing will run without seizing :—

COMPARISON OF THE FRICTION WITH THE DIFFERENT METHODS OF LUBRICATION, UNDER AS NEARLY AS POSSIBLE THE SAME CIRCUMSTANCES. LUBRICANT RAPE OIL, SPEED 150 REVOLUTIONS PER MINUTE.

Mode of Lubrication.	Actual Load Lbs. per sq. in.	Coefficient of Friction.	Comparative Friction.
Oil-bath, . .	263	·00139	1
Syphon lubricator, .	252	·00980	7·06
Pad under journal, .	272	·00900	6·48

Observations on the behaviour of the apparatus gave reason to believe that with perfect lubrication the speed of minimum friction was from 100 to 150 feet per minute; and that this speed of minimum friction tended to be higher with an increase of load, and also with less perfect lubrication. By the speed of minimum friction is meant, that speed in approaching which, from rest, the friction diminishes, and above which the friction increases.

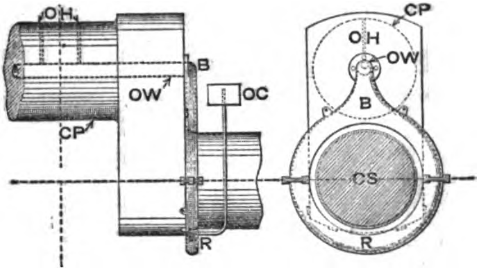
The following table gives the means of the actual frictional resistances at the surface of the journal per square inch of bearing, at a speed of 300 revolutions per minute, with all nominal loads from 100 lbs. per square inch up to 310 lbs. per square inch.

They also represent the relative thickness or body of the various oils, and (in their order, though perhaps not exactly in their numerical proportions) their relative weight-carrying power. Thus sperm oil, which has the highest lubricating power, has the least weight-carrying power; and though the best oil for light loads, would be inferior to the thicker oils if heavy pressures or high temperatures were to be encountered.

COMPARISON OF THE FRICTION WITH THE VARIOUS LUBRICANTS TRIED, UNDER AS NEARLY AS POSSIBLE THE SAME CIRCUMSTANCES. TEMPERATURE 90°, LUBRICATION BY OIL-BATH.

Lubricant.	Mean Resistance.	Per Cent.
	Lb.	
Sperm oil, . . .	0·484	100
Rape oil, . . .	0·512	106
Mineral oil, . .	0·623	129
Lard oil, . . .	0·652	135
Olive oil, . . .	0·654	135
Mineral grease, . .	1·048	217

Practical Examples of Lubricating Journals.—In the discussion which followed the reading of the foregoing report, a method of lubricating crank-pins was mentioned which has proved successful.



METHOD OF LUBRICATING CRANK PINS.

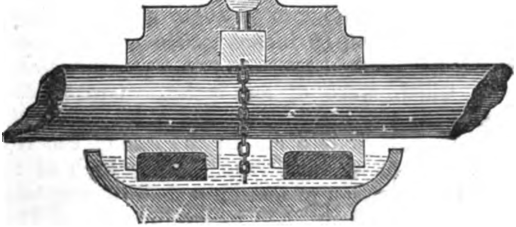
INDEX TO PARTS.

C S	represents	Crank-shaft.	B R	represents	Brass ring.
C P	„	Crank-pin.	O W	„	Oil way.
O C	„	Oil cup.	O H	„	Oil holes.

The oil from the oil cup, O C, passes into the hollow brass ring at R, and is driven outwards by centrifugal force to the point B, where it enters the oil way, O W. From thence it goes by the radial oil holes, O H, to form a film between the crank-pin, C P, and the surrounding brass bush of the connecting-rod end.

In discussing the third report, Mr. Daniel Adamson stated that his firm had adopted the method of cutting a flat on the shaft for the whole length of the journal of about $\frac{1}{8}$ inch wide for each inch in the diameter of the shaft up to 8-inch shafts and rather less for larger ones. In the case of heavy horizontal shafts, such as those supporting large flywheels, this method was found to effectually carry forward the oil into the bearing and thus produce smooth running.

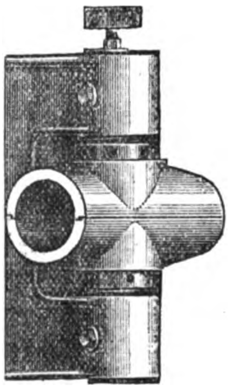
The two following figures show Messrs. J. Bagshaw & Son's



BAGSHAW'S SELF-OILING PEDESTAL.

self-oiling pedestal and swivel adjustable wall-bracket pedestal, which have been designed to minimise friction and the waste of oil.

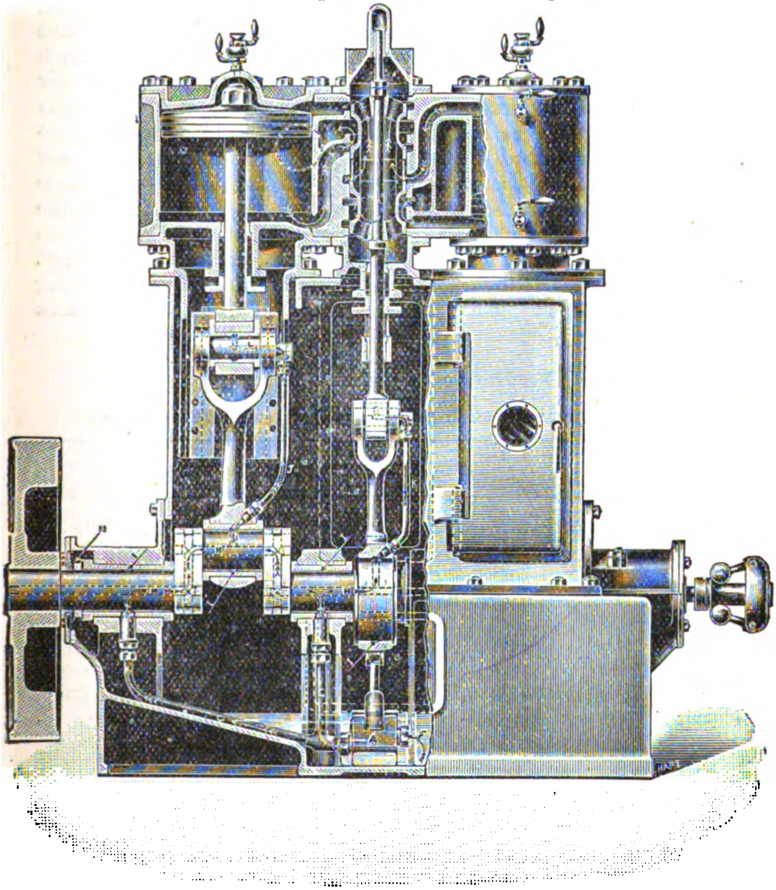
Another example, of the recent practice of lubricating several journals and slide blocks from one common source of supply under pressure, is furnished by Belliss' high-speed compound engines for the direct driving of dynamos. It will be observed from the figure on the next page, that not only the main crank-shaft bearings, but also the crank-pins, slide-blocks, the upper ends of the connecting-rods, the piston-valve eccentric and its rod, are all supplied with oil from a small pump worked by the same eccentric which moves the piston valve. The oil is thereby forced through *each* bearing under a pressure of 10 lbs. per square inch, and is again and again sent on its soothing mission for months at a time, without change or great loss in quantity. A heavy lubricating oil is used, and it always returns to the small pump through a filter which removes any grit which it may have picked up from the bearings. This is a very different state of matters from the old "travelling oil-can" system, when the quantity of oil applied and the times of application were as erratic as the judgment of the attendant.



BAGSHAW'S SWIVEL ADJUSTABLE WALL-BRACKET PEDESTAL.

Experiments on Collar Friction.—Mr. Tower also carried out some experiments on the friction of a collar bearing, for the purpose of ascertaining the friction in such cases as the thrust-bearings of propeller shafts. The results of these experiments constituted the third report of the Research Committee on Friction presented to the Institution of Mechanical Engineers in May, 1888. The collar or annular ring experimented with was made of mild steel, 12 inches inside and 14 inches outside diameter, and was pressed between two discs, the annular bearing surfaces of which were of gun-metal. Great difficulty was experienced with the lubrication, which was effected by means of four diametrical grooves cut in the face of the ring, $\frac{1}{4}$ of an inch wide and $\frac{3}{4}$ of an inch long. From each of these, there extended in the direction of motion a shallow serpentine groove $\frac{1}{10}$ of an inch wide and about $3\frac{1}{2}$ inches in length. These grooves were each supplied by a separate pipe into which oil was dropped

from a reservoir. The minimum amount of lubrication necessary to prevent excessive heating of the bearing varied from 60 to 120 drops of mineral oil per minute, according to the pressure



BELLISS & COY.'S HIGH-SPEED COMPOUND ENGINE WITH CONTINUOUS FORCED LUBRICATION TO ALL BEARINGS EXCEPT CYLINDERS, VALVES, AND THEIR GLANDS.

and velocity of the rubbing surfaces, but except with small pressures, it was found impossible to keep the bearing cool without water running over it. The pressure was varied from

15 to 90 lbs. per square inch, while the speeds ranged from 50 to 130 revolutions per minute. The results of these experiments seem to show, that (1) this kind of bearing is evidently much inferior to a cylindrical journal in its capability of carrying weight, in fact, 75 lbs. per square inch being the maximum that could be safely borne at the highest, and 90 lbs. per square inch at the lowest speed; (2) the friction in this case follows the law of solid friction much more nearly than that of liquids, or liquids and solids; (3) the coefficient of friction was independent of the speed but diminished slightly as the load was increased, and might be stated to be approximately ·05 at 15 lbs. per square inch, diminishing to ·033 at 75 lbs. per square inch. By far the most important factor, however, in determining the friction was the rate of lubrication, in fact, it was conclusively shown that the presence of friction meant non-lubrication. The following table shows the coefficient of friction at the different pressures and speeds:—

FRICTION OF A COLLAR BEARING.

(As Condensed by Prof. Unwin from Mr. Tower's Experiments.)

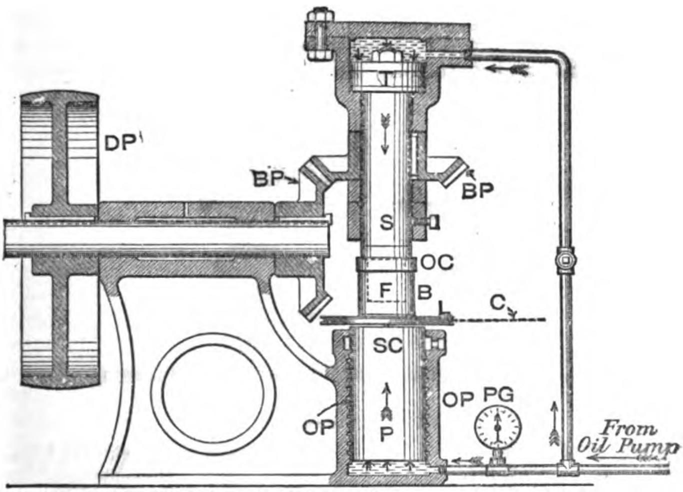
Intensity of Pressure, P, in lbs. per square inch.	SPEED IN REVOLUTIONS PER MINUTE.				
	50	70	90	110	130
15	μ = ·045	μ = ·065	μ = ·043	μ = ·054	μ = ·064
30	,, ·037	,, ·048	,, ·050	,, ·049	,, ·048
45	,, ·036	,, ·040	,, ·036	,, ·036	,, ·037
60	,, ·029	,, ·038	,, ·036	,, ·037	,, ·041
67	,, ·035	,, ·033	,, ·035	,, ·036	,, ·038
75	,, ·035	,, ·034	,, ·035	,, ·035	,, ·036
82	,, ·034	,, ·032	,, ·035	...	...
90	,, ·031	,, ·044	...	...	...

Mr. Thornycroft (the torpedo boat builder) said, that he limited the pressure on his thrust bearings to about 50 lbs. per square inch, and thus the limit of 70 to 80 lbs. arrived at by these experiments, received confirmation from his extensive practical experience of similar collar bearings. The pressures which can, however, be thus carried, depend (1) on the hardness and truth of the rubbing surfaces.* (2) On the freedom with

* Thus, hardened steel working in a dense cast-iron bearing when well lubricated is capable of withstanding a greater pressure per square inch than anything else.

which the lubricant can get in between the rubbing surfaces. This is often assisted by dithering or trembling or alternate pressure and relief, such as takes place at the thrust block of a steamer. (3) On the facilities for dissipating the heat generated through friction by admitting air freely to the bearing, since the rate at which heat was generated constituted the true limit to the load which a bearing will carry.

Friction of a Pivot Bearing.—The experiments on this kind of bearing formed the fourth report of the Research Committee, which was presented to the Institution of Mechanical Engineers in March, 1891. From the two following figures with index



Apparatus for Ascertaining the Friction of a Pivot Bearing.

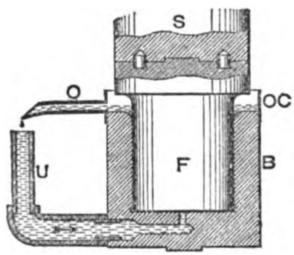
Index to Parts.

F	represents	Footstep.	P G	represents	Pressure Gauge.
B	,,	Bearing.	O C	,,	Oil Chamber.
O P	,,	Oil Press.	S	,,	Upright Shaft.
P	,,	Plunger.	T	,,	Top of Shaft.
S C	,,	Steel Centre.	B P	,,	Bevel Pinions.
L	,,	Lever Plate.	D P	,,	Driving Pulley.
C	,,	Chain.			

to parts, and the following abbreviated description, the student will have no difficulty in comprehending how these importa[nt] experiments were carried out. The footstep, F, and its bearing B, were flat-ended and of 3 inches diameter. They were presse[d]

together with a known force by the aid of a small hand oil pump. The oil from this pump (which was fitted with an air-vessel) passed below the plunger, P, of the oil press, O P, and, at the same time, it acted upon the top, T, of the vertical shaft, S, to the lower end of which the footstep was fixed in the manner shown by the smaller figure. The pressure of the oil thus supplied from the pump was indicated by the pressure gauge, P G. Into the top of the plunger, P, there was inserted a piece of hard steel having a conical centre or centre-pop, wherein rested a hard steel centre, S C, screwed into the under side of the lever plate, L, which carried the bearing, B.

A small chain, C, was fastened to this circular plate, L, and lay in the groove turned in its periphery. The other end of the chain was so connected to a spring-balance (not shown) that any tendency of the plate to rotate (due to the friction between the



METHOD OF LUBRICATING FOOTSTEP AND BEARING.

INDEX TO PARTS.

S represents Shaft.		U represents Upright Oil Pipe.	
F ,, Footstep.		O C ,, Oil Chamber.	
B ,, Bearing.		O ,, Overflow Pipe.	

footstep and the bearing), stretched the balance and thereby the frictional moment between the footstep and its bearing was measured in inch-pounds. The upright shaft, S, received motion through the two bevel pinions, B P, a horizontal shaft and the driving pulley, D P, which was connected by a belt to a suitable motor. The lubrication of the footstep and its bearing was carried out automatically; for the arrangement shown in the annexed figure acted like an oil pump. The mineral oil from the pipe, U, passed freely by gravity to the centre of the footstep, then radially along a diametrical groove, spirally over the flat surface, and finally it was forced up the sides of the bearing to the oil chamber, O C, from which it again passed to the pipe,

U, by the overflow, O. In fact, the faster the speed of rotation the quicker was the circulation of the oil.*

Results of the Experiments.—A series of experiments was first made with a steel footstep on a manganese bronze bearing, at speeds of 50, 128, 194, 290, and 353 revolutions per minute with loads varying from 20 to 160 lbs. per square inch of the flat surface. The manganese bronze bearing was then replaced by one with a white metal bearing surface, and observations of the friction at the various loads were made at 128 revolutions per minute. The coefficient of friction was obtained by dividing the readings of the spring balance as ascertained in inch-pounds by the total load on the bearing, or from the formula:—

$$\mu = \frac{S \times L}{P \times A}.$$

Where S = spring-balance reading in pounds.
„ L = leverage of chain.†
„ P = pressure on the gauge shown in pounds per square inch.
„ A = area of bearing in square inches.

From the results thus obtained, it was found that the coefficient of friction was slightly larger with the white metal than with the manganese bronze, but the difference was so small that the results may be looked upon as identical. Since the friction was mainly between *oil* and metal, instead of between *metal* and metal, it should be independent of the nature of the metal. Hence, it may be urged that with a perfect system of lubrication, applied under pressure to a bearing by means of a force pump, it should not matter much of what material the bearing and shaft or pivot are composed, so long as they are perfectly smooth and true. An examination of the results (see the accompanying set of curves) also show that the higher the speed the less the coefficient of friction became—*e.g.*, from ·0196 with a load of 20 lbs. per square inch at 50 revolutions per minute it fell to ·0167

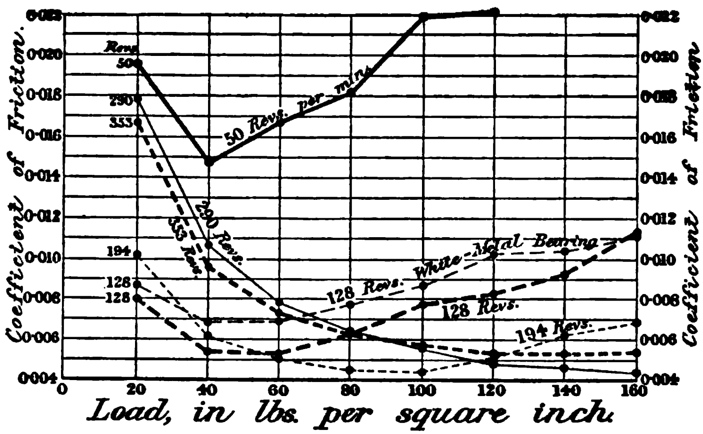
* It is worthy of mention that two opposite radial grooves were found to act better than three or four or any other number.

† The true leverage of the chain is the actual leverage divided by the distance from the centre of the shaft to the centre of frictional resistance, which was assumed to be in this case 1 inch from the centre of the shaft. The centre of frictional resistance being assumed to be 1 inch from the centre of the shaft, we have:—

$$F \times 1 = S \times L$$

i.e., the moment of frictional resistance = S × L.

at 353 revolutions per minute with the same load; and from ·0221 at 50 revolutions and 120 lbs. load to ·0054 at 353 revolutions and the latter load. Unfortunately, we think it was not proved how much of this reduction of the coefficient was due to increase of speed *per se;* or whether the lower coefficient could not have been got at the lower speeds if equally good lubrication had been maintained. Some of this reduction at least (in the



CURVES SHOWING RESULTS OF EXPERIMENTS ON PIVOT FRICTION.*

absence of direct observation on the point) may be put down to the better lubrication which the bearing or pivot automatically received at the higher speeds. Also, in actual practice, whenever dithering or trembling comes into play, the lubricant gets more readily between the surfaces and thus produces more thoroughly the effect of liquid friction.

In the discussion which took place on the above reports, it was pointed out that the introduction of two or more loose, hardened, steel washers between the bottom of the footstep and its bearing, or between a collar and its bearing, enabled heavier loads to be carried than without them. A difference of opinion was expressed as to their precise action. We think, however, that Mr. Tower's explanation was the best, viz.:—that only one pair of

* It would have been better, if after marking the points showing the several observations, smooth curves had been drawn through the mean positions. We have, however, reproduced the above figure direct from the *Proceedings of the Institution of Mechanical Engineers.*

these interposed discs were rubbing at one time, but when these became heated the smallest tendency to seize occasioned more friction between the working pair than between some other pair; consequently, these latter took up the work and thus the work was alternately divided between the several pairs of discs, giving time for each pair to cool before they again come into action.

Experiments on the Friction of Railway Brakes.*—In 1878, Captain Douglas Galton and Mr. George Westinghouse carried out some careful experiments on the friction of railway brakes. The brake blocks were made of cast-iron and the wheels had steel tyres. The pressure, and also the friction, between the brake blocks and peripheries of the wheels were automatically recorded by means of hydraulic gauges. Two series of experiments were made; the first, to determine the coefficient of friction between the brake blocks and the tyres, which we shall term the "brake coefficient"; and the second, to determine the coefficient of friction between the wheels and the rails, when the former were "skidded," or prevented from rotating, which we shall term the "rail coefficient." From these experiments, the brake coefficient was generally greater with low than with high speeds. Thus, immediately after the application of the brakes, the brake coefficient was 0·18 for a speed of 17 miles per hour, while at a speed of 47½ miles per hour, the coefficient was only 0·132. After the brakes had been on for 5 seconds the coefficients at these speeds were 0·157 and 0·07 respectively. When the brakes had been on for 15 seconds, the coefficients were further reduced to 0·11 and 0·055 respectively. Thus we see, that the brake coefficient not only diminished as the speed increased, but diminished the longer the brake had been in contact with the wheel. As the speed decreased, the friction between the wheel and the brake continued to increase, until it became equal to the friction between the wheel and the rail. Then the wheel ceased to rotate and skidded along the rail. The "rail coefficient" was much lower than the "brake coefficient" and increased as the speed decreased. This increase was slow at first but increased greatly as the speed got less, until, when the carriage was about to stop, or just before skidding, it became even greater than the "brake coefficient." The rail coefficient was also found to be greater with steel tyres on iron rails at high speeds, than with steel tyres on steel rails.†

* See *Proc. Inst. M.E.*, June and October, 1878, and April, 1879.

† In *The Practical Engineer* for July 20, 1894, p. 49, it is stated that soft forged steel brake shoes have been proved to last much longer, wear the wheels less, and to be quite as effective as cast-iron.

Friction between Water and Bodies Moving through it.*— The frictional resistance between water and bodies passing through it has been investigated by Col. Beaufoy in a memorable series of experiments carried out in the Greenland Dock, near London, early in this century, and more recently by Dr. William Froude at Chelston Cross, and by Dr. Tideman at Amsterdam.

Dr. Froude's experiments, being the most thorough and conclusive, and those most commonly referred to in the calculations involved in the resistance of ships, we shall briefly describe them, as well as give a few of his results.

They were made in a still water tank 278 feet long, 36 feet wide, and 8 feet 9 inches deep.

The surfaces experimented upon were wooden planks, $\frac{3}{16}$ inch thick, varying from 1 foot to 50 feet long, having both bow and stern sharpened so as to eliminate resistances other than frictional. These planks (with their upper edges $1\frac{1}{2}$ inches below the water level) were suspended from a carefully balanced framework (free to swing without friction fore and aft) attached to a dynamometric truck (set in motion by an endless steel rope) on rails fixed over the tank and running its entire length. The resistance of the specimen was communicated through a spiral spring to a lever, actuating a pen which recorded the tension on a cylinder revolving synchronously with the wheels of the truck. The time was separately registered by means of a pen connected with a chronometer.

Dr. Froude set himself to determine :—

1. The law of the variation of the resistance in terms of the velocity.

2. The law of the variation of the resistance in terms of the length of surface.

3. Variations of resistance with varying qualities of surface.

The second of these problems requires a word of explanation. Whereas, with short lengths, the resistance varied sensibly as the squares of the velocities, the rate of variation was found to fall continuously as the lengths were increased, until it became as low as the 1·83 power of the velocities with a specimen of 50 feet, which was the greatest length experimented upon. Dr. Froude has pointed out, that the cause of this diminution is to be sought in the effect of the forward motion, imparted by the friction of the surface to the stream lines in contact with it, or nearest to it. These stream lines have consequently a lower velocity, relatively

* This latter part of Lecture VI. was kindly contributed by Mr. Robert Caird, of Messrs. Caird & Co., Shipbuilders and Engineers, Greenock.

TABLE I.

Surface.	Length of the Surface or Distance from Cutwater.											
	2 Feet.			8 feet.			20 feet.			50 feet.		
	A.	B.	C.	A.	B.	C.	A.	B.	C.	A.	B.	C.
Varnish, . .	2·00	0·41	0·390	1·85	0·325	0·264	1·85	0·278	0·240	1·83	0·250	0·226
Paraffin, . .	1·95	0·38	0·370	1·94	0·314	0·260	1·93	0·271	0·237	...	...	...
Tinfoil, . .	2·16	0·30	0·295	1·99	0·278	0·263	1·90	0·262	0·241	1·83	0·246	0·232
Calico, . .	1·93	0·87	0·725	1·92	0·626	0·504	1·89	0·531	0·447	1·87	0·474	0·423
Fine Sand, . .	2·00	0·81	0·690	2·00	0·583	0·450	2·00	0·480	0·384	2·06	0·405	0·337
Medium Sand, .	2·00	0·90	0·730	2·00	0·625	0·488	2·00	0·534	0·465	2·00	0·488	0·456
Coarse Sand, . .	2·00	1·10	0·880	2·00	0·714	0·520	2·00	0·588	0·490	...	...	...

to the specimen being towed past them, than the undisturbed water.

The law of variation in terms of length of surface has not yet been satisfactorily investigated for lengths beyond 50 feet, and further experiments are very desirable.

The foregoing table sums up the results of these frictional experiments. The columns A, B, and C respectively refer to:—

A. The power of the velocity, to which the resistance is sensibly proportional.

B. Resistance in pounds per square foot of surface taken as a mean resistance over the whole length.

C. Resistance per square foot taken at the specified distances abaft the cut water which are given at the head of the columns.

The resistances in this Table are those due to a velocity of 600 feet per minute.

The following table is deduced from these experiments and is in a form suitable for the Naval Architect :—

TABLE II.

FROUDE'S FRICTIONAL CONSTANTS FOR SALT WATER, PARAFFIN OR SMOOTHLY PAINTED SURFACES.

Length of Vessel or Model in Feet.	Coefficient of Friction. μ	Power according to which Friction Varies. n	Length of Vessel or Model in Feet.	Coefficient of Friction. μ	Power according to which Friction Varies. n
8	·01197	1·825	80	·00933	1·825
9	·01177	,,	90	·00928	,,
10	·01161	,,	100	·00923	,,
12	·01131	,,	120	·00916	,,
14	·01106	,,	140	·00911	,,
16	·01086	,,	160	·00907	,,
18	·01069	,,	180	·00904	,,
20	·01055	,,	200	·00902	,,
25	·01029	,,	250	·00897	,,
30	·01010	,,	300	·00892	,,
35	·00993	,,	350	·00889	,,
40	·00981	,,	400	·00886	,,
45	·00971	,,	450	·00883	,,
50	·00963	,,	500	·00880	,,
60	·00950	,,	550	·00877	,,
70	·00940	,,	600	·00874	,,

Frictional Resistance of a Ship Propelled through Sea Water.—The frictional resistance of a ship is readily calculated from Table II. by the formula:—

$$R = \mu S V^n$$

Where, R = Resistance in pounds.
μ = Coefficient of friction.
S = Wetted surface in square feet.
V = Velocity in knots.
n = The power according to which friction varies.

The total resistance of a ship when propelled through water is composed of:—

1. Frictional resistance.
2. Eddy-making resistance.
3. Wave-making resistance.

The total resistance of a model is measured by a dynamometric apparatus identical with that described above. From it, the frictional resistance calculated from the table is deducted and the balance is the eddy-making and wave-making resistance (or residuary resistance). The calculation of the residuary resistance of an actual ship from that of a model follows what is called Froude's Law of Comparison, which, briefly stated, is:—

If the linear dimensions of a ship are λ times those of its model, and if, at the velocities v_1, v_2, v_3 . . . of the model in water, the resistances are r_1, r_2, r_3 . . . then the resistances R_1, R_2, R_3 . . . of the ship, at the velocities V_1, V_2, V_3 . . . (which are respectively equal to $v_1\sqrt{\lambda}$; $v_2\sqrt{\lambda}$; $v_3\sqrt{\lambda}$ . . .) will be $R_1 = \lambda^3 r_1$; $R_2 = \lambda^3 r_2$; $R_3 = \lambda^3 r_3$ . . .

Example.—Applying the foregoing to the following case:—

Model 10 ft. long;	1·194 ft. broad;	0·555 ft. draught of water.	
Ship 360 „ ;	43 „ ;	20 „	„

$$\lambda = 36;\ \sqrt{\lambda} = 6;\ \lambda^3 = 46{,}656.$$

Calculation of Frictional Resistance:—

μ = ·00888 (from Table II.)
S = 24,500 square feet.
V = 12 knots.
$V^{1·825}$ = 93·219.
$R = \mu S V^{1·825}$ = 20,280·7 lbs. = 9·053 tons.

Residuary Ship Resistance.—If at a velocity of 3·378 feet per second (equal to 2 knots) the tank trial of the model gives a residuary resistance $r = \cdot 23$ lb., the corresponding speed of the ship will be:—

$$v\sqrt{\lambda} = 3\cdot378 \times 6 = 20\cdot268 \text{ feet per second.}$$
$$\text{,,} \qquad \text{,,} \qquad = 1216 \text{ feet per minute.}$$
$$\text{,,} \qquad \text{,,} \qquad = 12 \text{ knots.}$$

And the residuary resistance of the ship at that velocity will be:—

$$\lambda^3 r = 46{,}656 \times \cdot23 = 10{,}731 \text{ lbs.} = 4\cdot79 \text{ tons.}$$

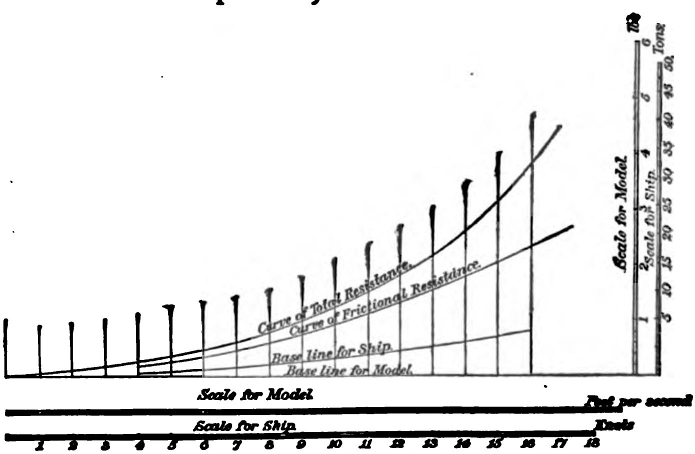
Total Ship Resistance:—

Frictional resistance (as above) =	9·053 tons.
Residuary resistance (,,) =	4·790 ,,
Total resistance =	13·843 ,,

Reducing this to horse-power it becomes :—

$$\frac{13\cdot85 \times 2240 \times 1216}{33{,}000} = 1142\cdot6 \text{ Effective Horse-power.}$$

And, with a propulsive co-efficient of ·5 or 50 per cent. } = 2285·2 Indicated Horse-power.



TOTAL AND FRICTIONAL RESISTANCE OF A SHIP PROPELLED THROUGH SEA WATER.

The calculation may, otherwise, be made directly from the total resistance of the model, correcting for surface friction by the method elaborated by Mr. R. E. Froude in his paper read before the Institute of Naval Architects in 1888.

The above diagram shows the results of a similar calculation in graphic form. This figure is reproduced from an actual diagram worked out from a tank trial where λ (or the linear ratio of dimensions between ship and model) was equal to 28.

Lecture VI.—Questions.

1. Give a concise account of General Morin's, Hirn's, and Thurston's experiments on friction, and explain wherein their conclusions fell short of the results arrived at by the Institution of Mechanical Engineers.

2. Give an account of some experiments on the friction of a well lubricated journal, and state what has been ascertained as to the magnitude of the friction under varying loads, temperature, and velocity. Also state what you know as to the intensity of the pressure at different points of the bearing surface. (S. & A. Mach. Const. Hons. Exam., 1885).

3. Give the results of some experiments which have been made to determine the coefficient of friction in a well lubricated bearing, and the greatest pressure to which the bearing may be subjected. State also at what portion of the surface of the bearing the lubricant should be introduced. Describe and give a sketch of the construction of a bearing for a shaft which is required to run at a very high speed, assuming that the revolving parts cannot be perfectly balanced. (S. & A. Mach. Const. Hons. Exam., 1890).

4. Describe and sketch any form of machine for measuring the friction of lubricated journals. Show how you would deduce from it the coefficient of friction. (C. & G. of L. Mech. Eng. Hons. Exam., 1892).

5. Give a short account of Mr. Beauchamp Towers experiments and results on the friction of collar bearings.

6. State what you know about the friction of pivot bearings. Sketch and describe any apparatus which has been used for determining the same.

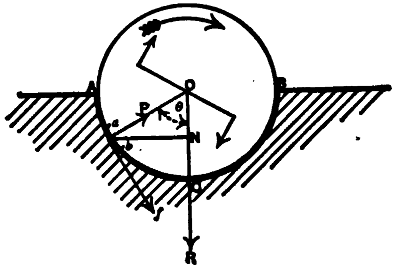
7. What is your idea of the most perfect system of lubricating engine bearings? Give reasons for your answer, with sketches.

8. State what you know about the friction between railway carriage wheels and their brakes and the permanent way rails.

9. How is the frictional resistance between a moving ship and sea water determined? What is meant by residuary and total ship resistance? Work out an original example to determine the effective horse-power required to propel a ship 400 feet long, 50 feet broad, and 25 feet draught at 15 knots, assuming the constants, &c., given in the lecture.

LECTURE VII.

Calculation of Work Lost by Friction in Journals.—If Coulomb's laws of friction be applied to the case of cylindrical surfaces, then the frictional resistance to rotation of a cylindrical journal in its bearing (measured along its tangent to the surfaces in contact) would be $F = \mu R$; where, R is the total normal pressure acting on the journal. This would probably be the case if the journal and its bearing were in contact along a very narrow longitudinal area parallel to the axis of the journal. If, however, the journal and bearing are well worn and a good fit, then contact will take place over a considerable area, and the distribution of pressure will vary from point to point.



Friction of Journals.

Suppose we consider a well fitting horizontal cylindrical journal and its bearing, acted on by vertical forces, the resultant of which is R.

Let p = intensity of the normal pressure on any longitudinal area—for example, $a\,b$.

„ f = the friction along the narrow longitudinal area at $a\,b$.

„ l = the length of the journal.

Then, $p \times a\,b \times l$ = Total normal pressure on the area $a\,b$.

Resolving vertically, we get:—

$$R = \Sigma\,(p \cos\theta \times a\,b \times l).$$

Or,

$$R = l\,\Sigma\,(p \cos\theta \times a\,b) \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If, μ = Coefficient of friction between the journal and its bearing.

And, F = Total friction over the whole bearing.

Then, $F = \Sigma f = \mu l \Sigma p \times ab$ . . . . (2)

If we knew the law according to which p varies from point to point, it would be an easy matter to calculate the frictional resistance in this case. When the journal and its bearing are well worn and a good fit, we may assume that the intensity of pressure at any point will vary as the vertical distance of the point below the diameter A B :—

i.e., $$p \propto \mathrm{ON}.$$

But this is the same law as that which would be followed by a heavy liquid enclosed in the semi-cylindrical space A C B, the total weight of the liquid being R.

We know, that in such a case, the total normal pressure on the cylindrical surface A C B, would be = *area of surface* A C B × *depth of c.g. of surface below* A B × *weight of a cubic unit of the liquid.*

Let, d = diameter of journal.

Then, $$\left.\begin{array}{c}\textit{Area of semi-cylindrical}\\ \textit{surface } \mathrm{ACB}\end{array}\right\} = A = \tfrac{1}{2}\pi d l.$$

And, *Depth of c.g. of surface below* A B $= \bar{x} = \dfrac{d}{\pi}$.

We now require to find the weight of a cubic unit of the hypothetical liquid in A C B.

Let, w = weight of a cubic unit.

Then,

Vol. of semi-cylinder A C B × w = R.

$$\therefore \qquad \tfrac{1}{2}\frac{\pi}{4}d^2 l \times w = R.$$

$$\text{Or,} \qquad w = \frac{8R}{\pi d^2 l}.$$

Hence, $$\left.\begin{array}{c}\textit{The total normal pressure}\\ \textit{over whole surface } \mathrm{ACB}\end{array}\right\} = A\bar{x}w.$$

i.e., $$\Sigma p \times ab = \tfrac{1}{2}\pi d l \times \frac{d}{\pi} \times \frac{8R}{\pi d^2 l} = \frac{4}{\pi}R.$$

But from Equation (2) . . . . $F = \mu\, l \sum p \times a\, b.$

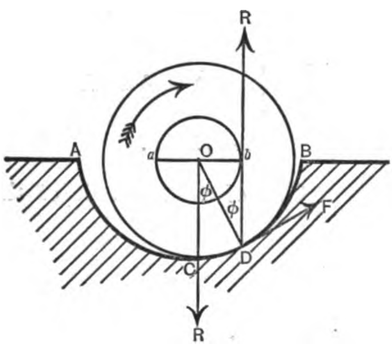
Hence, ... $F = \mu \frac{4}{\pi} R = \frac{4}{\pi} \mu R.$

$\therefore$ The frictional moment, $M = F \times \frac{d}{2} = \frac{2}{\pi} d\, \mu\, R.$

$\therefore$ Work lost in friction in one turn of journal $\Big\} = M \times 2\pi = 4\, d\, \mu\, R.$ (I)

This result is only true on the assumption that the normal pressure varies as the depth of the point below A B, and since the surfaces are cylindrical, it is doubtful if μ has the same value as in the case of plane surfaces.

If the bearing be so well worn that its radius of curvature is slightly greater than that of the journal, we may suppose contact to occur along a narrow longitudinal area parallel to the axis of the journal. This state of affairs is shown in an exaggerated manner, by the annexed figure. The small area on which the journal bears, is not situated exactly at C (the lowest part of the bearing), but at or near D, such that the $\angle$ C O D = ϕ, the "angle of friction." This is due to the shaft tending to climb out of its bearing. When the inclination of the surfaces in contact is equal to the angle of friction, then slipping takes place. This occurs when the shaft bears at D, since the tangent plane at that place is inclined at an angle, ϕ, to the horizon.



FRICTION OF A WORN JOURNAL.

Let R = Resultant force acting on the shaft, which for the present may be supposed to act vertically downwards through the centre of the shaft. At D, introduce a force equal, parallel, and opposite in direction to R. Then this is the *resultant reaction* (*i.e.*, the resultant of the *normal* reaction and friction) at the point D. Through O, draw the common perpendicular O b, to the forces R. Then those two forces form a couple, whose moment = R $\times$ O b. This frictional moment resists the rotation of the shaft.

$\therefore$ $$M = R \times O\,b.$$

But, $$O\,b = O\,D \sin\varphi = \frac{d}{2}\sin\varphi.$$

$\therefore$ $$M = \tfrac{1}{2}\,R\,d\sin\varphi.$$

Hence,

$$\left.\begin{array}{r}\textbf{Work lost in friction in}\\ \textbf{one turn of journal}\end{array}\right\} = M \times 2\pi = R\,\pi\,d\sin\varphi. \quad (\mathrm{II}_a)$$

Where, $$\sin\varphi = \frac{\tan\varphi}{\sqrt{1+\tan^2\varphi}} = \frac{\mu}{\sqrt{1+\mu^2}}.$$

For well lubricated journals φ is very small, so that we may assume that $\sin\varphi = \tan\varphi = \mu$.

Then, $$\left.\begin{array}{r}\textbf{Work lost in friction in}\\ \textbf{one turn of journal}\end{array}\right\} = M \times 2\pi = \pi\,d\,\mu\,R. \quad (\mathrm{II}_b)$$

For practical purposes it is more convenient to use formula (II_b) than (I) or (II_a), remembering that μ is a special coefficient for journals, to be determined by experiment.

We have seen, that the resultant reaction of the bearing at D is R and acts vertically upward. This reaction is tangential to a small circle, $a\,b$, which can be described about O, and is spoken of as the "*friction circle.*" Let d_1 be the diameter of the friction circle.

Then, $$R \times \frac{d_1}{2} = M = \tfrac{1}{2}\,d\,\mu\,R.$$

$\therefore$ $$d_1 = \mu\,d \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\mathrm{III})$$

i.e., Diameter of friction circle = μ × diameter of journal.

Hence, in cases of journal friction it is often convenient to draw the friction circle, and then the direction of the resultant reaction of the bearing may be easily determined. This resultant reaction is always a tangent to the friction circle.

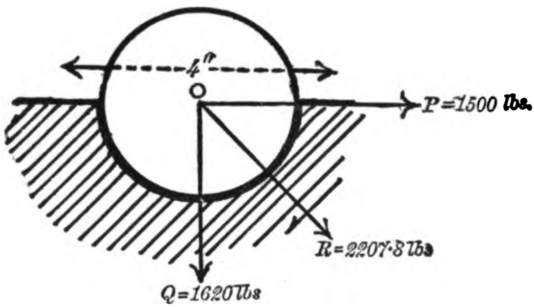
If the bearing has a cover, the resultant force, R, must be increased by the tension in the bolts holding down the cover when calculating M.

EXAMPLE I.—A horizontal shaft, 4 inches in diameter, resting in bearings at its ends, transmits power to various machines by means of belts passing over pulleys keyed to the shaft. The tension in the belts causes a horizontal force of 1,500 lbs., and a vertical downward force of 500 lbs. in a plane at right angles to the shaft. The weight of the shaft and pulleys is 10 cwts.

Coefficient of friction between the shaft and its bearings is 0·07. Find the horse-power lost in friction, the shaft making 100 revolutions per minute.

ANSWER.—The forces acting on the shaft are (1) a horizontal force of 1,500 lbs.; (2) a vertical force of $500 + 10 \times 112 = 1,620$ lbs. Since these forces act at right angles to each other, the resultant pressure on the bearings will be:—

$$R = \sqrt{P^2 + Q^2} = \sqrt{1500^2 + 1620^2} = 2207 \cdot 8 \text{ lbs.}$$



FORCES ACTING ON A SHAFT.

Each bearing may or may not sustain equal shares of this resultant load. This will depend on the arrangement of the pulleys on the shaft. Nevertheless, we may consider only one bearing of the shaft, and suppose the resultant load on this bearing to be R = 2207·8 lbs. Hence, from the formula already obtained, we get:—

$$\left.\begin{array}{l}\textit{Work lost in friction in} \\ \textit{one turn of shaft}\end{array}\right\} = \pi d \mu R.$$

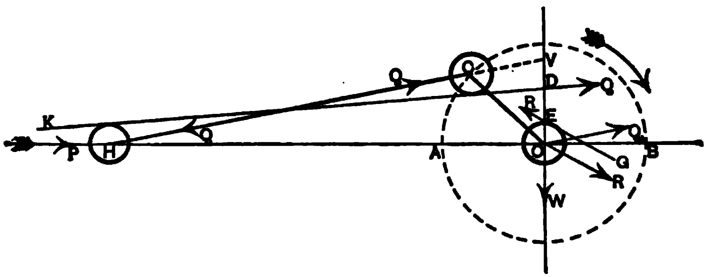
$$\therefore \text{ H.P. lost in friction } = \frac{\pi d n \mu R}{33,000},$$

$$\text{,, } \quad \text{,, } \quad = \frac{\frac{22}{7} \times \frac{4}{12} \times 100 \times \cdot 07 \times 2207 \cdot 8}{33,000},$$

$$\text{,, } \quad \text{,, } \quad = \cdot 49.$$

Corrections for Twisting Moment on Crank Shaft of Engine.—Let O C be the crank, and H C the connecting-rod of an ordinary direct acting engine. Let P = total effective pressure on cross-head pin at H. Then, it is shown in the author's *Text-Book on*

Steam Engines, p. 158, that the twisting moment on the crank shaft at O is P × O V; where V is the point of intersection of the centre line of the connecting-rod with the line through O perpendicular to centre line, H B, of the engine. This method of calculating the twisting moment at any point of the stroke is adopted when we wish to neglect the friction of the journals at H, C, and O. We shall now determine the effect on the



FINDING THE TWISTING MOMENTS ON CRANK SHAFT.

twisting moment when the friction of the journals is taken into account.

Draw the friction circles for the journals at H, C, and O, as described above.*

Then the resultant pressure on a bearing must be tangential to its friction circle. Therefore, the thrust, Q, along the connecting-rod must be tangential to the friction circle at H, and also to the friction circle at C. If the student considers the direction of motion at H and C, he will observe, that the line of thrust (which is a common tangent to the circles at H and C) must be drawn as shown. Thus, the thrust is actually along K D instead of H V. This takes friction into account so far as the crosshead and crank-pin bearings are concerned.

Let W represent the total vertical load on the crank-shaft, including the weight of the shaft itself. The crank-shaft is then acted on by two forces, Q_1 and W, as shown, Q_1 being equal and parallel to Q along K D. The resultant of these two forces is R, and the reaction of the bearing, which is also R, acts in the direction, G R, parallel to O R, and tangential to the friction circle at O.

* On the figure the friction circles for these three journals have been drawn to a very much exaggerated scale for the sake of clearness.

Let G R cut O V at the point E. Then :—

Effective twisting moment on crank shaft = P × E D.

For, from what has been already said:—

Twisting moment due to thrust Q *along* K D = P × O D.

Where P is the horizontal component of Q, or, what is the same thing, P is the total pressure acting on the piston.

Similarly, since the horizontal component of R = horizontal component of Q = P.

∴ *The resisting moment due to* R *along* G R *is* = P × O E.*

$$\therefore \left.\begin{array}{c}\textbf{Effective twisting moment}\\ \textbf{on crank shaft}\end{array}\right\} = \mathrm{P(OD - OE) = P \times ED.} \quad \mathrm{(IV)}$$

It is evident, that as D approaches E, the effective twisting moment on the crank shaft diminishes, until when D coincides with E there will be no effective moment acting on the crank. This will occur at four positions of the crank; two on either side of centre line of the engine. The angle which the crank makes with the centre line of the engine when D coincides with E, is called the "*dead angle*," and when the crank lies within this angle (on either side of engine centre line), no pressure, however great, applied to the piston will move the engine. If the position of the crank had been taken in any other quadrant, say, in the quadrant V O B, then the direction of the thrust Q_1 would be such that K D, the common tangent to the friction circles at H and C, would be drawn *below* the centre line H C. The direction of the common tangent H D should, however, present no difficulty if the student only pays attention to the direction of rotation of the journal in its bearing.

Rolling Friction.—The resistance which is experienced when a wheel or cylinder is rolled along a rough horizontal plane is called *Rolling Resistance*, or *Rolling Friction*. This resistance is in general much less than sliding friction. It depends on the radius and breadth of the wheel and also on the nature of the surface over which the wheel rolls. It is found by experiment, that the resistance to rolling on a horizontal plane is expressed by a formula of the form

$$\mathrm{P} = \frac{\mathrm{W}}{r}\, c.$$

* The thrust R along E R is in a similar condition to the thrust Q along K D; that is to say, the friction circle at O can be looked upon as a small crank-pin circle and E R the direction of thrust of the connecting-rod.

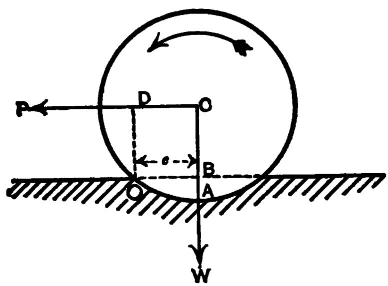
Where, P = Pull required to overcome the resistance (as measured by a horizontal force at the axis of the rolling body).

" W = Total weight of the rolling body.

" r = Radius of rolling body.

" c = Constant *length* measured in the same units as r (depending on the nature of the surfaces in contact).

We can best explain the nature of this resistance, and how the above formula is obtained, by considering the case of a cylinder or broad wheel rolling along an ordinary road. In such a case, the wheel sinks into the ground and leaves a rut along its course. The depth of this rut will depend on the total weight, W, on the wheel, the radius, r, and the softness of the ground. The result of the sinking is, that the force, P, applied at C is employed in continually drawing the wheel over an obstacle at O, in front of the wheel.



ROLLING FRICTION.

Let the height of this obstacle (or, what is the same thing, the depth of the rut) be B A = h. Then, by taking moments about O, we get :—

$$P \times OD = W \times OB,$$

$$\therefore \qquad P = \frac{W}{OD} \times OB = \frac{W}{r - h} \times OB.$$

Put O B = c. Then, since h will, in general, be small compared with r, we may neglect it in the denominator on the right-hand side of the equation, and write

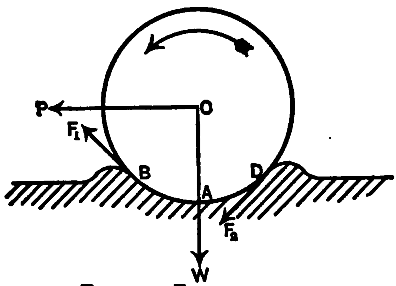
$$P = \frac{W}{r} c.$$

Hence, we see where the constant c comes from, and why it is a *length* of the same units as r.

From the results of the few experiments which have been carried out on rolling resistance on ordinary roads and rails, the

above formula appears to hold good, and c is found to be independent of r. Thus, when r is expressed in *inches*, c may be taken at from ·02 inch to ·025 inch for iron wheels and rails; ·06 inch to ·1 inch for iron wheels on wood, the lower values being taken for the harder woods. For carriage wheels on good macadamised roads c may be taken at ·5 inch; but this value will vary considerably with the nature of the ground, being as high as 3 inches or even 5 inches with soft ground.

If the surface over which the wheel rolls be very soft and elastic the expression for the resistance is more complex and is very difficult of explanation. Take, for example, the case of a wheel rolling along a thick sheet of india-rubber. In such a case the rubber will take the form shown by the figure, being heaped up both in front of and behind the wheel. When the wheel moves it tends to surmount and compress the rubber in front at B. But the rubber at B tends to avoid this compression, and, as a consequence, heaps itself up in front as shown. During the action, the rubber "creeps" over the surface of the wheel at B, and in doing so, a frictional resistance, F_1, is set up which opposes the onward motion of the wheel. Again, as the wheel moves onward, the heaped-up rubber in the rear at D tends to regain its normal state of flatness of surface, and in doing so creeps down the surface of the wheel introducing a frictional resistance, F_2, in the direction shown, which resistance also opposes the progress of the wheel. Thus the wheel is retarded by the two frictional resistances, F_1 and F_2, in addition to the other force necessary to overcome the obstacle in front, as explained in the case of the wheel on ordinary ground.



ROLLING FRICTION ON A PLIABLE SURFACE.

Tractive Force.—By *tractive force* or *traction* is meant the effort necessary to draw a carriage or train along a level road or rail against the total frictional resistances. This total resistance includes axle and rolling frictions, the friction between the flanges of the wheels and the rails and the resistance of the air, &c.

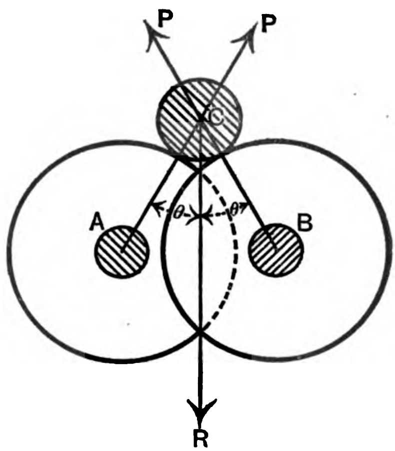
The following table gives the tractive force in lbs. per ton for different roads:—

TRACTIVE FORCE FOR DIFFERENT ROADS.

Paved roads,	33 lbs. per ton.
Macadamised roads,	44 to 67 ,, ,,
Gravel roads,	140 ,, 150 ,, ,,
Common earth roads,	200 ,, 250 ,, ,,
Railway trains (moving slowly),	2 ,, 8 ,, ,,

In the case of railway trains, the resistance of the air increases very rapidly with the speed of the train and with the velocity of the wind, so that at high speed the tractive force required to keep the train moving along a horizontal line may be as high as 50 lbs. per ton.

Anti-Friction Wheels.—These wheels are so arranged that their circumferences form a bearing for an axle or shaft. The axles A and B of the two anti-friction wheels are placed near to each other, and the axle, C, whose friction it is desired to reduce, rests on their circumferences. By this means, the resistance to the rotation of C is greatly diminished, and hence the arrangement is often resorted to in the case of philosophical and other delicate apparatus where the frictional resistances have to be reduced to a minimum. The contrivance is neither sufficiently simple nor compact and strong to be adopted for the ordinary bearings of large heavy shafts; but a modification thereof is often met with in the ball bearings of the best constructed driving shafts for foot lathes, for American electric elevators, and for bicycles, tricycles, &c.* With ball bearings the friction is wholly that of rolling friction, but in the ordinary anti-friction wheels the rolling friction may



ANTI-FRICTION WHEELS.

* See pages 91, 92, and 159 of the author's *Elementary Manual of Applied Mechanics* for these applications in the cases of Sir Wm. Thomson's Siphon Recorder and Atwood's Machine, &c. Also the *Electrical Engineer* of New York, Nov. 2nd, 1892, for a description of the ball bearings in the Sprague-Pratt Electric Elevator for High Service Duty.

be neglected and then the friction of the axles, A and B, considered in the following way :—

Let R = Resultant vertical force acting on axle C.
„ D = Diameter of each wheel A and B.
„ δ = „ „ axle A and B.
„ d = „ of the shaft C.
„ 2θ = Angle A C B.
„ P = Pressure on A or B due to R on C.
„ μ = Coefficient of friction between axles A and B and their bearings.

Then, since R must be the resultant of P at A and P at B, we get:—

$$R = 2 P \cos \theta.$$

Or,
$$P = \frac{R}{2 \cos \theta}.$$

Suppose the wheels A and B to make one complete turn. Then C will make n turns, so that:—

$$\pi d n = \pi D.$$

Or,
$$n = \frac{D}{d}.$$

Hence, neglecting friction between C and circumferences of wheels A and B, we get :—

$$\left.\begin{array}{l}\textit{Work absorbed in friction} \\ \textit{in one turn of wheels A} \\ \textit{and B}\end{array}\right\} = \left\{\begin{array}{l}\textit{Work absorbed in friction at axle} \\ \textit{A + Work absorbed in friction} \\ \textit{at axle B.}\end{array}\right.$$

$$\text{„} \qquad \text{„} \qquad = 2 \pi \delta \mu P.$$

$$\text{„} \qquad \text{„} \qquad = \pi \delta \frac{\mu R}{\cos \theta}.$$

But, had the axle C been resting in an ordinary bearing instead of on the circumferences of the wheels A and B, the

$$\textit{Work absorbed in friction would be} = n \pi d \mu R,$$

$$\text{„} \qquad \text{„} \qquad \text{„} \qquad = \pi D \mu R.$$

$$\therefore \frac{\text{Friction with anti-friction wheels}}{\text{Friction without anti-friction wheels}} = \frac{\pi \delta \dfrac{\mu R}{\cos \theta}}{\pi D \mu R} = \frac{\delta}{D \cos \theta}$$

In general, θ ($\frac{1}{2} \angle$ A O B) will be small, so that cos θ will be unity very nearly.

$$\therefore \frac{\text{Friction with anti-friction wheels}}{\text{Friction without anti-friction wheels}} \doteq \frac{\text{dia. of axle A or B}}{\text{dia. of wheel A or B}} = \frac{\delta}{D}.$$

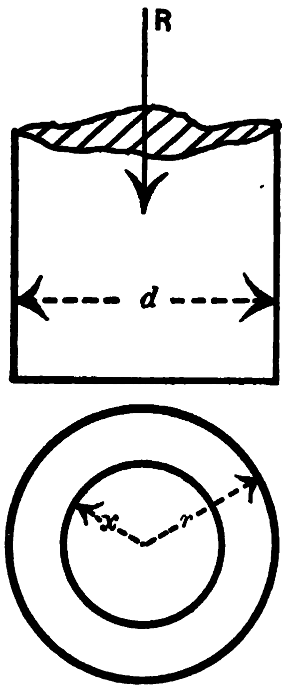
This result agrees closely with experiments carried out on the friction of anti-friction wheels.

Friction of Flat Pivots and Collar Bearings.—When a shaft is subjected to forces parallel to its axis the end of the shaft may terminate in a footstep bearing, or if the shaft has to be continued through its bearing, this axial pressure is provided for by having a collar or collars made on the shaft. If the shaft terminates in a bearing at its end, the end of the shaft may be flat, rounded, or conical, according to circumstances.

We shall first show how to calculate the work lost in friction in the case of a flat pivot or collar bearing.

Let R = Total thrust on shaft.

„ p = Intensity of pressure over bearing $= \dfrac{R}{\text{area of bearing.}}$



Friction of Flat Pivots and Collar Bearings.

Suppose the thrust, R, to be equally distributed over the whole bearing surface, and let us consider a small annular ring of breadth, $d\,x$, and mean radius, x (see plan).

Then, *Pressure on elementary ring* $= p \times 2\,\pi\,x\,d\,x$.

$\therefore$ *Moment of friction for the elementary ring* $= \mu\,p \times 2\,\pi\,x\,d\,x \times x.$

„ „ „ $= 2\,\pi\,\mu\,p\,x^2\,d\,x.$

$\therefore$ *Moment of friction for whole bearing* $= M = 2\,\pi\,\mu\,p \int x^2\,d\,x.$

Where, $\int x^2\,d\,x$ means the sum of all such terms, as $x^2\,d\,x$, taken over the whole bearing. Consequently, $M = 2\,\pi\,\mu\,p\,\dfrac{x^3}{3}.$

If the bearing be an annular ring or collar of outside radius, r_1, and inside radius, r_2, then:—

$$p = \frac{R}{\pi\left(r_1^2 - r_2^2\right)}.$$

And $$M = 2\pi\mu p\int_{r_2}^{r_1} x^2\,dx.$$

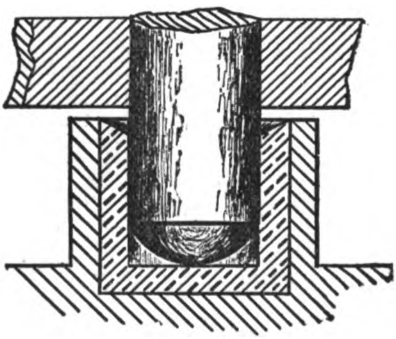
Or, $$M = 2\,\frac{\mu R}{r_1^2 - r_2^2} \times \frac{r_1^3 - r_2^3}{3}.$$

i.e., $$M = \frac{2}{3}\left(\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right)\mu R$$

Or, $$M = \frac{1}{3}\left(\frac{d_1^3 - d_2^3}{d_1^2 - d_2^2}\right)\mu R \quad . \quad . \quad . \quad . \quad . \quad . \quad (V)$$

Where d_1, d_2 refer to the inside and outside diameter respectively.

Hence, Work lost in friction in one turn of collar journal $= M \times 2\pi = \frac{2}{3}\pi\,\frac{d_1^3 - d_2^3}{d_1^2 - d_2^2}\mu R$. (VI)



Rounded Pivot.

In the case of a flat pivot $d_2 = 0$, and then :—

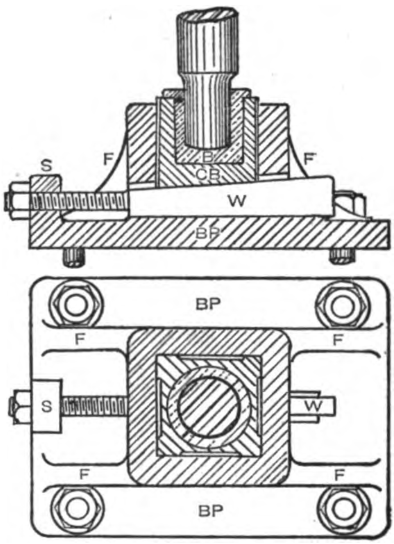
$$M = \frac{1}{3} d_1 \mu R. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (V_a)$$

$\therefore$ Work lost in friction in one turn of flat pivot $= \frac{2}{3}\pi\, d_1 \mu R$. . . . . (VI_a)

Thus, we see that the frictional moment of a flat pivot in its footstep is only $\frac{2}{3}$ of that for a horizontal journal of equal diameter.

By diminishing the diameter of the pivot, the frictional moment will be diminished in the same proportion. Hence, we often find that small pivots are rounded at their lower ends and rest on a flat step in the manner shown by the accompanying figure.

In the case of turn-tables and the vertical posts of cranes where the motion is slow this plan is also adopted. But for the bearings and footsteps of large shafts carrying a heavy load and moving at high speeds, the following style of adjustable footstep and bearing has been found best.



ADJUSTABLE FOOTSTEP.

INDEX TO PARTS.

B	represents	Bearing.
CB	,,	Cast bush.
F	,,	Footstep.
BP	,,	Base plate.
W	,,	Wedge.
S	,,	Screw.

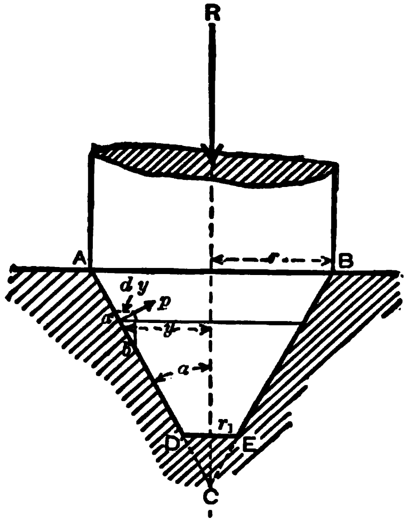
Friction of Conical Pivots.—Sometimes the end of the vertical shaft is made conical—instead of flat or hemispherical—and turns in a step or bearing of corresponding shape. In this case the expression for the frictional moment will be different from the one just obtained for a flat pivot.

Let A D E B represent a section of a conical pivot, or frustrum of a cone, with r and r_1 for its greatest and least radii respectively.

Let the total axial thrust, R, be uniformly distributed over the transverse section at A B. The manner in which this pressure is distributed over the step is not definitely known. The normal pressure may be constant at all points on the step, or it may vary according to some other law. If, however, we assume that the step always remains a good fit for the pivot,

and that the wear is uniform throughout, we can then explain how the pressure, p, varies from point to point.

The wear over any small conical surface on the step depends on the product of the normal pressure, p, and the velocity of



CONICAL PIVOT.

rubbing, v. Hence, if the wear be everywhere the same, we must have:—

$$p\,v = \text{a constant.}$$

But the velocity of rubbing depends on the distance of the small area in question from the axis of rotation. If this distance be y, then:—

$$p\,v \propto p\,y.$$

$$\therefore \qquad p\,y = \text{a constant.}$$

Consequently, the pressure at any point on the step varies inversely as the distance of the point from the axis of rotation.

Hence, if we consider a small conical area of breadth or slant length, $\overline{a\,b}$, and mean radius, y, and if the projection of $\overline{a\,b}$ on a

horizontal plane be denoted by dy; then, if α = half the conical angle ACB; $dy = \overline{ab} \sin \alpha$.

$$\left.\begin{array}{l}\textit{Total normal pressure on}\\ \quad\textit{elementary frustrum}\\ \quad\textit{area bounded by } ab \textit{ as}\\ \quad\textit{a slant side}\end{array}\right\} = p \times 2\pi y \times \overline{ab} = 2\pi py \times \overline{ab}.$$

Now, the sum of the vertical components of all such normal pressures must balance R.

$$\therefore \qquad R = \Sigma\, 2\pi py\, \overline{ab} \sin \alpha = 2\pi py \sin \alpha\, \Sigma\, \overline{ab}.$$

Since py has previously been shown to be a constant it may be written outside the summation sign.

But, $\quad \Sigma\, \overline{ab}$ is clearly $=$ AD.

And, $$AD = \frac{r - r_1}{\sin \alpha}.$$

$\therefore$ $$\Sigma\, \overline{ab} = \frac{r - r_1}{\sin \alpha}.$$

Consequently, $$R = 2\pi py\,(r - r_1).$$

And, $\therefore$ $$py = \frac{R}{2\pi (r - r_1)}.$$

Substituting this value of py in equation (1), we get:—

$$\left.\begin{array}{l}\textit{Total normal pressure on}\\ \quad\textit{elementary frustrum}\\ \quad\textit{area}\end{array}\right\} = \frac{R}{r - r_1}\,\overline{ab} = \frac{R}{(r - r_1)\sin \alpha}\,dy.$$

$$\therefore \left.\begin{array}{l}\textit{Moment of friction for}\\ \quad\textit{elementary frustrum}\\ \quad\textit{area}\end{array}\right\} = \frac{\mu R}{(r - r_1)\sin \alpha}\,y\,dy.$$

$$\left.\begin{array}{l}\textbf{And, Moment of friction}\\ \quad\textbf{for whole bearing}\end{array}\right\} = \frac{\mu R}{(r - r_1)\sin \alpha}\int_{r_1}^{r} y\,dy.$$

Or, $$M = \left(\frac{\mu R}{(r - r_1)\sin \alpha}\right)\frac{r^2 - r_1^2}{2}$$

i.e., $$M = \frac{1}{2}\,\frac{r + r_1}{\sin \alpha}\,\mu R. \quad . \quad . \quad . \quad \text{(VII)}$$

$$\therefore \left.\begin{array}{l}\textbf{Work lost in friction in}\\ \quad\textbf{one turn of conical}\\ \quad\textbf{pivot}\end{array}\right\} = M \times 2\pi = \frac{\pi(r + r_1)}{\sin \alpha}\,\mu R. \quad \text{(VIII)}$$

If the pivot comes to a point at O, then $r_1 = 0$, and then:—

$$M = \frac{1}{2}\frac{r}{\sin\alpha}\mu R \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (IX)$$

EXAMPLE II.—A vertical shaft, 4 inches in diameter, turns on a flat pivot. The weight of the shaft with its wheels, &c., is 2,500 lbs. Find the horse-power lost in friction between the end of the shaft and its footstep, the shaft making 140 revolutions per minute, and the coefficient of friction being taken at 0·08.

ANSWER.—Here $d = 4'' = \frac{1}{3}$ ft., $n = 140$, $R = 2{,}500$ lbs.

Hence, assuming formula VI_a, we get:—

$$\left.\begin{array}{l}\textit{Work lost in friction}\\ \textit{in one turn of}\\ \textit{pivot}\end{array}\right\} = \frac{2}{3}\pi d\,\mu R.$$

$$\therefore \text{ H.P. lost in friction} = \frac{\frac{2}{3}\pi d n\,\mu R}{33{,}000}$$

$$\text{,,} \qquad \text{,,} \qquad = \frac{\frac{2}{3}\times\frac{22}{7}\times\frac{1}{3}\times 140 \times \cdot 08 \times 2{,}500}{33{,}000} = \cdot 59.$$

EXAMPLE III.—If, in the last example, the pivot had been conical instead of flat, the angle at the vertex of the cone being 60°, and the smallest diameter of the pivot $1\frac{1}{2}$ inches, what would then be the H.P. lost in friction, assuming the pressure on the step to vary according to the law stated in the text?

ANSWER.—Here $d_1 = 1\frac{1}{2}''$, $\alpha = 30°$, the other data being same as in last example. Then, by formula VIII. :—

$$\left.\begin{array}{l}\textit{Work lost in friction}\\ \textit{in one turn of pivot}\end{array}\right\} = \frac{\pi(r + r_1)}{\sin\alpha}\,.\,\mu R.$$

$$\text{,,} \qquad \text{,,} \qquad = \frac{\pi(d + d_1)}{2\sin\alpha}\,.\,\mu R.$$

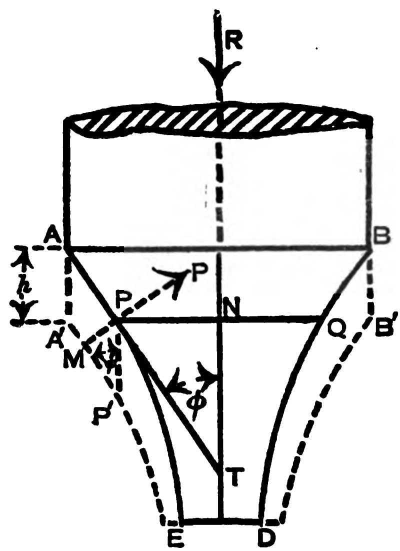
$$\therefore \text{ H.P. lost in friction} = \frac{\pi(d+d_1)n}{2\sin\alpha}\cdot\frac{\mu R}{33{,}000}$$

$$\text{,,} \qquad \text{,,} \qquad = \frac{\frac{22}{7}\times\left(\frac{4}{12}+\frac{1\frac{1}{2}}{12}\right)\times 140\times\cdot 08\times 2500}{2\times\frac{1}{2}\times 33{,}000}$$

$$\text{,,} \qquad \text{,,} \qquad = 1\cdot\dot{2}.$$

Schiele's Anti-Friction Pivot.—The assumption made in our last investigation regarding the uniform wear of the ordinary conical pivot and step is not strictly correct. When such pivots have been at work for some time it is found that the contact between the pivot and its step is very imperfect, due to the unequal wear which naturally arises from the difference of velocities at different parts of the conical surface. Since this is a matter of some importance, especially with certain kinds of instruments and machines, we shall here investigate the proper form to be given to the pivot and its step in order that the vertical wear of the latter may be everywhere the same.

The figure represents a section of such a pivot. During wear of the step let the pivot sink through a vertical depth, A A′ = h. Then, by hypothesis, the vertical wear everywhere will be h; so that any point, P, will, after wear, be at P′, where P P′ = h. The dotted curves represent the outline of the pivot or step after wear, and it is evident that they will be similar to the full curves A P E, B Q D.



SCHIELE'S ANTI-FRICTION PIVOT.

Consider a point, P, on the curve. Draw P N perpendicular to the axis of the shaft. Let P T be the tangent to the curve at the point P, and P M the normal at that point.

Let y = Ordinate P N.
„ ω = Angular velocity of shaft.
„ p = Intensity of normal pressure.
„ μ = Coefficient of friction between pivot and step.

The normal wear at P per unit area is:—

$$PM \propto \mu p \omega y.$$

Now, for a small vertical wear, we may consider the triangles T P N, P′ P M, as similar.

$$\therefore \qquad PT : PN = PP' : PM.$$

Or,
$$PT = \frac{PP' \times PN}{PM},$$

$$,, \quad = \frac{h \times y}{\mu p \omega y}.$$

In this pivot, the vertical wear is supposed to be everywhere the same, therefore, the intensity of normal pressure, p, must be constant for all points on the pivot.

$$\therefore \qquad PT = \frac{h}{\mu p \omega} = \text{a constant.}$$

Consequently, the curve is such, that the length of the tangent, P T, at any point, P, is constant. The curve having this property is known as the "*Tractrix*" or "*Tractory Curve.*" Hence, a suitable form of pivot is obtained by the revolution of such a curve about its axis. This form of pivot was invented by C. Schiele, and is called "*Schiele's Anti-Friction Pivot.*" Such pivots are well adapted for high-speed machinery, the wear being perfectly uniform throughout and giving a very smooth motion.

Calculation of Friction Moment in Schiele's Pivot.

Let R = Total axial thrust on shaft.
,, r = Largest radius of pivot = radius of shaft.
,, r_1 = Smallest ,, ,,

We shall now show, that $p = \dfrac{R}{\pi(r^2 - r_1^2)}$.

Consider a small conical area of the pivot, the mean diameter of which is P Q = $2y$, and length of slant side, ds.

Then, *Area of elementary ring* = $2\pi y\, ds$.

But,
$$ds = \frac{dy}{\sin\varphi}.$$

$$\therefore \qquad \textit{Area of elementary ring} = \frac{2\pi}{\sin\varphi}\, y\, dy.$$

Or,
$$\left.\begin{array}{r}\textit{Total normal pressure}\\ \textit{on elementary ring}\end{array}\right\} = P = \frac{2\pi p}{\sin\varphi}\, y\, dy.$$

Resolving vertically, we get:—

Vertical component of $P = P \sin \varphi = 2 \pi p \; y \, d y.$

$$\therefore \qquad R = 2 \pi p \int_{r_1}^{r} y \, d y = \pi p (r^2 - r_1^2).$$

And, therefore,
$$p = \frac{R}{\pi (r^2 - r_1^2)}.$$

Next, the moment of friction for the elementary ring is:—

$$d M = \mu P y = \frac{2 \pi \mu p}{\sin \varphi} y^2 d y.$$

We have seen that the length of the tangent, P T, at any point, P, on the curve is constant, hence:—

Let t = Length of tangent, P T.

Then,
$$t = \frac{y}{\sin \varphi}.$$

$$\therefore \qquad d M = 2 \pi t \mu p y \, d y.$$

Or,
$$M = 2 \pi t \mu p \int_{r_1}^{r} y \, d y.$$

$$\therefore \qquad M = 2 \pi t \times \frac{\mu R}{\pi (r^2 - r_1^2)} \times \frac{r^2 - r_1^2}{2}.$$

Or,
$$M = \mu R t \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (X).$$

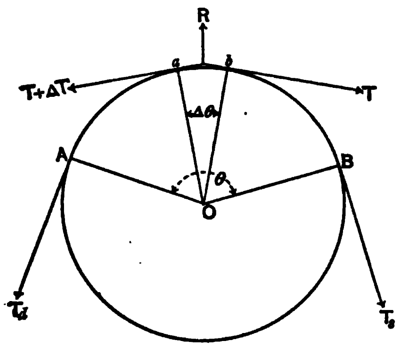
an equation which shows that the friction-moment depends only on the thrust along the shaft and the length of the constant tangent.

The minimum length of the tangent is $t = r$, for then the line, A B, is a common tangent to the curves, A P E, B Q D. Hence:—

$$M_{min} = \mu R r \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (X_a).$$

The friction-moment with this pivot is thus one-half greater than with a flat pivot of the same diameter, but, as already said, the wear is more uniform throughout the bearing surface.

Frictional Resistance between a Belt or Rope and a Flat Pulley.—Let the figure represent a belt stretched over a pulley. Let T_d, T_s denote the tensions in the two parts of the belt not in contact with the pulley. Suppose the belt to be just on the point of slipping on the pulley in the direction B to A, so that



FRICTION BETWEEN BELT AND PULLEY.

$$T_d > T_s.$$

Let θ = A O B = angle subtended at centre, O, by part of belt in contact with pulley; and let μ = coefficient of friction between belt and pulley rim.

Consider the equilibrium of the part of the belt on a very small arc, *a b*, anywhere between A and B, the arc, *a b*, subtending an angle, $\Delta\,\theta$, at the centre, O, of the pulley.

Let the tensions in the belt at *a* and *b* be T + Δ T, and T respectively, so that the increase in tension over the arc, *a b*, is Δ T. The directions of T + Δ T and T will be along the tangents at *a* and *b* respectively.

Let R = Resultant reaction between part of belt, *a b*, and the pulley rim, due to tensions in belt at *a* and *b*.

Then, neglecting the small difference, Δ T, between these tensions, we get:—

$$R = 2\,T\cos\left(90^\circ - \frac{\Delta\,\theta}{2}\right) = 2\,T\sin\frac{\Delta\,\theta}{2}.$$

Since $\Delta\,\theta$ is a very small angle, we may write:—

$$\sin\frac{\Delta\,\theta}{2} = \frac{\Delta\,\theta}{2} \text{ very approximately.}$$

$$\therefore \qquad R = T\,\Delta\,\theta.$$

But, since slipping is about to take place, Δ T must be a measure of the friction over the arc, *a b*.

$$\therefore \qquad \Delta\,T = \mu\,R = \mu\,T\,\Delta\,\theta.$$

Or,
$$\frac{\Delta\,T}{T} = \mu\,\Delta\,\theta.$$

Hence, in the limit, when the arc, $a\,b$, is indefinitely small, we get:—

$$\frac{d\,\mathrm{T}}{\mathrm{T}} = \mu\, d\,\theta.$$

Therefore, for the whole arc, A B, of contact, we get:—

$$\int_{\mathrm{T}_s}^{\mathrm{T}_d} \frac{d\,\mathrm{T}}{\mathrm{T}} = \mu \int_o^\theta d\,\theta.$$

i.e., $$\log_e \mathrm{T}_d - \log_e \mathrm{T}_s = \mu\,\theta.$$

Or, $$\left.\begin{aligned} \log_e \frac{\mathrm{T}_d}{\mathrm{T}_s} &= \mu\,\theta \\ \frac{\mathrm{T}_d}{\mathrm{T}_s} &= e^{\mu\,\theta} \end{aligned}\right\}^{*} \quad \ldots\ldots \quad \text{(XI)}$$

Or,

Since $e = 2{\cdot}7182$ (the base of the Napierian system of logarithms) it is a constant; and since μ is also a constant, we may write the last equation in the form

$$\frac{\mathrm{T}_d}{\mathrm{T}_s} = k^\theta. \quad \ldots\ldots\ldots \quad \text{(XII)}$$

thus showing that:—**The ratio of the tensions increases as the power of the number representing in circular measure the angle subtended by the belt at the centre of the pulley.**

The ratio is thus independent of the radius of the pulley.

The above results are true for a rope wound round a post and also for friction brakes wherein the strap encircles the friction wheel or pulley.

From these results we can understand why a sailor has such a power of holding back a ship at a quay by merely coiling the rope two or three times round a post. For example, when a rope is coiled *once* round a post, let $\mathrm{T}_d/\mathrm{T}_s = 4$. Then, when coiled *twice* round the post, $\mathrm{T}_d/\mathrm{T}_s = 4^2 = 16$; when *thrice* round the post, $\mathrm{T}_d/\mathrm{T}_s = 4^3 = 64$, and so on. This shows how rapidly the resistance increases.

In the formula $\log_e \frac{\mathrm{T}_d}{\mathrm{T}_s} = \mu\,\theta$ we may change into common logarithms by multiplying both sides by 0·434.

$$\therefore \qquad \mathrm{Log}\, \frac{\mathrm{T}_d}{\mathrm{T}_s} = 0{\cdot}434\,\mu\,\theta. \quad \ldots\ldots \quad \text{(XIII)}$$

* θ in these equations is always expressed in circular measure.

EXAMPLE IV.—A rope is wound thrice round a post, and one end is pulled with a force of 20 lbs., what is the greatest pull which can be resisted on the other end of the rope when the coefficient of friction is 0·4?

ANSWER.—Here $T_s = 20$ lbs.; $\theta = 3 \times 2\pi = 6\pi$; $\mu = 0{\cdot}4$.

$$\therefore \qquad \text{Log}\,\frac{T_d}{T_s} = {\cdot}434\,\mu\,\theta.$$

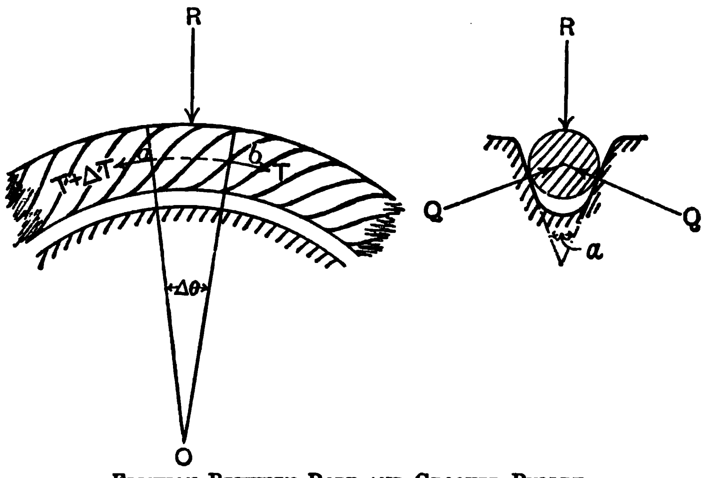
$$\text{,,} \quad \text{,,} = {\cdot}434 \times {\cdot}4 \times 6 \times 3{\cdot}1416$$

$$\text{,,} \quad \text{,,} = 3{\cdot}27229.$$

$$\therefore \qquad \frac{T_d}{T_s} = 1871, \text{ nearly.}$$

$$\text{Or,} \qquad T_d = 20 \times 1871 = \mathbf{37{,}420\ lbs.}$$

Consequently, a force of 20 lbs. at one end of the rope is able to resist a force of about $16\frac{3}{4}$ tons at the other end.



FRICTION BETWEEN ROPE AND GROOVED PULLEY.

Resistance to Slipping of a Rope on a Grooved Pulley.—The grooves round the rims of the pulleys used for hemp or cotton rope drives are usually V-shaped and of such dimensions that the rope, instead of resting on the bottom of the grooves, gets wedged into them and presses on the sides only. By this means a greater resistance is offered to slipping between the rope and the pulley.

Consider a small length, $a\,b$, of the rope subtending an angle, $\Delta\,\theta$, at centre of pulley. Let $T + \Delta\,T$ and T denote the tensions at a and b respectively. Let Q be the pressure between element $a\,b$ of rope and sides of groove, α the angle between the sides of the grooves.

Then, from right-hand figure, we get:—

$$R = 2\,Q \sin \frac{\alpha}{2}.$$

But, in the previous investigation, we saw that

$$R = T\,\Delta\,\theta.$$

$$\therefore \qquad 2\,Q \sin \frac{\alpha}{2} = T\,\Delta\,\theta.$$

Now, the resistance of friction for the element, $a\,b$, is:—

$$\Delta\,T = \mu \times 2\,Q.$$

$$\therefore \qquad \Delta\,T = \mu \times \operatorname{cosec} \frac{\alpha}{2} \times T\,\Delta\,\theta.$$

$$\text{Or,} \qquad \frac{\Delta\,T}{T} = \mu \operatorname{cosec} \frac{\alpha}{2}\,\Delta\,\theta.$$

Proceeding to the limit and integrating for the whole arc embraced by the rope on the pulley, we get:—

$$\int_{T_s}^{T_d} \frac{d\,T}{T} = \mu \operatorname{cosec} \frac{\alpha}{2} \int_0^{\theta} d\,\theta.$$

$$\textit{i.e.,} \qquad \log_e \frac{T_d}{T_s} = \mu\,\theta \operatorname{cosec} \frac{\alpha}{2} \quad . \; . \; . \; . \; . \; . \; . \; . \quad \text{(XIV)}$$

Compared with our previous results for a flat pulley, we see that the logarithm of the ratio of the tensions is increased in the proportion of $\operatorname{cosec} \frac{\alpha}{2} : 1$.

Generally, the groove angle is about 45°, for hemp or cotton ropes, and then:—

$$\operatorname{cosec} \frac{\alpha}{2} = \operatorname{cosec} 22\tfrac{1}{2}^\circ = 2{\cdot}6.$$

Lecture VII.—Questions.

1. Find an expression for the work absorbed in one revolution of an axle on its bearing. An axle is 2 inches in diameter, and the weight pressing it on the bearing is 1,000 lbs., find the number of units of work lost in 100 revolutions. (Take the coefficient of friction as 0·08). *Ans.* 4188·8.

2. Deduce a formula for obtaining (approximately) the work lost in friction in one revolution of a horizontal shaft in its bearing. A horizontal shaft, 9 inches in diameter, is acted on simultaneously by a horizontal force of 3 tons and a vertical force (including its own weight) of 4 tons. Find the horse-power lost in friction when the shaft makes 100 revolutions per minute. The coefficient of friction is 0·07. *Ans.* 5·6 H.P.

3. Explain what is meant by "dead angle" when applied to a steam engine. Explain how you would find it.

4. Explain the nature of "rolling friction," and deduce a formula which approximately represents its amount.

5. What are anti-friction wheels? Find an expression for the work saved by their use over ordinary bearings.

6. Explain, by aid of sketches, how friction is reduced to a minimum in the cases of large crank-shaft bearings and in bicycle bearings.

7. Discuss the advantages and disadvantages of ball bearings. Describe with sketches the construction of an adjustable ball bearing. (S. & A. Mach. Const. Hons. Exam., 1885.)

8. Write down the formula for finding the work lost in friction in one revolution of a vertical shaft turning on a flat pivot. A vertical shaft 3 inches in diameter, and weighing 30 cwt. (including wheels, &c.), turns on a flat pivot. Find the horse-power lost in friction when the shaft makes 100 revolutions per minute, the coefficient of friction being taken at 0·07. *Ans.* ·373 H.P.

9. Explain a formula by means of which the loss by friction on the pivot of a vertical shaft may be calculated, and apply the formula in the following case:—Weight on pivot, 3 tons; diameter of pivot, 4 inches; coefficient of friction, ·01; revolutions per minute, 75. Find the horse-power lost in friction. (C. & G. Mech. Eng. Hons. Exam., 1890.) *Ans.* ·106.

10. Investigate an expression for the moment of friction of a flat pivot, stating the assumption made as to distribution of pressure. Find the H.P. lost by the friction of a footstep bearing, the diameter of which is 4 inches, the total load on it being 3,000 lbs., the number of revolutions 100 per minute, and coefficient of friction ·06. (S. & A. Mach. Const. Hons. Exam., 1887.) *Ans.* ·38.

11. Explain the theory of the anti-friction pivot, and deduce a formula for the work lost in friction in one revolution of the shaft. Draw the curves representing the outline of the pivot.

12. A string is stretched on the circumference of a rough circle; when one of the forces is on the point of preponderance, find the pressure on the circumference, and the tension of the string, at any assigned point. (S. & A. Theor. Mechs. Hons. Exam., 1889).

13. A thread is stretched by two forces, P and Q, over a rough plane curve; when P is on the point of overcoming Q show that $P = Q\,e^{\mu\theta}$, where θ denotes the angle between the normals at the extreme points where the thread is in contact with the curve. A rope is wrapped three and a-half times round a horizontal cylinder, the coefficient of friction between the rope and the cylinder is $\frac{1}{11}$; a weight of 1,000 lbs. is fastened to one end

of the rope, what weight must be fastened to the other end to prevent sliding, the weight of the rope being neglected. Take $e = 2{\cdot}72$. (S. & A. Theor. Mechs. Hons. Exam., 1883.) *Ans.* $Q = P/e^{\cdot} = 135{\cdot}14$ lbs.

14. Deduce a formula for the resistance offered to slipping of a rope round a grooved pulley in terms of groove angle, angle subtended at centre of pulley by the rope and coefficient of friction. Find the maximum ratio of tensions in the tight and slack ends of a rope passing over a grooved pulley, the angle of the groove being 40°, coefficient of friction ·15, arc of pulley embraced by rope $\frac{5}{8}$ of circumference. *Ans.* 5·22.

15. Establish a formula giving the ratio between the tensions at the extremities of a rope which is coiled through a given angle round a post. A rope has its direction changed through two right angles by passing round a grooved guide pulley whose diameter is 12 inches, the diameter of the axle of the pulley being $1\frac{1}{4}$ inches, and the coefficient of axle friction being 0·07. How is the efficiency of the pulley affected by axle friction when a load of 2,500 lbs. is being raised? If the pulley was fixed so that it could not turn, how would the efficiency be affected by the friction of the rope on the pulley, taking the coefficient as 0·6? (S. & A. Hons. Exam., 1895). *Ans.* Efficiency = 98·4 per cent.; efficiency = 15·2 per cent.

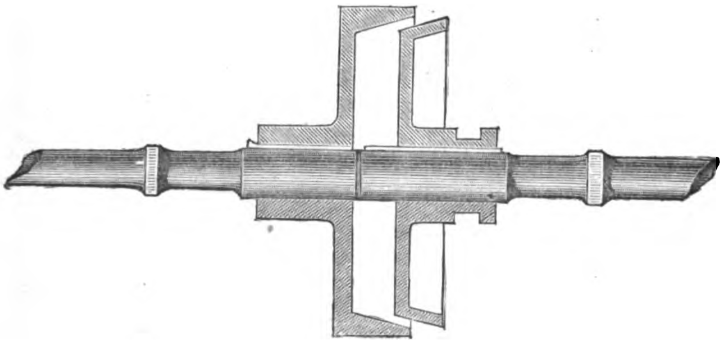
LECTURE VIII.

CONTENTS.—Friction Usefully Applied—Friction Clutches—Frustrum, Addyman's, and Bagshaw's—Weston's Friction Coupling and Brake—Grooved Disc Friction Coupling—Weston's Centrifugal Friction Pulley—Brakes Defined and Classified—Block Brakes—Flexible Brakes—Proper Direction of Brake-Wheel when Lowering a Load—Mathematical Proof—Example I.—Paying-out Brake for Submarine Telegraph Cables—Differential Brake for Lord Kelvin's Deep-Sea Sounding Machine—Dynamometers—Absorption Dynamometers—Prony Brake—Method of Taking Test for Brake Horse-Power—Example II.—Improved Prony Brake—Appold's Compensating Lever—Semicircular Strap Dynamometer—Society of Arts Rope Dynamometer—Advantages of Rope Brake—Tests of Engines with Rope Brake—Transmission Dynamometers—von Hefner-Alteneck or Siemens'—Rotatory Dynamometers—Epicyclic Train; King's, White's, or Webber's—Spring Dynamometers—Ayrton and Perry's and van Winkle's—Hydraulic Transmission Dynamometers—Flather's and Cross'—Tension Dynamometer for Submarine Cables—Questions.

Friction Usefully Applied.*—As we remarked in Lecture X. of our *Elementary Manual of Applied Mechanics*, friction has its advantages as well as its disadvantages. And, although it is the duty of the engineer to reduce friction to a minimum in the case of the bearings of engines, shafting, and machines in general; yet, he has, nevertheless, to devise means for producing a maximum of friction in the case of brakes, blocks, and grips whereby motion has to be arrested gradually or suddenly; or, in the case of friction gearing, pulleys, and clutches whereby power has to be transmitted from one shaft to another. Or, he may have to arrange for a more or less constant retarding force as in the case of absorption dynamometers when used for paying-out submarine cables or for determining the brake-horse power of motors. It will, therefore, be our endeavour, in this Lecture, to describe these various methods of usefully employing friction with suitable illustrations applicable to each case.

* The student should refer to Thomas W. Barber's *Engineers' Sketch Book of Mechanical Movements*, published by E. & F. Spon, London, for a large variety of Brakes and Retarding Appliances in Section 5, Friction Clutches in Section 15, and Friction Gear in Section 38, and to an excellent book on *Dynamometers and the Measurement of Power*, by John F. Flather, Ph.B., published by John Wiley & Sons of New York, as well as to the several papers referred to by footnotes in this Lecture.

Friction Clutches.*—When two light shafts are in line with each other, one of which has to be set in motion or stopped at any time, whilst the other one is kept rotating, they may be conveniently coupled together by the simple form of clutch illustrated by the accompanying figure. To the left-hand shaft there is keyed



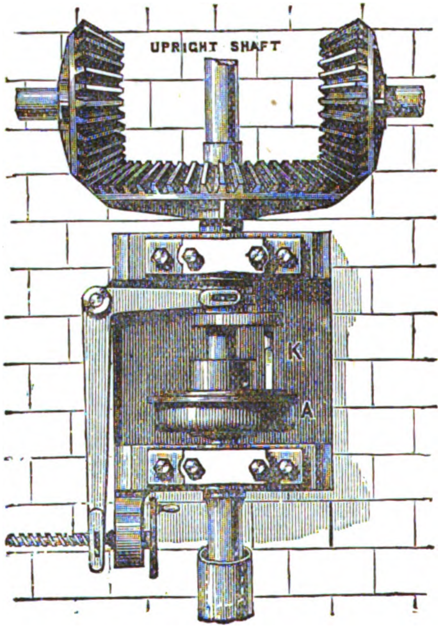
SIMPLE FRUSTRUM FRICTION CLUTCH.

a hollow disc, truly bored out to a certain taper, which depends upon the materials, and must be great enough to prevent jamming. On the right-hand shaft another disc fits freely on a feather-key, so that it may be forced to the right or left by a forked lever (not shown) which engages the groove in its boss. This right-hand disc is turned to the same taper as the left-hand one; consequently, when it is pressed home thereon and held in position by a locked or weighted lever, the friction between the two conical surfaces is sufficient to transmit power from the left-hand to the right-hand shaft. In order that the shafts may remain in line, each disc is supported by a bearing immediately behind its own boss.

The following figure shows how this same sort of clutch might be applied to a vertical shaft gearing with, and transmitting motion to, one or two horizontal shafts. Here the lower end of the bell-crank lever is forked and fitted with a nut which engages a screw worked by a wheel and handle. The upper end of the bell-crank is also forked, and should engage the grooved collar of

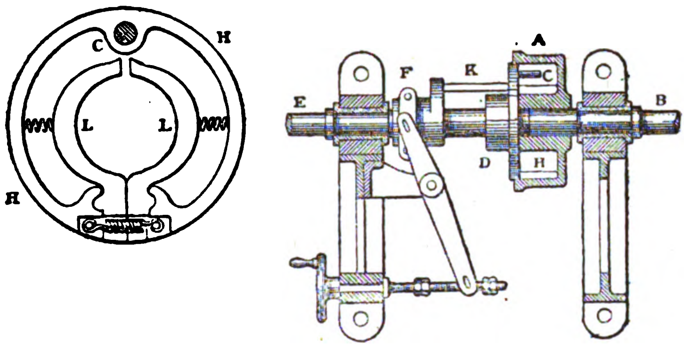
* We are indebted to Messrs. J. Bagshaw & Sons, Batley, for three of the following figures, and to the Council of the Institution of Civil Engineers for the third and fourth figures, which are taken from Mr. Walter Bagshaw's paper on "Friction Clutches." See *Proc. Inst. C.E.* for 1886-87, vol. lxxxviii., p. 368.

the movable part of the clutch. The makers, however, prefer the method shown in the next and following illustrations, whereby the wedge-pointed rod, K, on being depressed, forces open two internal levers, L L, which, in turn, expand a split friction ring, H, until it grips the inside of the hollow clutch, A.



ADDYMAN'S FRICTION COUPLING FOR UPRIGHT SHAFTING.

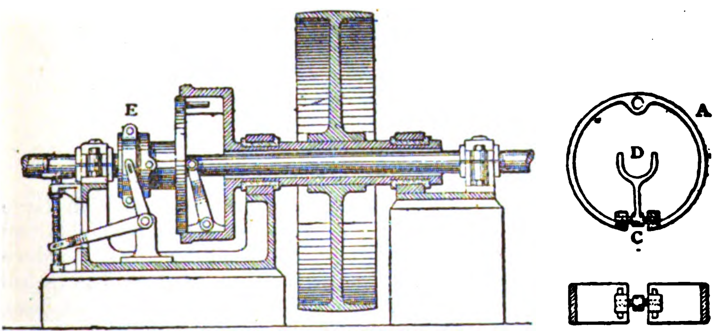
Another method by the same firm is shown in the second figure on the following page. Here, one internal forked lever, D (which clears the shaft), is fixed at its lower end to a right- and left-handed screw. When this lever is forced forward the screws are turned in one direction, and press out the split cast-iron ring, A, until it bites the internal surface of the hollow clutch, and starts the machine connected therewith. On the other hand, when the lever is pulled back, the screws are turned in the reverse direction and pull the ends of the split-iron ring together, thus freeing it from the clutch and permitting the driven machine to come to a stand-still.



ADDYMAN'S FRICTION COUPLING FOR HORIZONTAL SHAFTING.

INDEX TO PARTS.

A	for Hollow shell of clutch.	F	for Collar.
B	,, Driving shaft.	H	,, Friction ring.
C	,, Crank pin or driver.	K	,, Wedge-pointed rod.
D	,, Disc.	L	,, Levers.
E	,, Driven shaft.		

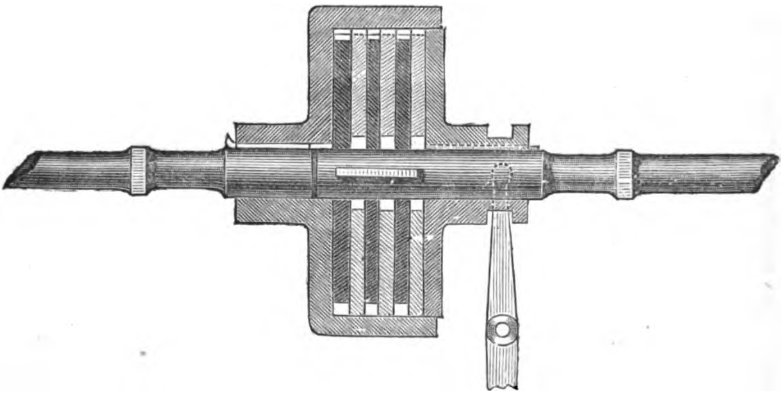


BAGSHAW'S HOLLOW SLEEVE CLUTCH.

Weston's Friction Coupling and Brake.—This apparatus consists of a shell keyed on one of the shafts, in which are fitted two series of friction discs free to slide lengthwise towards or from each other. The series, shaded black in the figure, is made to rotate with the right-hand shaft by means of feather-keys, and the other series (shaded light) with the outside shell. When no compression is applied to the discs through the lever, the shafts are free to revolve independently of each other, but upon com-

pressing the discs, the rotary motion of one series is transmitted wholly to the other, and the apparatus acts as a coupling. When the pressure is slightly relieved, the friction between the two sets of discs acts as a brake and thus controls the speed.

It will be observed that this coupling is designed on the principle of multiple-gripping surfaces, and, as friction increases in proportion to the number of pairs of surfaces in contact, it is possible to so increase their number and the extent of the surface in contact, so as to multiply the resistance due to friction to any desired amount. Weston's arrangements of alternate discs of iron or steel and gun-metal, or metal and wood or leather, are used for

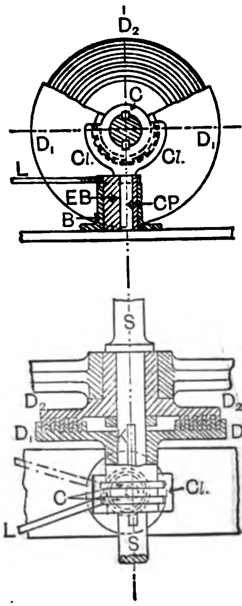


WESTON'S FRICTION COUPLING AND BRAKE.

a variety of devices, such as lowering and holding brakes in large cranes, elevating gear in large guns, &c. The resistance obtained in this way is very remarkable. It is stated by Prof. Goodeve in his *Principles of Mechanics*, that "six discs of iron, 14½ inches in diameter, riding between wooden discs and used in a windlass, are recorded to have sustained a direct pull on the cable of 34 tons without yielding."

Concentric Grooved Disc Friction Coupling.—Another form of friction coupling is that known as "Robertson's Wedge and Groove Friction Clutch," which will at once be understood from the accompanying sectional plan and elevation. If the discs, D_1, D_2, forming the two parts of the coupling, are to be brought into contact, then the lever, L, is moved to the right, which turns the forked claw or clip, C *l*, connected to the central circular pin, C P, inside the eccentric boss, E B, of the bearing, B. This claw in turn forces forward the collar, C (along the feather of the shaft),

connected to the disc, D_1, and thus brings D_1 into gear with D_2. This latter disc is fixed to the pulley or wheel which is connected to the machine to be set in motion. To bring the discs out of gear the lever, L, is moved to the left, and precisely the reverse action takes place. The advantage of all these several forms of friction couplings is, that they transmit power without jar and will slip under an excess of force or shock beyond that which they are designed to transmit. The forces transmitted are, however, limited by the coefficient of friction and the number, extent, and exact fit and freedom from oil or moisture of the surfaces in contact. We shall have to refer to spur and bevel frictional gearing later on, and to investigate the limits to which such appliances may be adapted.



CONCENTRIC GROOVED DISC FRICTION COUPLING.

Weston's Centrifugal Friction Pulley.*—Two forms of this friction pulley are shown in the following figures, the first suitable for rope and the second for belt driving. This pulley is specially well adapted for driving high speed machines, such as centrifugals and dynamos.

The principal advantages claimed for it are:—(1) A number of machines may be driven direct from the same shaft; and any one of them may be started or stopped independently of the others.

(2) Both starting and stopping are performed gradually and easily.

(3) No sudden shocks or stresses are caused to the pulley, the belt, or any part of the driven machine, since the necessary friction for imparting the motion is applied automatically by centrifugal action.

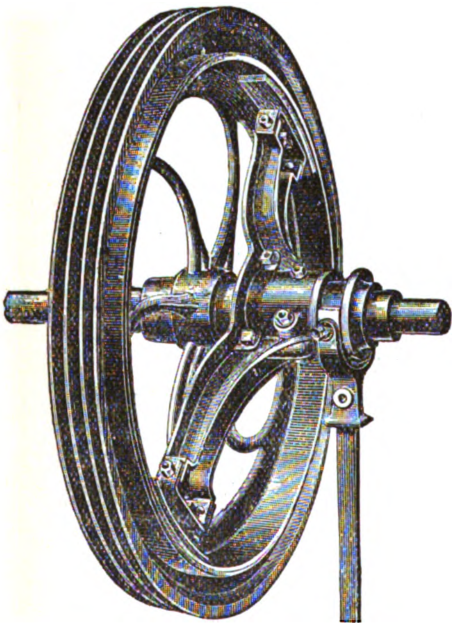
(4) When the friction pulley is the driver no loose pulleys are required, and consequently there is no wear and tear of belting from shifting forks, &c.

(5) The pulley may be so adjusted as *not* to transmit more than the desired power without slipping or giving warning that an extra load has been brought into circuit.

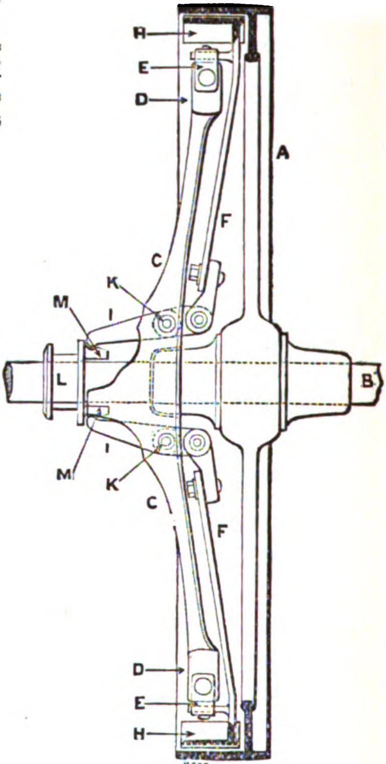
The construction and action of this pulley will be readily

* We are indebted to Messrs. Watson, Laidlaw & Co., of Glasgow, the makers of this friction pulley, for the two following illustrations.

understood by reference to the following figures. A, is the pulley proper, which carries the belt, and rides loose on the shaft B. C, is an arm bolted or keyed to the shaft, and revolves with it. This arm carries two toe-levers, I, pivoted at K. To these toe-levers are attached the friction arms, F, the latter being again connected to the arm, C, at the ends, D, by means of flexible springs, E. The gripping surfaces, H, of the arms, F, are faced with leather, and turned to the same curvature as the inside of the rim of the pulley. It



Friction Pulley for Rope Drive.



Vertical Section of Friction Pulley for Belt Drive.

will thus be seen that when the arm, C, is in motion it will carry round with it the two friction arms, F. The latter will tend, under the influence of centrifugal action, to fly outwards, and thus to bind themselves against the rim of the pulley, A, and carry the latter round with them. In the condition shown in the illustration, this tendency is restrained by the toes, M, of the

sliding sleeve, L. All that is necessary, therefore, in order to permit the friction arms to transmit their motion to the pulley is to withdraw the toes, M, by moving the sleeve, L, a very short distance along the shaft by means of a hand lever, such as is shown in the left-hand figure. The springs, E, prevent the motion of the friction arms being too suddenly communicated to the pulley; hence, as we said before, there is no sudden stress put upon the belt, and it gradually acquires its full speed.

In order to get the best and most economical results, the highest convenient speed should be arranged for this friction pulley.*

Brakes Defined and Classified.—The contrivances comprised under the general title of brakes, are those by means of which friction is intentionally opposed to the motion of a machine, in order to stop it, retard it, or employ some of its superfluous energy with the view of producing uniform motion.

Brakes may be classified as follows :—

1. *Block brakes* are those in which one solid body is simply pressed, and rubs, against another.

2. *Strap, or flexible, brakes* are those which embrace the periphery of a drum or pulley.

3. *Pump brakes* are those in which the friction amongst the particles of a fluid produces resistance when the fluid is forced through restricted passages.

4. *Turbine, or fan, brakes* are those in which the resistance employed is that of a fluid to a fan rotating in it.

Block Brakes.—The most familiar example of the use of the ordinary block brake is its application to road vehicles and railway rolling stock. Its effect is to retard or stop the rotation of the wheels, and thus to make them slip instead of rolling on the road or railway. The resistance caused by such a brake to the motion of a carriage may be less than, but can never be greater than, the friction between the stopped wheel and the road or rail.

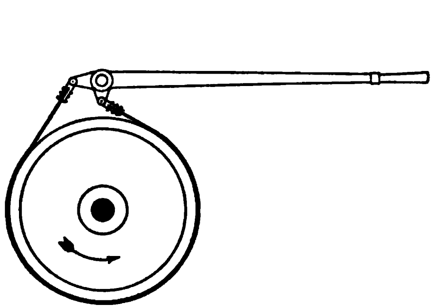
Flexible Brakes.—In hoisting and lowering machinery, such as crabs, winches, cranes, and colliery winding engines, a form of brake called the friction strap or flat brake is generally employed for holding the load when raised to the desired height, or for gradually arresting its motion on being lowered to the required

* There are an immense number of patented "Friction Power Transmitters," but we have given sufficient illustrations to show the application of these useful devices. The student should, however, consult the pages of *The Engineer*, *Engineering*, and other similar periodicals—*e.g.*, he should refer to *The Engineer* of April 18, 1890, for a description and complete set of sectional figures of Shaw's "Coil Friction Power Transmitter," which is an interesting departure from the usual methods of bringing machines into circuit with their drivers.

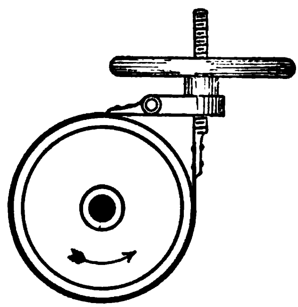
depth. In the case of ordinary small crabs, winches, and cranes, this brake takes the form of a flexible steel strap, which, to a greater or less extent, encircles a strong flat-faced cast-iron wheel or pulley. It is made of sufficient length to clear the wheel when slack. The strap is tightened by means of a lever actuated by the hand or foot. If the load to be arrested is comparatively small, then the brake-strap may be conveniently fixed to the barrel-shaft of the crab or crane, and it naturally takes the form shown by the left-hand figure, where the inner end of the lever terminates in a bell-crank.*

The extremities of the arms of this bell-crank are attached to the ends of the brake-strap. This brake-strap is simple, can be readily fitted to most hoisting gear, and possesses great gripping force.

In double or treble purchase crabs and cranes where the load to



SIMPLE BRAKE-STRAP AND LEVER.



SIMPLE BRAKE-STRAP AND SCREW.

be held in position or arrested is greater than in the case of single purchase ones, the brake is usually fixed to the second or third motion shaft, and may, for convenience of manipulation, take the form shown by the right-hand figure. As in the previous case, the brake-wheel is usually made of cast-iron, with a solid central web between its boss and rim. The upper end of the thin flexible steel strap is fixed to a projecting arm or stretcher, and the other end to the lower terminal of a vertical screw.

This screw is raised or lowered by a horizontal hand-wheel whose boss is a nut fitting the vertical screw. When lowering a load rapidly by means of a brake, it is usual to throw the first

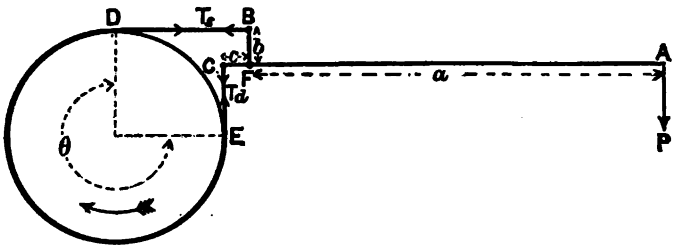
* Also, see the figures at pp. 118 and 121, of Lecture XIII., in the author's *Elementary Manual of Applied Mechanics*, as well as the figures of the Crab in Lecture XI. of this Text-book.

motion shaft out of gear by aid of a pawl, &c., as explained in Lecture XIII. of our *Elementary Manual.**

Proper Direction of Rotation of Brake-Wheel when Lowering a Load.—In fixing a brake-strap to any piece of hoisting machinery, care should be taken that the brake-wheel turns (as shown by the arrows on the two previous figures) in a direction so as to produce the greater stress upon that end of the brake-strap which is attached to the fixed end of the lever or projecting arm; for, if fitted in the reverse order, it will be found much more difficult to control the motion of the load.

Mathematical Proof of the Previous Statement.—The truth of the previous statement may be proved by calculating the frictional moments about the centre of the brake-wheel.

Let T_d and T_s denote the tensions in the tight and slack ends of the brake-strap. Which is the tight and which the slack end will, of course, depend upon the direction of rotation of the brake-wheel. In the accompanying figure the direction of rotation is



GEOMETRICAL DIAGRAM OF BRAKE-WHEEL, STRAP, AND LEVER.

indicated by a curved arrow; and, consequently, C E is the tight and B D the slack end.

Let θ = Angle subtended by strap at centre of wheel.
" a, b, c = Lengths of arms, A F, B F, C F respectively.
" r = Radius of brake-wheel.
" P = Force exerted at end, A, of the lever, A C.
" μ = Coefficient of friction between strap and wheel.
" M = Frictional moment about centre of brake-wheel.

Then, $$M = (T_d - T_s)\,r. \quad \ldots \quad (1)$$

But, as we proved in equation (XI), Lecture VII., $$\frac{T_d}{T_s} = e^{\mu\theta} = k. \quad \ldots \quad (2)$$

* See *Notes on the Construction of Cranes and Lifting Machinery*, by Edward C. R. Marks. Published by John Heywood, Deansgate, Manchester, first edition, pp. 78 to 81, and Fig. 83, for an improved form of lowering brake-strap, which is lined with leather and fitted to the third motion shaft of the crane gear.

Taking moments about F, the fulcrum of the lever, we get:—

$$P a = T_s b + T_d c \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

By eliminating T_d and T_s from these three equations, we can express M in terms of P, a, b, c, k, and r, all of which have known values.

Thus, from (2), $T_d = k\, T_s$.

Substituting this value of T_d in (1) and (3), we get:—

$$M = (k-1)\, r\, T_s \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

And,

$$P a = (b + c k)\, T_s \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Dividing (4) by (5), $\dfrac{M}{P a} = \dfrac{(k-1)\, r}{b + c k}$,

$$\therefore \qquad M = \frac{P a r (k-1)}{b + c k} \quad . \quad . \quad . \quad . \quad . \quad . \quad (\mathrm{XV})^*$$

Very often the *tight* end, C, of the strap is immovable, for it may be fixed by a pin to the fulcrum, F. In that case $c = 0$, and we have:—

$$M = \frac{P a r (k-1)}{b} \quad . \quad . \quad . \quad . \quad . \quad . \quad (\mathrm{XV}_a)$$

If, however, the direction of rotation be the opposite of that given in the figure, then BD becomes the tight end of the strap, and it is easily proved that:—

$$M = \frac{P a r (k-1)}{b k} \quad . \quad . \quad . \quad . \quad . \quad . \quad (\mathrm{XV}_b)$$

Now, k is always greater than unity, hence we see from (XV_a) and (XV_b) that the resistance to friction is less for one direction of motion than for the other. An example will make this point still clearer.

Example I — A treble-purchase crab is fitted with a strap friction-brake worked by a lever. The shaft on which the brake-wheel is keyed, carries a pinion of 12 teeth which gears with a wheel of 48 teeth on the next shaft. This second shaft has a pinion of 12 teeth gearing with another wheel of 60 teeth on the drum or barrel shaft. The diameter of the drum or barrel is 14 inches; diameter of brake-pulley 2½ feet; length of brake handle 3 feet. One end of the brake-strap is immovable, the other end being fixed to the shorter arm of the brake-lever, which is 3 inches long. The angle subtended by the strap at centre of brake-pulley is 270°. If a force of 50 lbs. be applied at the end of the brake-lever, what is the greatest load for each direction of

* The Roman Numbers for these equations follow those in Lecture VII.

rotation of the brake-wheel which could be supported on the end of the rope that passes round the drum? Take $\mu = \cdot 1$.

ANSWER.—The friction moment at the brake-pulley is :—

$$M = \frac{P\,a\,r\,(k-1)}{b} \quad \ldots \ldots \ldots \ldots \ldots \quad (1)$$

Or, $$M = \frac{P\,a\,r\,(k-1)}{b\,k} \text{ (according to the direction of rotation) } (2)$$

Since, $\theta = 270^\circ = \frac{3\,\pi}{2}$; and, $k = e^{\mu\theta}$.

$\therefore\ \log k = \cdot 4343\,\mu\,\theta = \cdot 4343 \times \cdot 1 \times \frac{3 \times 3\cdot 1416}{2} = \cdot 20466.$

$\therefore\quad k = 1\cdot 602$, nearly.

Substituting $P = 50$ lbs.; $a = 36$ ins.; $b = 3$ ins.; $r = 15$ ins. in (1), we get :—

$$M = \frac{50 \times 36 \times 15\ (1\cdot 602 - 1)}{3} = 5{,}418 \text{ in.-lbs. } (3)$$

Or, from (2), $$M = \frac{50 \times 36 \times 15\ (1\cdot 602 - 1)}{3 \times 1\cdot 602} = 3{,}382 \text{ in.-lbs. } (4)$$

Now, the moment of the couple at the brake-wheel, due to the load, W, at the drum, is :—

$$M = W \times 7 \times \frac{12}{60} \times \frac{12}{48} = \frac{7\,W}{20} \text{ in.-lbs.}$$

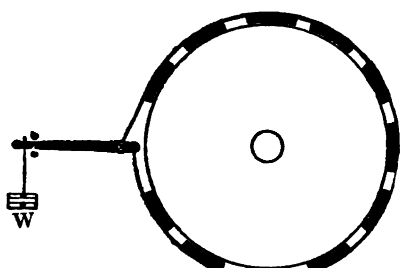
Hence, from (3), $$W = \frac{5418 \times 20}{7} = 15{,}480 \text{ lbs.}$$

Or, from (4), $$W = \frac{3382 \times 20}{7} = 9{,}663 \text{ lbs.}$$

This example shows at once the importance of attending to the direction of rotation of the brake-wheel when the load is being lowered, before fitting up the brake appliance. The rule is, therefore, *to make the tight end of the strap the fixed or immovable end, and to attach the slack end to the shorter arm of the brake lever.* By an inspection of the arrangement, it becomes evident that for one direction of rotation of a brake-wheel, the friction between the strap and its pulley assists the effort on the lever, whilst it opposes it for the reverse direction.

Paying-Out Brake for Submarine Cables.—When a brake-strap exceeds 2 or 3 feet in diameter, it is usually fitted throughout its inner surface with wooden blocks—preferably of hornbeam or beach. These are screwed to the steel strap from the outside thereof. The "paying-out gear" for the laying of submarine

cables (see accompanying illustration) always contains one or two of these larger brakes of from 6 to 8 feet in diameter. The cable as it comes from the tanks of the ship is coiled four or five times round the paying-out drum, from which it then passes aft under the dynamometer pulley and over the stern sheaves into the sea. Now, a restraining force must be applied to the cable in order that it may be laid as evenly as possible along the irregular bed of the ocean, with just the desired amount of slack, so as not to put too great an initial stress thereon, and to permit of the cable being lifted for future repairs, without having recourse to cutting the same, when in moderate depths say up to 1,000 fathoms. This restraining force cannot be directly put upon the cable without injuring it, so recourse is had to the device of fixing one or two large brakes, of the Appold type, to an extension of the paying-out drum shaft. The accompanying figure will serve to indicate to the student the kind of brake generally employed for this purpose. The engineer in charge calculates the necessary stress required for the particular type of cable, depth of water, and speed of ship; and after making his calculations he applies the desired weights, W, at the end of the brake-lever.



APPOLD'S BRAKE FOR THE PAY-OUT GEAR OF SUBMARINE CABLES.

The brake runs in a trough of water, so that the heat generated between the wooden blocks and the brake-wheel may be carried off quickly, as well as to ensure that the coefficient of friction may remain as constant as possible. In order to test whether too much or too little slack is being paid out at any time, the engineer adds a slight excess of weight over his calculated amount, W, for a short time, and then subtracts a slight amount for an equivalent time; when, by aid of the tachometer or speed counter on the paying-out drum, and the ship's log, as well as the known length of cable for each revolution of the drum, he is able to permanently adjust the required amount of brake-weight during a run of several nauts for a uniform speed of the ship.

Differential Brake for Lord Kelvin's Deep-Sea Sounding Machine.—Another interesting illustration of the application of a brake to marine purposes is contained in Lord Kelvin's deep-sea pianoforte-wire sounding machine. The object to be attained by

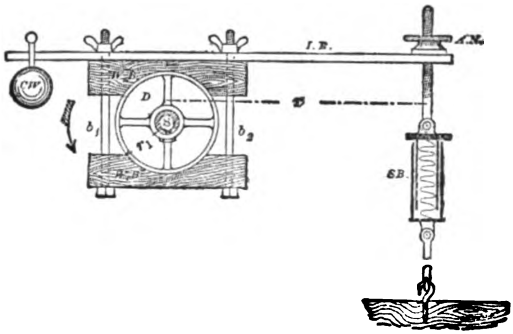
this differential brake is to put the necessary restraining force upon the sounding wire as it is being lowered, and to suddenly stop the same immediately the lead reaches the bottom of the ocean, so that the correct depth may be registered by the counter attached to the wire-drum. The brake weights are attached to a rope which passes over, and is fixed to the larger pulley (shown at the right-hand top corner of the figure), whilst the brake-wire is attached to a smaller pulley on the same spindle. This brake-wire passes round a V-groove on the side of the drum containing the pianoforte wire, and its other end is fastened to the framing. When heaving up the pianoforte wire from a great depth with the "lead" attached thereto, it was found necessary to take a turn of the sounding-wire round the "strain-relieving pulley" in order to prevent the drum being damaged by the constant tension on the sounding-wire.

Dynamometers.*—It is frequently of great practical importance to ascertain by direct experiment the "nett" power developed by motors or expended in driving machines. For example, steam, gas, and oil engines, turbines, water wheels, and electric motors, &c., are being designed, made, and sold every day to drive machinery of one kind or another. It is, therefore, surely far better, both for the buyer and the seller, to know the *exact* "Brake horse-power" (B.H.P.) which a generator will develop at a certain speed and under a certain mean pressure, than to vaguely talk of the "Nominal horse-power" (N.H.P. whatever that may mean); or even to speak of the "Indicated horse-power" (I.H.P.) in the case of engines; or the "Gross horse-power" in regard to hydraulic machines; or the "Electrical horse-power" (E.H.P.) with respect to dynamos. It is also surely far better to know exactly the Brake horse-power which a certain machine, or even a section or the whole of a factory requires when worked under certain conditions, than to make a rough guess at the amount from vague data or even previous general experience. For, it is by aid of such tests that the "mechanical efficiency" of the motors and machines can be accurately determined and further improvements take place in raising their efficiency.

It is, therefore, with a certain pleasure that the author places before the student a few of the many devices that have been invented for obtaining the brake horse-power developed by motors or required for driving machinery; because, he has been persistently advocating the adoption of this system of gauging power for many years. Ever since the introduction of electric lighting and the transmission of power by electricity, this view of the case

* The word dynamometer is derived from the Greek words δύνἄμις, signifying *force*, and μετρεω, to *measure*.

has year by year become more appreciated and been taken advantage of. It is not too much to say, that the general engineer has been greatly indebted in this respect, to his colleague the electrical engineer. For all powers up to 200 H.P. or so, there is no practical difficulty in making exact brake trials; and probably in the near future, we shall see engines of 1,000 H.P. and upwards tested and paid for by this uniform and reliable standard. There are two main types of mechanical dynamometers—(1) Absorption or Friction Brake; (2) Transmission. Absorption dynamometers absorb the work which they help to measure and dissipate the same as heat. Whereas, transmission dynamometers pass on the work which they help to measure, and only waste a small



ORIGINAL PRONY BRAKE DYNAMOMETER.

INDEX TO PARTS.

W B	for Wooden blocks.	I B	for Stiff iron bar.
D	,, Drum or pulley.	S B	,, Salter's balance.
S	,, Driving shaft.	C W	,, Counter weight.
b_1 b_2	,, Bolts with ram's horn nuts.	A N	,, Adjusting nut.

fraction of the total work delivered to them. In the first instance, we shall describe with an example the Prony brake, because it will form an easy introduction to more recent and perfect kinds.

Absorption Dynamometers.—Prony Brake.—This dynamometer is only a particular application of the friction brake already mentioned in this Lecture. As will be seen from the following figure and "index to parts," a pulley or drum, D, keyed to the shaft, S, is gripped between two wooden blocks, W B, which may be tightened or loosened (as required to produce more or less friction between them and the pulley) by turning the ram's horn nuts of the bolts, b_1 b_2. An iron bar fixed across the top of the upper block (or, if preferred, along the bottom of the lower one) is balanced by a counter weight, CW, and has either an adjustable

weight or a Salter's spring-balance, S B, fixed to the other end at a known distance or radius, r, from the centre of the shaft. Or, the counter weight may be dispensed with, and the bar extended to the left side instead of to the right, and allowed to press upon the platform or table of an ordinary Pooley weighing machine.*

Method of Taking Test for Brake Horse-Power. — 1. Adjust position of C W until it balances the weight of I B, A N, and S B, with the wooden blocks slack on the pulley.

2. Start machinery and tighten blocks, W B, by the ram nuts until the desired speed is attained. At the same time, adjust S B by nut, A N, until a balance is obtained, taking care to keep I B level by aid of a length-rod or pointer.

3. Note number of revolutions per minute by a tachometer or speed indicator if great, or a counter and stop-watch if slow.

4. Note the stress indicated by spring balance.

Then, the horse-power absorbed by the brake is obtained from the formula :—

$$\text{B.H.P.} = \frac{2\,\pi\,r\,n\,\text{P}}{33{,}000}.$$

Where, r = Radius or horizontal distance from centre of balance to centre of shaft in feet.

„ n = Number of revolutions per minute.

„ P = Pull indicated by the Salter's balance.

Since, $$\frac{2\,\pi}{33{,}000} = \cdot 0001904 = \text{a constant.}$$

We get, $$\text{B.H.P.} = \cdot 0001904 \times r \times n \times \text{P}.$$

Example II. — A small fast-speed Westinghouse engine was fitted with a Prony brake of the form just described. The fly-wheel was 2 feet diameter and 6 inches broad. The horizontal distance from the centre of the crank-shaft to the centre of the spring-balance was 2·5 feet; the mean revolutions per minute

* This latter method of registering the forces produced by the friction between the revolving pulley and the stationary wooden blocks is very handy, when engines are having their steam consumption registered for long continuous runs, during the process of getting their bearing surfaces into good working condition. Any alteration in the balancing force is easily effected by simply shifting the small adjusting weight along the light arm of the weighing machine, and little or no attention need be paid to the brake lever. Besides which, there can be no danger to the attendant in the case of the pulley firing and seizing the wooden blocks.

were 624, and the mean pull on the spring-balance was 48 lbs. Find the brake horse-power.

	Or, by logarithms :—
Here, $r = 2{\cdot}5$ ft.; $n = 624$; $P = 48$ lbs.	Log. $\cdot 0001904 = \bar{4}{\cdot}2797$
$\therefore$ H.P. $= \cdot 0001904 \times r \times n \times P$	„ $2{\cdot}5 = 0{\cdot}3979$
	„ $624 = 2{\cdot}7952$
$\therefore$ H.P. $= \cdot 0001904 \times 2{\cdot}5 \times 624 \times 48$	„ $48 = 1{\cdot}6812$
$\therefore$ **H.P. = 14·26.**	Antilog. of $1{\cdot}1540 =$ **14·26.**

It is important to note, that neither the diameter of the pulley nor the pressure of the friction blocks on the same (due to the weight of the apparatus or the tightening of the ram nuts), nor the coefficient of friction enter into the formula for obtaining the horse-power. The only data required being the horizontal length of lever, r, the number of revolutions per minute, n, and the pull, P.

For, let p, be the pressure, and f, the coefficient of friction between the face of the drum D, and two brake blocks W B, then the twisting moment T, tending to turn the brake blocks round with the shaft is

$$T = 2pf \times r_1$$

Where r_1 is the radius of the pulley or drum, D, in feet.

But this twisting moment is balanced by the pull on the spring balance, P, multiplied by its leverage, r.

$$\therefore \qquad 2pfr_1 = Pr.$$

The angle turned by the pulley or drum, D, per minute $= 2\pi n$ radians, and since the work done by a couple, is the product of its moment into the angle through which the body turns :—

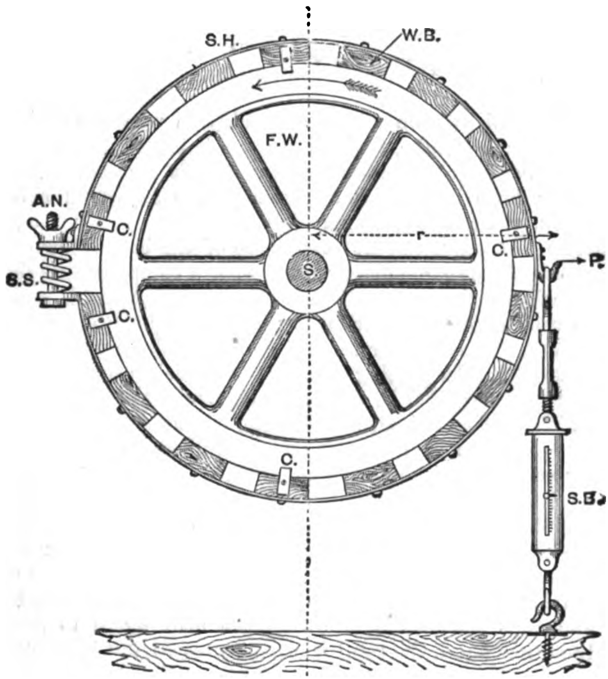
The work absorbed by friction = The work done per minute in foot-pounds.

$$\text{Or,} \quad 2pfr_1 \times 2\pi n = Pr \times 2\pi n$$

$$\therefore \qquad \text{B.H.P.} = \frac{Pr \times 2\pi n}{33{,}000} = \frac{2\pi r n P}{33{,}000}.$$

Improved Prony Brake.—Another very useful and practical form of Prony brake is that shown in the following figure. It is more suitable for larger powers and larger pulleys or flywheels than that shown by the previous illustration.

In this form of Prony brake the counterbalance weight, iron bar, and two wooden blocks, &c., are replaced by one or two thin steel straps, S H, fitted with a large number of hard wood blocks, W B, placed about 2 inches apart. These blocks are generally made of the same width as the flywheel, F W, upon which they bear, and they are kept from slipping to one side or the other by a number of metal clips, C, C, screwed on each



Improved Prony Brake Dynamometer.

side of them. On starting the engine, the adjusting nut, A N, is left quite slack until the desired speed has been attained. It is then gradually turned until the necessary pull is registered by the Salter's balance, S B. Should this tightening up of the brake-strap raise the pointer, P, above the level line, P S, then the adjusting link between S B and P will have to be turned in a direction that will bring P down a little, when a slight slackening of A N will probably let P down to the level mark. The spiral

spring, S S, between the outstanding lugs of the brake-strap, serves to give the brake a little more elasticity than it would otherwise have, and also keeps these lugs hard against the head and nut of the adjusting bolt. After the desired speed and pull have thus been rendered fairly constant, a set of readings should be taken every ten or fifteen minutes over a period of several hours, and the mean B.H.P. obtained from the mean speed, n, and pull, P, and horizontal distance, r, in exactly the same way as in the previous example, viz.:—

$$\text{B.H.P.} = \frac{2\pi r n P}{33,000}.$$

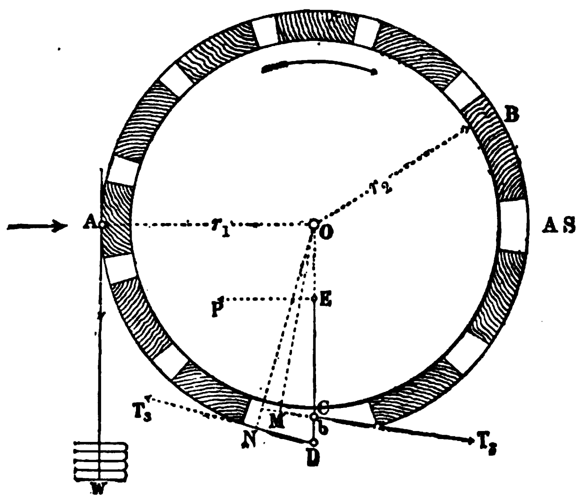
It will be evident from the figure, and from what we said about the ordinary Prony brake, that the Salter's balance may be replaced by a Pooley weighing machine resting on the ground, and a stiff vertical bar fixed between the bottom of the adjusting bolt and the platform of the weighing machine.

Appold's Compensating Lever.*—One of the best known forms of friction-brake dynamometers fitted with a compensating device, is that designed by Mr. C. E. Amos and Mr. Appold, and was the form used at one time for large powers by the Royal Agricultural Society. It is similar to that shown by the previous figure; but, besides a hand-adjusting screw at A S, it is provided with a compensating lever, E C D, by means of which the rise or fall of the load, W, is attended with a decrease, or increase, in tension on the brake-strap, so that a position of equilibrium may be automatically attained without causing inaccuracy in the indications. With a given tension in the brake-strap, and with the load, W, carried so that its point of suspension, A, is opposite the pointer mark, ⟶, the lever, E C D, takes a vertical position; but as soon as the load, W, is lifted, the lever pivoted at E, moves round to the left hand, and virtually increases the length of the brake-strap, and thus slackens it, allowing the load again to descend. If, on the other hand, the total friction decreases and is insufficient to carry the load in its normal position, the descending load presses round the point of the compensating lever to the right,

* The following four figures are from *The Proc. Inst. C.E.*, vol. xcv., Session 1888-89, by kind permission of the Council, from a Paper by W. W. Beaumont, M.Inst.C.E., on "Friction Brake Dynamometers," which the student should consult, not only for the information contained in the paper, but also for that derivable from the excellent and extensive discussion. For a description of "Froude's Turbine Brake," see Dr. Edward Hopkinson's and Mr. R. E. Froude's remarks on Mr. Beaumont's paper, as well as Prof. Osborne Reynold's paper "On the Triple-Expansion Engines and Engine Trials at the Owens College, Manchester," *Proc. Inst. C.E.*, vol. xcix., Session 1889-90.

thus tightening the strap and increasing the frictional grip until the conditions are again such as will enable the lever to reassume the vertical position. If the change in the position of the point of suspension of the load has been due to a temporary cause, this automatic action may restore the balance without further adjustment; but if the departure from the normal position is not small, then adjustment by hand-screw at A S must be resorted to. It will be seen that the compensating action cannot come into play except by the rise or fall of the weight from its proper position, and hence the value of the device is confined to its power of limiting that rise and fall.

So long as the Appold brake is not used for more than 15 H.P., and is sufficiently, but still sparingly, lubricated with tallow or suet, the friction between the wooden blocks and iron wheel is



Appold's Compensating Brake, as used by the Royal Agricultural Society.

sufficient at ordinary speeds to balance the load without tightening the belt-strap. Under these conditions the compensating lever does not sensibly affect the results, since the pull on it will not be more than a few pounds. The conditions are the same as, or very similar to, those which would obtain if the brake were without a compensating lever, but with a strap so slack that the bottom blocks barely touch the wheel.

In the correspondence upon Mr. Beaumont's paper, Professor

T. Alexander and Mr. A. W. Thomson considered that the Appold brake gave accurate results when it was used properly.

Let W = Load on brake-strap (see foregoing figure).
„ $T_2\,T_3$ = Tensions at two ends, C and D, of strap connected to lower ends of compensating lever.
„ P = Pull on upper end, at E, of this lever.
„ $r_1\,r_2$ = Radii of brake-strap and wheel respectively
„ F = Total friction of brake-strap.

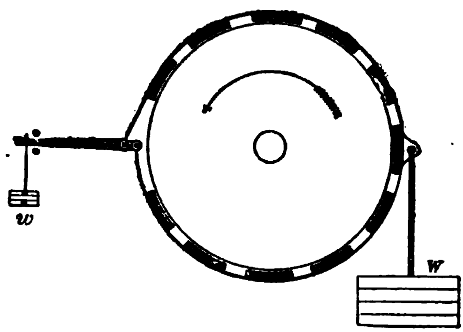
Suppose the lever, E C D, to take some definite fixed position, say to the left of the vertical when the engine is working smoothly. In this position, the lever may be supposed to be fixed to the ground. The tension of the brake-blocks on the lever towards the right at C, and left at D, are represented in the figure by T_2 and T_3. On the other hand, the reactions of the lever on the brake-blocks are T_2 towards the left at C, and T_3 towards the right at D. Then, since there is equilibrium, the sum of the moments round the centre, O, of the weight, W, the friction of brake-blocks, and the tensions, T_2 and T_3, is zero. Now, if we consider the lever as not fixed to the ground, but pivoted at E, then R, the resultant of T_2 and T_3, must pass through E. T_2 and T_3 may now be replaced by R, and the sum of all the moments round O is again zero. Resolve R into vertical and horizontal components, V and P, acting at the point E. Since E is vertically under O, the line of action of V passes through O, and its moment is zero; and, therefore, the sum of the moments round the centre, O, of the weight, W, the friction of brake-blocks, and the horizontal force, P, acting towards the left at E, is zero; that is:—

$$W\,r_1 = F\,r_2 + P \times OE.$$

The amount of this horizontal force, P, can be easily measured by a spring-balance. With a low coefficient of friction, the tension on the brake-strap has to be increased; and since the ratio existing between T_2 and T_3 is constant, depending on the proportions of the lever, it follows that P may be of considerable amount; and any quantitative results calculated without taking it into account will be erroneous. With a high coefficient of friction the force, P, may be small, and the results might probably be not far wrong, even if P were left out of account. In every case, however, where accuracy is desired, the moment of P must be considered.

Professor A. B. W. Kennedy in his paper on the "Use and

Equipment of Engineering Laboratories"* says, that if the Appold pendulum-lever is used in any form for the automatic adjustment of the brake, it should be so arranged that its own pressure can be measured and allowed for. He prefers to use this brake in the manner illustrated by the accompanying figure where the small weight, *w*, is adjusted from time to time in order to keep the brake always floating freely. The changes in this weight have to be noted, and the necessary allowance made in the calculation for the B.H.P. He also believes that the side pressure, P, of the upper end of the pendulum-lever, as it was arranged in the Royal Agricultural Society brakes, if not measured and allowed for, causes a very considerable error in the calculated power. He also thinks that the brake should be large enough to run dry, as it is much more easily kept under control under these circumstances.†



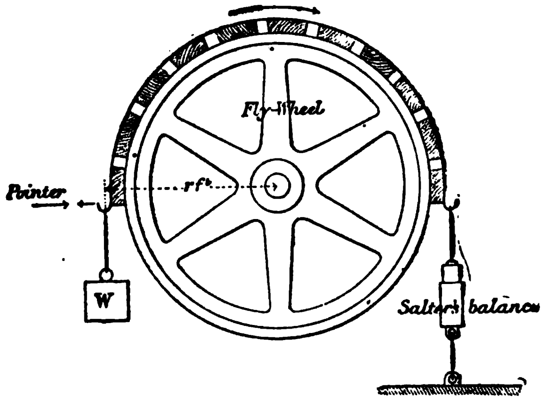
IMPROVED METHOD OF USING THE APPOLD BRAKE AS A DYNAMOMETER.

Semicircular Strap Dynamometer.—One very simple form of dynamometer which avoids the objections previously mentioned in regard to the Appold compensating lever, is shown by the following figure. Here, a semicircular strap of leather, or a number of $\frac{1}{8}$- to $\frac{1}{4}$-inch wires or steel bands, lined with hard wood, are attached at one end to a constant weight, W, lbs., and at the other end to a Salter's balance. The weight, W, should be tethered to some fixed bolt in the floor by a slack piece of flexible

* See vol. lxxxviii. (21st Dec., 1886) of *The Proceedings of the Institution of Civil Engineers*, London, for Prof. Kennedy's paper. Also see *The Mechanics of Machinery*, by Prof. Kennedy, published by Macmillan & Co., p. 632.

† The student will observe that this method of using the Appold brake as a dynamometer, is the same as that referred to at the beginning of this Lecture, for restraining a submarine cable from passing too rapidly to sea.

rope in order to prevent the possibility of it being carried bodily over to the injury of the attendants in the case of the pulley firing and gripping the brake. If the flywheel or pulley revolve in the direction shown by the arrow, and the pointers, ⟶⟵, are kept level with each other, then the net pull on the brake will be (W − S) lbs., where S is the stress registered by the Salter's balance. Hence, if r be the horizontal distance in feet from the centre of the shaft to the vertical centre line of the weight, W,



SEMICIRCULAR STRAP-BRAKE DYNAMOMETER.

as well as to the centre line of the Salter's balance, and n the number of revolutions per minute. Then :—

The work done per minute on the pulley and dissipated in heat $\Bigg\} = 2\pi r n(W - S)$ foot-lbs.

And, the B.H.P. $= 2\pi r n (W - S) \div 33{,}000$.

This form of absorption dynamometer has several objections :—

1. The lubrication requires considerable attention. (This fault is also common to all the previously mentioned dynamometers.)

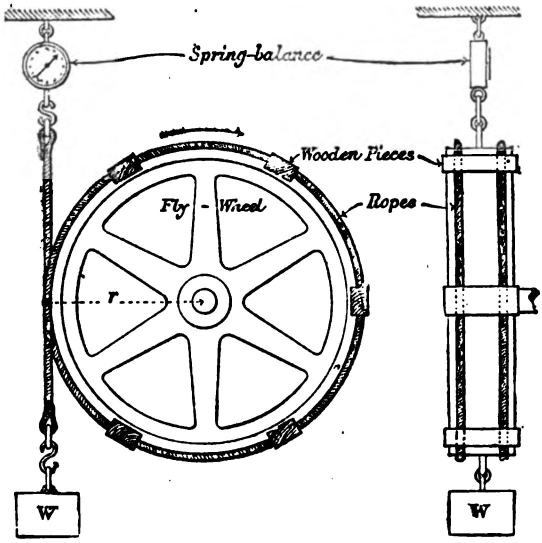
2. The oil, grease, or soapy water used for lubricating the face of the brake-wheel bespatters the floor, &c., and the observer's clothes, unless the precaution is taken to thoroughly encase the lower half of the wheel. (This objection is also common to the previously mentioned dynamometers.)

3. If everything is not perfectly adjusted and running quite smooth, oscillations producing a hunting up and down action set in, due to variations in the friction; and consequently, considerable

guessing and frequent observations have to be taken of the Salter's balance.

These several objections are entirely obviated by adopting the rope-brake which we shall now illustrate and describe.

Society of Arts Rope Dynamometer.*—The jurors for the famous gas engine trials, held under the auspices of the London "Society of Arts" in 1888, were the first to publicly use a rope-brake in any extensive series of competitive trials, and hence the general name which has been given to this very simple and excellent form of brake. But, as will be seen from our footnote,



SOCIETY OF ARTS ROPE DYNAMOMETER.

rope-brakes had been designed and used prior to these tests by at least four well-known persons. As will be gathered from the

* The first rope-brake of which we have any record was invented by Sir Wm. Thomson, in 1872, and applied to his deep sea sounding machine as previously described in this Lecture. Prof. James Thomson, of Glasgow University, devised a rope-brake ergometer prior to 1880, see *Engineering*, Oct. 29, 1880, p. 379. An almost identical compensating rope-brake was also invented by M. Carpenter, of Paris, in 1880, see *Proc. Inst. C.E.*, vol. lxiii. (1881, part I.), p. 404. For a modification of this brake, by Prof. Barr, of Glasgow University, which is very similar to that afterwards used by the "Society of Arts," see *Proc. Inst. C.E.*, vol. lxxxviii. (1887, part II.), p. 110, and also vol. xcv. (1889, part I.), p. 31.

following three sets of figures, this brake consists of an endless flexible rope, doubled round a pulley or the flywheel of an engine, and fitted with several ⊓ shaped wooden distance pieces, in order to keep the two parts of the rope uniformly apart and also to prevent them slipping off the wheel. These distance pieces or clips should be secured to the rope by soft copper wire lacing, drawn in from the outside of the clips and then through the centre of the rope, instead of being fastened thereto by nails or screws from the inside; for such latter metal fastenings are liable to part, to heat, and, consequently, char the rope. The rope should be thoroughly stretched and treated with castor oil or grease and black lead powder, prior to its being fitted to the wheel and to the clips, whenever long and important tests are desired. No further lubrication is required, and consequently the first and second defects mentioned on a previous page as pertaining to strap-brakes are entirely avoided. If large powers are to be demanded from a wheel of limited size, then it should have its rim of ⊔ section, so that a small stream of water may be played into the inside of the hollow part of the rim, which water will help very materially by its evaporation to dissipate the heat generated by the friction between the brake rope and the outer surface of the wheel. The surface of the pulley should be flat instead of rounded, in order to get the rope to work perfectly smooth, and a trial run of a few hours prior to the special test is advisable, in order to bring about a small flat glazed surface on the rope, which glazing is materially assisted by the previous application of the black lead powder. For anything up to 5 B.H.P. at 1,000 or more feet per minute of friction surface speed, the author has found that a flexible ship's log-line about ·3 inch in diameter with a double turn round the wheel forms an excellent brake rope. From 5 to 10 B.H.P. a ·5-inch diameter manilla rope serves the purpose. From 10 to 30 B.H.P. a ·6-inch rope will do, and for 100 to 150 B.H.P. (at about 4,000 feet per minute) four turns of 1-inch rope on a large 16 feet diameter flywheel runs quite cool, as may be seen from the last example on absorption dynamometers in this Lecture.

Advantages of the Rope-Brake.—The author has tested a large number and variety of motors with the rope-brake, and he considers that it has the following advantages:—

1. It can be constructed on short notice, from materials always at hand, in a factory or workshop, and at little expense.
2. It is so self-adjusting that very accurate fitting is not required.
3. It can be put on and taken off the brake-wheel in a very short time.
4. Being comparatively light and of small bulk, it can be hung up on the wall of the testing room, or laid past in a cupboard for future use.

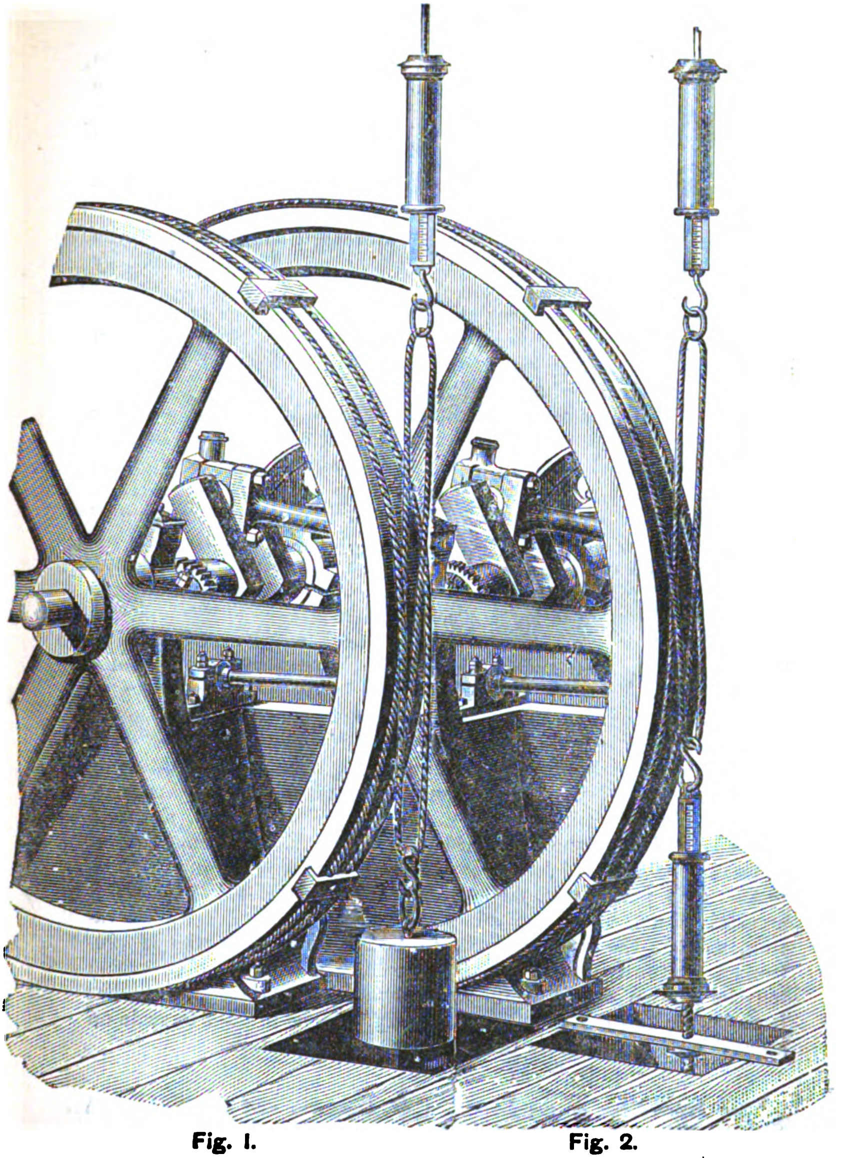


Fig. 1. Fig. 2.

THE TWO FORMS OF ROPE-BRAKE

USED BY PROF. JAMIESON IN TESTING THE "ACME GAS ENGINE," AND "BROWN'S ROTARY ENGINE" FOR BRAKE HORSE-POWER.

5. It requires no attention whatever for lubrication, if the previously mentioned precautions as to treating and fitting the same are attended to.

6. The back pull registered by the spring-balance may be rendered very steady and of small amount by properly adjusting the weight, W, prior to the commencement of the recorded brake trials.

7. The brake-wheel, if of the proper size, soon attains a maximum temperature, so that the radiated heat equals that generated by the friction.

8. It may be used for very small as well as for large powers.

9. For large powers more and stronger ropes are only required on a comparatively larger wheel, and with the water cooling device mentioned in the previous section.

Tests of Engines with the Rope-Brake.—After what we have stated, three examples of such tests will suffice to show the student the wide variety of cases to which the rope-brake may be applied. The first is that of an Acme gas engine of about 19 B.H.P.; the second, that of a Brown's fast-speed rotary steam engine;* and the third, that of "Field's combined steam and hot-air engine."† The results of the first two are given in the first table, and those of the third in the second table, together with a graphic diagram of the more important conclusions.

Fig. 1 shows the arrangement of dead weight and Salter's balance used by the author in testing the "Acme gas engine," and Fig. 2 the way in which he applied two spring balances to the brake rope in case of "Brown's rotary engine." The latter plan has, under certain circumstances, particular advantages over the former. By the selection of two spring balances with different periods of oscillation, the tendency to jerk or "hunt" may be considerably reduced, or even entirely checked. Also, the nett brake load (*i.e.*, P, the difference between the simultaneous indications on the two balances) may be kept constant throughout the test. This permits of the logarithm for $2\pi r P \div 33{,}000$ being ascertained and written down as a constant, prior to each observation, so that the only variable to be recorded is, n, the revolutions per minute. Consequently, the B.H.P. for each observation may be known within a minute or two after the mean speed has been noted, and the complete data may then be plotted to scale on a graphic diagram before taking the next observation.

* See *Proceedings of the Institution of Engineers and Shipbuilders in Scotland*, vol. xxxv., Session 1891-92.

† *Ibid.*, vol. xxxviii., Session 1894-95, for the author's papers on these tests.

B.H.P. TESTS OF AN "ACME GAS ENGINE," AND OF A "BROWN'S ROTARY ENGINE."

DATA.	"Acme Gas Engine" by Alex. Burt & Co., Glasgow.	"Brown's Rotary Engine" by Lang & Sons, Johnstone.
Duration of tests in hours,	4	5
Initial gas or steam pressure in lbs. per sq. in. above atmosphere,	150	95
Final gas or steam pressure in lbs. per sq. in. above atmosphere,	1	1·5
Radius of brake load in feet,	2·771	2·042
Mean revolutions per minute,	154	574·5
Mean nett brake load in lbs.,	231	93·2
Mean B.H.P.,	18·77	20·8
Gas in cb. ft. or steam in lbs. per B.H.P.-hour,	19·13 cb. ft.	37·9 lbs.

The author was recently requested to test and report upon a new departure in the use of steam in steam engines. He has, therefore, much pleasure in placing the results of his experiments on "Field's combined steam and hot-air engine" before the student, because (1) they show one direction in which economy may be attained by preventing the condensation of steam in the cylinder; (2) the brake used was one of the largest in this country; (3) the table and the graphic diagram of results will form a useful example in case the student should be called upon to undertake similar tests.

This invention is the joint design of Mr. Edward Field, M.Inst.M.E. (inventor of Field's well-known tubular boiler), and Mr. F. Saunders Morris, M.Inst.M.E., working in conjunction with Messrs. Musgrave & Co., of Bolton, and Mr. George Dixon, their chief engineer.*

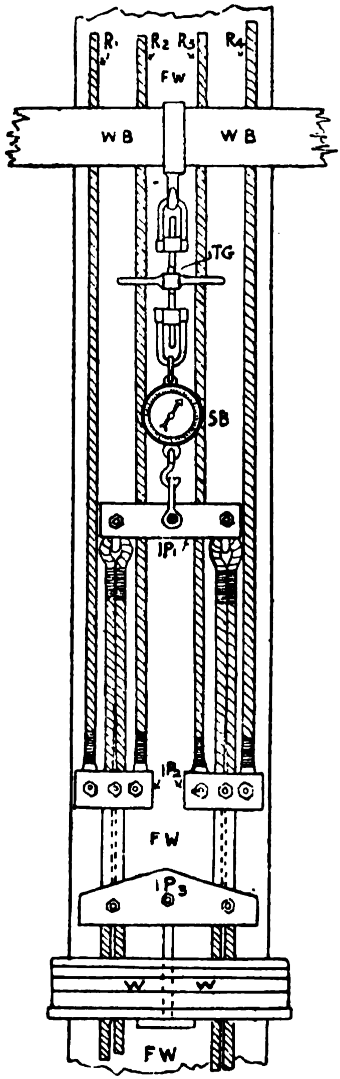
It consists of a hot-air pipe connection to the jacket and to each end of a single cylinder non-condensing engine.

A Roots' blower, driven by the engine, draws fresh cold air from the engine-room, and forces the same through a series of heating pipes placed in the main flue between the boiler or boilers and the chimney. This heater therefore occupies pretty much

* For a complete set of sectional figures of the cylinder, general arrangement of plant, and indicator diagrams, see the author's paper on this subject, vol. xxxviii., *Proceedings of the Institution of Engineers and Shipbuilders in Scotland*, Session 1894-95, from which we have been kindly permitted by the Council to use the following two figures and extract of results.

the same position as a Green's economiser. The air was maintained, in my experiments, at a mean pressure of $1\frac{3}{4}$ lbs. on the square inch, and delivered to the ends of the cylinder at a mean temperature of 553° F., and to the valve-casing jacket at about 380° F. This hot air was admitted to the cylinder through special cylinder covers, each containing five inlet valves, which automatically opened inwards as soon as the exhaust steam commenced to escape. These valves continued open until compression commenced, being held close to their seats by light spiral springs. Consequently, the whole internal surface of the cylinder was heated up to a temperature far exceeding that of the steam, thus preventing the possibility of condensation taking place within the cylinder. Under these circumstances, the excellent result of 18·6 lbs. of steam per I.H.P.-hour was obtained from a single cylinder non-condensing engine—a result which, as far as the author can learn, has never been equalled by any other method of using steam in a single cylinder, and without subsequent condensation.

Brake Gear.—The large flywheel, of fully 50 feet in circumference and 20 inches in width, was used as a brake-wheel.



End View of Brake Gear.

Index to Parts.

FW	represents	flywheel.
R_1 to R_4	,,	ropes.
WB	,,	wooden beam.
TG	,,	tightening gear.
SB	,,	spring balance
IP_1, IP_2, IP_3,	,,	iron plates.
W	,,	weights.

D STEAM

eathers, Green

ective piston area a
L × A ÷ 33,000 = 2·4
Cornish. (13) Nor
ed scale marked off

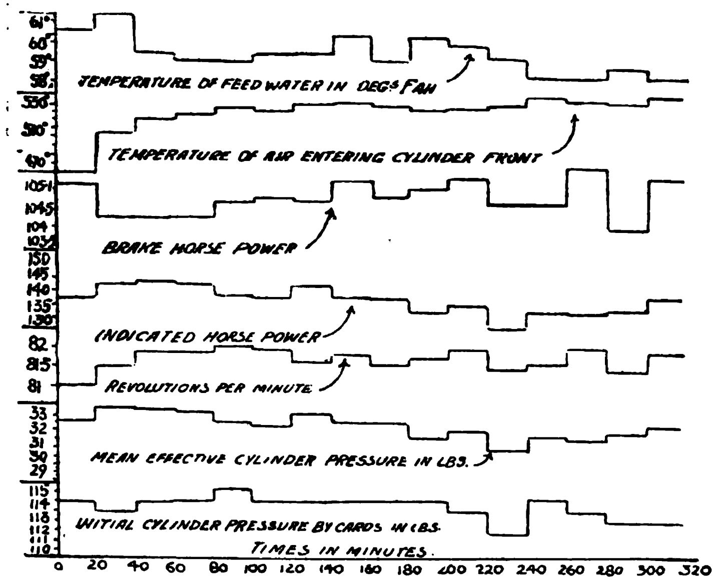
Re HOT A

1	18	19	20
Time.	ean essure of t Air pplied lbs.)	Temp. of Air entering Blower.	Temp. of Air entering Jacket.
p.m. 1.5	·0	...	395
2.1	·1	...	415
2.3	·9	74° F.	426
2.5	·0	...	410
3.1	·75	...	405
3.3	·75	...	385
3.5	·7	78° F.	375
4.1	·7	...	370
4.3	·65	...	375
4.5	·6	...	365
5.1	·6	...	360
5.3	·6	80° F.	358
5.5	·6	...	372
6.1	·6	...	370
6.3	·6	81° F.	365
6.5	·6	...	360
Mea	·74	...	381·6

† Rise of temperatur

This flywheel was encircled by four parts of a strong and flexible rope, 1 inch in diameter. The inner ends of this rope were attached to a spring-balance, tightening gear, and wooden beam, while the outer ends were connected to a fixed weight, consisting of nearly 1,000 lbs. of cast iron for the first day's trial, and about one-third of that for the second day's run.

In other words, the dynamometer was an excellent and large example of what has now come to be termed the "Society of Arts' brake." It worked perfectly, and there was no undue heating anywhere. This was no doubt partly due to the stream of water which played on the inside of the ⊔ shaped flywheel, to the large surface, and to the strong draught caused by the fan action



GRAPHIC DIAGRAM OF THE CHIEF RESULTS OF THE TESTS OF FIELD'S COMBINED STEAM AND HOT-AIR ENGINE.

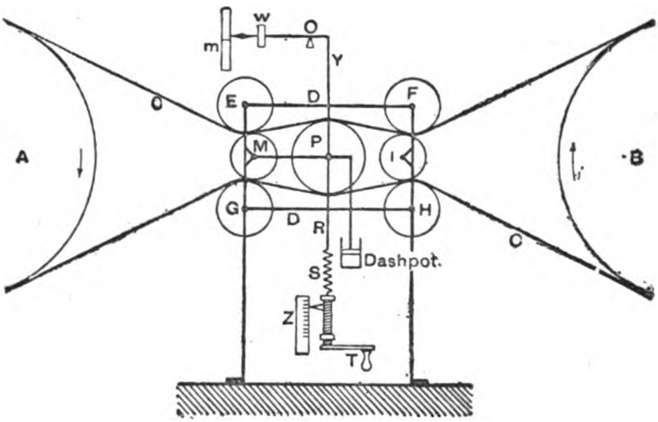
of the wheel. As far as possible, however, no water was permitted to get between the rope and the flywheel, and no lubrication of any kind was applied to these parts.

Counter.—The number of strokes and revolutions per minute were obtained by aid of a "Harding's counter" fixed to the crank shaft.

Tests.—First of all the permanent data marked at the top of the

table were carefully checked. Then, simultaneous observations were taken every twenty minutes of each of the items marked in columns 1, 2, 3, 7, 8, 11, 12, 13, 14, 18, 20, 21, 22, 23, and 24. The figures in columns 4, 5, and 6, relating to the mean pressure in the cylinder, were obtained from the indicator cards which were also taken from each end of the cylinders every twenty minutes. The figures in the other columns were either calculated or observed as required at the times stated in the respective columns. The more important observations and calculations are drawn to scale on the foregoing graphic diagram.

Transmission Dynamometers*—von Hefner-Alteneck or Siemens' Dynamometer.—Transmission dynamometers may be divided into two classes—(1) those which help to measure the work transmitted by a belt or set of ropes; (2) those which help to measure the work transmitted by a shaft. Of the first class, one form which has been used largely in dynamo tests is the Alteneck-Siemens' dynamometer. The general arrangement of this instrument is shown by the following diagram.† Power is transmitted from the



ALTENECK-SIEMENS' TRANSMISSION DYNAMOMETER.

* For a description of Morin's Traction, Rotatory, and Integrating Dynamometers, see Prof. Macquhorn Rankine's *Manual of the Steam Engine and other Prime Movers.* For a description of Morin's, Webber's (similar to White's), Brigg's (modification of the Alteneck-Siemens'), Tatham's, Brackett's, Webb's, Hartig's, Emmerson's, Van Winkle's, and Flather's transmission dynamometers, see *Dynamometers and the Measurement of Power*, by John J. Flather, Ph.B., published by John Wiley & Sons, New York.

† The above figure is reproduced by permission from "The Electrician Series" of *Motive Power and Gearing*, by E. Tremlett Carter.

pulley A to the pulley B, through a flexible leather belt, C, which passes through the dynamometer. The apparatus consists of a rectangular framework, D, with idle pulleys pivoted at the four corners, E, F, G, H, and two other idle pulleys, I and M, also pivoted to the frame. In addition, there is a movable pulley, P, attached to a lever having its fulcrum at M, and its other end connected to a dash-pot or pump-brake, whereby any sudden jerking motion is lessened. The tight side of the belt tends to lift this pulley, P, while the slack side presses it down. Attached to the lever at the centre of P, are two vertical rods, R and Y. The rod, R, terminates at its lower end in a spiral spring, S, the tension of which is adjusted by the handle, T, and indicated on the scale, Z. The rod, Y, terminates at its upper end in a small lever, pivoted at O. This lever carries a weight, W, and is provided with a pointer which travels over a scale on which there is a zero mark, *m*. The instrument having been fixed in position, the handle, T, is turned so as to bring the pointer of the top lever to the mark, *m*. The reading on the scale, Z, is then noted, and the engine started. Instantly, the upper pointer will fall below the mark, *m*, on account of the pulley, P, being lifted; but this must now be rectified by turning the handle, T, so as to increase the pull of the spring, S, until the pointer again stands at *m*. The reading on the scale, Z, is again noted, and the former reading subtracted from it. Simultaneously with these observations, the velocity of the belt must be observed by measurement of the speed of the pulley B. Let r be the radius of the pulley B in feet, n its revolutions per minute, P the difference between the two readings on the spring scale, Z, and k a constant which is marked on the instrument. Then:—

$$\text{H.P.} = \frac{2\,\pi\,r\,n\,\text{P}}{33{,}000\,k},$$

which is the same formula we had before in the case of the Prony brake and the other forms of absorption dynamometers with the exception of the constant k.

The difference between the two readings on the scale, Z, constitutes the difference of the tensions in the rod R when the belt is doing work and idle, and is proportional to the difference of tensions in the two sides of the belt when driving the pulley B. The constant k is necessary from the fact, that only the vertical components of the tensions in the belt affect the rod R. If θ be the angle between the belt (where it leaves the pulley P) and the vertical diameter through P; T_d and T_s the tensions in the direct

or tight and slack sides of the belt respectively, and v the velocity of the belt in feet per minute (or $2\pi r n$), then :—

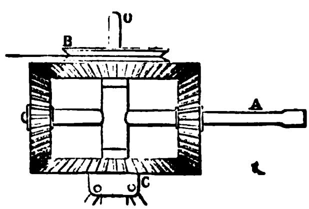
Nett pull on spring S = P = Difference of tensions in rod R.

$$\therefore \qquad P = 2\,T_d \cos\theta - 2\,T_s \cos\theta = 2\cos\theta\,(T_d - T_s).$$

$$i.e., \qquad T_d - T_s = \frac{P}{2\cos\theta},\ \text{and } k = 2\cos\theta.$$

$$\text{Again, the} \quad H.P = \frac{v\,(T_d - T_s)}{33{,}000} = \frac{2\pi r n \times P}{33{,}000 \times 2\cos\theta} = \frac{2\pi r n P}{33{,}000\,k}.$$

Rotatory Transmission Dynamometers — Epicyclic Train,* or King's, White's, and Webber's Dynamometers.—The term "epicyclic train dynamometer" is applied to those of the second class of transmission dynamometers which help to measure the work in a shaft by transmitting the same through an epicyclic train. The effort exerted is measured by means of the force required to keep the train-arm at rest. The following figure will serve to explain the principle (although not the full details) of King's, White's, and Webber's transmission dynamometers. The bevel wheel, B, is driven by a motor, and it transmits its motion through the intermediate wheels on the arm, A, to the bevel wheel, C, which is connected to the working machinery. The train-arm, A, is kept steady and level by a weight or spring attached thereto. There is usually a counter weight on a short extension of this arm, A, on the left-hand middle bevel wheel, which weight serves to balance the longer arm, A. Suppose the arm, A, to be permitted to revolve, then no work would be transmitted from B to C, and C would, therefore, remain stationary. In this case, the number of rotations of A, in a given time, would only be half that of B. Consequently, a weight placed on B at a certain radius from its centre would balance double that weight at the same radius on the arm, A. Therefore, the moment of the force applied to the arm, A (relatively to the common axis of A, B, and C), must be double the moment of the force transmitted from A to C when the arm, A, is balanced. Hence, if r be



EPICYCLIC TRAIN DYNAMOMETER.

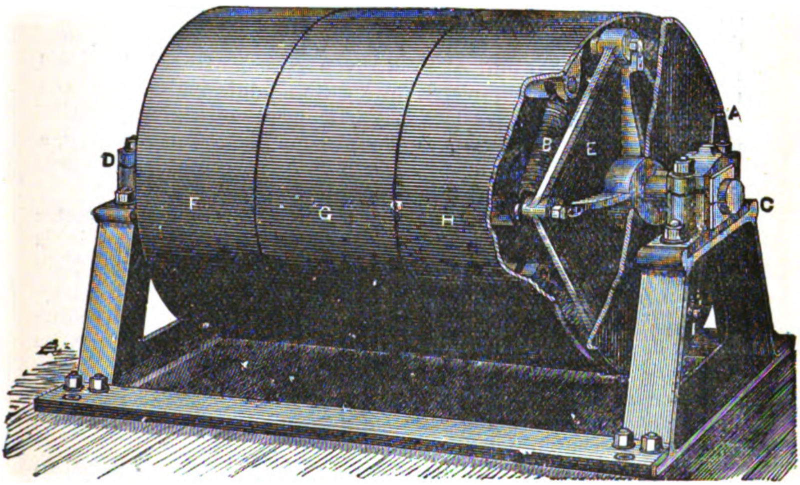
* The word epicyclic is derived from the Greek words ἐπί, signifying *upon*, and κύκλος, a *circle*. Hence this term for a wheel or wheels travelling around a circle or another wheel.

the radius of the pull, P, applied to A, and n the number of revolutions per minute of B and C, then :—

$$\left.\begin{array}{l}\text{The work transmitted}\\ \quad\text{per minute}\end{array}\right\} = \frac{2\,\pi\,r\,n\,P}{2}.$$

$$\text{And, the} \qquad \text{H.P.} = \frac{2\,\pi\,r\,n\,P}{2 \times 33{,}000} = \frac{\pi\,r\,n\,P}{33{,}000}.$$

Spring Dynamometers—Ayrton and Perry's and Van Winkle's Transmission Dynamometers.—Another kind of dynamometer belonging to the second class is that wherein springs, placed at



PROFS. AYRTON & PERRY'S TRANSMISSION DYNAMOMETER.

a certain radius from the centre of the rotating shaft, help to measure the torque therein when transmitting power.

One of the simplest and most easily understood is that devised by Profs. Ayrton and Perry, of the City and Guilds of London Technical Institute. A very similar instrument has been constructed by Mr. Van Winkle of U.S. America, and supplied to a firm in Chili for measuring up to 600 horse-power at 120 revolutions per minute. This one is believed to be the largest transmission dynamometer ever constructed.

The apparatus illustrated by the foregoing figure, consists of a pulley, F, rigidly fixed to the shaft, C D, a loose pulley, G, and

a pulley, H, joined by the spiral springs, B, to the ribbed plate, E, which is also rigidly fixed to the shaft, C D. If the motor belt be on F, and the belt to the dynamo or driven machine on H, or *vice versâ*, the springs, B, will be stretched, depending on the "torque" or twist transmitted. The extension of these springs by means of a small link-motion (seen at the lower right-hand corner of the figure) causes the bright bead, A (at the end of a long arm), to approach towards the centre of the shaft. Hence, the smaller the radius of the circle described by this bright bead as it revolves, the greater the torque.* Consequently, the horse-power transmitted is at once obtained from observing the indicated torque and the speed of rotation. The arm carrying the bead is slightly flexible, and when no power is being transmitted the bead is pressed with a certain force against the rim of the front plate, hence the bead does not commence to move until a certain pre-arranged horse-power is being transmitted at a given speed. Its whole radial motion is, therefore, completed for a certain additional transmitted horse-power. The necessary addition depends on the strength of the springs and the leverage of the link-motion. Consequently, a large change in the radius of the circle of the bright bead is produced by a small change in the transmitted horse-power.

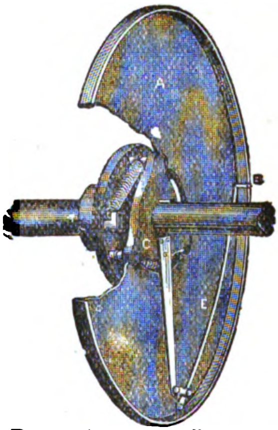
The next figure shows Profs. Ayrton and Perry's dynamometer coupling, which differs only from the preceding in that it is intended to be used with machinery driven directly by shafting where belting is not employed. For instance, this coupling may be used to measure the horse-power given by a fast-speed engine to a dynamo or other machine driven directly by it, or it may be employed to measure the power given by a marine engine to the screw or to the paddles, or generally the horse-power transmitted along any line of shafting; the spring coupling, in fact, replacing the ordinary coupling used with such shafts.

One of the halves of the coupling seen in the figure is keyed to the driving shaft—for example, the shaft of a fast-speed engine; and the other to the driven shaft—for example, that of the dynamo. The half, C, is attached to the other half by means of the spiral springs, and the stretching of these is therefore a measure of the torque. The angular motion of the one relatively to the other causes the bright bead, B, to approach the centre, and, as before, the radius of the circle of light helps one to measure the horse-power transmitted at any particular speed.

The transmission dynamometer and dynamometer coupling just

* The word torque was first suggested by Prof. James Thomson of Glasgow University, and means the turning moment or the turning force multiplied by its distance from centre of shaft.

described have the great advantage over any sort of laboratory dynamometers, in that the former have not to be put into position and adjusted for each particular experiment, but are always ready, and are always indicating the power transmitted at any given speed. If, for example, a dynamometer coupling be inserted in the shafting of a factory in place of the ordinary coupling, a glance at it, at any time, will show the power that is being transmitted by it. If two such dynamometer couplings be inserted at two places in the same set of shafting, the difference between the transmitted powers indicated by them is the power utilised by the machinery driven by that portion of the shafting that is between them.



PROFS. AYRTON & PERRY'S DYNAMOMETER COUPLING.

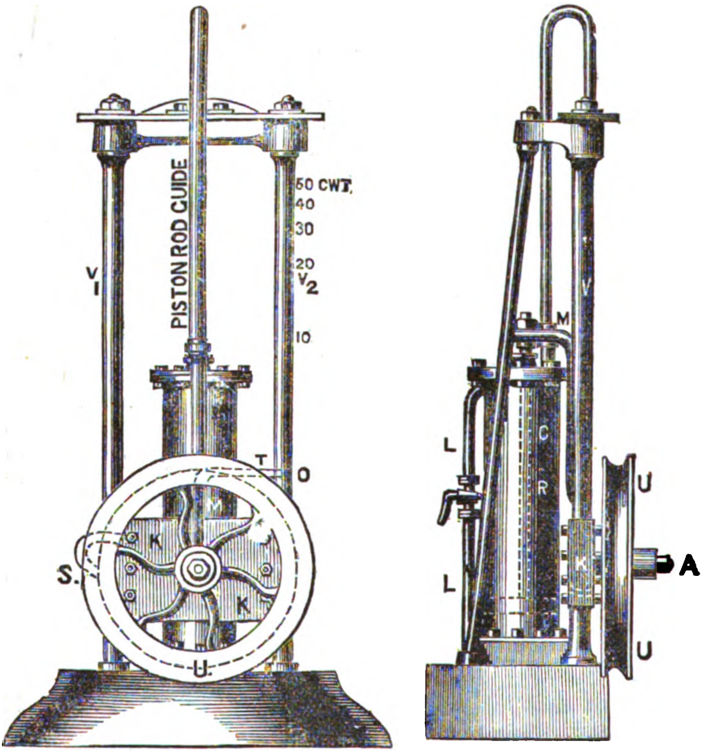
Hydraulic Transmission Dynamometers — Flather's * and Cross's. — Owing to a want of confidence in results obtained by aid of spring dynamometers, Prof. Flather and Mr. J. A. Cross (both of the U.S. America) have independently perfected two forms of hydraulic transmission dynamometers, which are said to be reliable. The construction and action of these instruments are as follows:—The power shaft is keyed to a boss or pulley with two or more arms carrying hydraulic cylinders. The projecting ends of the plungers of these cylinders, bear upon the arms of a loose pulley on the same shaft. The torque imparted by the driving belt to the loose pulley is thus transmitted to the shaft through the liquid in the cylinders. The pressure thus caused in the liquid is conveyed by radial pipes to a common central trunnion, and from thence to a pressure gauge or indicator.

This apparatus has many advantages—(1) It is simple. (2) It is not affected to any great extent by the velocity of the shafting. (3) It requires no counter shaft, and no change of driving belt. (4) It takes the place of an ordinary driving pulley, and is driven by the same belt. (5) It may be connected to a recording gauge, and thus a continuous diagram of the load may be obtained with-

* Professor Flather's arrangement is described in his book on *Dynamometers and the Measurement of Power*, already referred to by two footnotes, and Mr. Cross's apparatus in *The Electrical Engineer* of New York, July 4, 1894, p. 3. An earlier form by von Hefner Alteneck is described in *Industries* of February 3, 1888, at page 122.

out any special attention. (6) It does not require to be displaced after a test is completed, for, by the simple closing of a cock or valve, the recording apparatus may be disconnected and the remainder left as an ordinary pulley.

Tension Dynamometer for Submarine Cables.—By referring to the figure of a telegraph steamer in a previous article of this



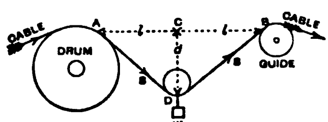
TENSION DYNAMOMETER FOR SUBMARINE CABLES.

Lecture, the student will understand the positions and use of the dynamometer on board a cable ship. As will be seen from the following figure, this apparatus consists of a vertical cylinder, C, filled with oil or soapy water, and containing a piston and a piston-rod, R, passing through a gland and stuffing box at M. This piston-rod is connected to a crosshead, K, which is free to move

up and down between the upright guides, V_1 V_2. On the outstanding turned pin, A, of the crosshead, K, there is carried an unkeyed grooved pulley, U, under which the cable is passed. To the top of the crosshead is fixed a pointer, T, which indicates the height to which the pulley, U, may be elevated, and consequently the stress on the cable or grapnel rope.

In order to prevent sudden jerks and oscillations of the moving parts, the top and bottom of the cylinder, O, are connected by a pipe, L, with a cock at its centre. The cylinder piston and liquid thereby act as a "pump-brake" or dash-pot, with greater or less freedom according to the opening of this cock. When paying out a heavy cable, or one in deep water, additional weights may be attached to the arm, A, in order to render the dynamometer less sensitive. In order to keep the pulley, U, always clean a curved scraper, S, is applied to the groove when desirable. The vertical scale may be marked off by calculation according to the following formula, but it should also be verified by an actual test, since this rule does not take friction into account.

In order that the friction between the dynamometer-crosshead, and its guides, shall be a minimum, the dynamometer (D in the following figure) *must* be placed midway between the point, A, where the cable bears on the paying-out or picking-up drum, and the point, B, where it bears on the guide-pulley next to the stern or bow sheaves, and these bearing points, A and B, should be in a horizontal line.



STRESS DIAGRAM FOR A SUBMARINE CABLE DYNAMOMETER.

Let S = Stress on cable or rope (in cwts.) to be found.

„ W = Weight (in cwts.) of all moving parts in dynamometer.

„ l = Distance A C or C B (in inches).

„ d = Deflection of cable from horizontal (in inches).

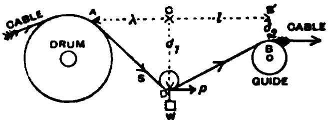
Then by the parallelogram of forces :—

$$S = W \frac{\sqrt{l^2 + d^2}}{2\,d} \text{ (cwts.)}$$

$$\therefore \quad d = \frac{W\,l}{\sqrt{4\,S^2 - W^2}} \text{ (inches).}$$

Since the stresses and deflections of the cable are approximately in inverse ratio to one another, and W and l are constant, it is only necessary to work out one example for d, plot it off on the dynamometer scale, and mark the others in the inverse ratio—*e.g.*, for double the stress half the deflection, and so on.*

If the points, A and B, are not in a horizontal line, and the dynamometer, D, not midway between them, as shown in the following figure, then the calculation becomes more complicated.



SPECIAL STRESS DIAGRAM FOR A SUBMARINE CABLE DYNAMOMETER.

Let S and W = Same as before (in cwts.)
„ p = Horizontal pressure on guides of D (in cwts.)
„ λ = Horizontal distance A C.
„ l = „ „ C B'.
„ d_1 = Deflection C D (in inches).
„ d_2 = Vertical height B B' (in inches).

Then, $$S = \frac{W}{\dfrac{d_1}{\sqrt{d_1^2 + \lambda^2}} + \dfrac{d_1 - d_2}{\sqrt{(d_1 - d_2)^2 + l^2}}}$$

And, $$p = S\left(\frac{l}{\sqrt{(d_1 - d_2)^2 + l^2}} - \frac{\lambda}{\sqrt{d_1^2 + \lambda^2}}\right)$$

If, however, the points, A and B, are in horizontal lines—*i.e.*, $d_1 = d$, and $d_2 = O$, but D not midway between them.

Then, $$S = \frac{W}{\dfrac{d}{\sqrt{d^2 + \lambda^2}} + \dfrac{d}{\sqrt{d^2 + l^2}}}$$

* The distance, l, between the guide pulley and the dynamometer is so great compared with the deflection, d, of the dynamometer, that the above rule is practically correct.

Lecture VIII.—Questions.

1. State the various ways in which friction may be applied usefully. Sketch and describe a good friction clutch or coupling. State the advantages and disadvantages of friction clutches.

2. Sketch, with an index to parts, and give a concise description of, the following pieces of mechanism—(1) Addyman's friction coupling; (2) Bagshaw's hollow sleeve clutch.

3. Sketch and describe (1) Weston's friction coupling and brake; (2) Weston's centrifugal friction pulley; (3) Robertson's grooved disc friction coupling.

4. Sketch and describe a friction brake as applied to a crane. The lever applied to the strap is a bent lever, of which one arm is 2 feet 11 inches long, and the other arm, which is at right angles to it, is $3\frac{3}{4}$ inches long, the diameter of the friction drum being 2 feet; find the tension of the movable end of the strap when a pressure of 100 lbs. is applied to the handle, and the tension at the fixed end for a given coefficient of friction. *Ans.* 933·3 lbs.; 1,495 lbs. or 582·6 lbs. according to direction of rotation, taking $\mu = 0{\cdot}1$, and $\theta = 270°$. (S. & A. Hons. Exam., 1886.)

5. With the assistance of sketches describe the construction of two kinds of brakes, one in which a resisting force of moderate magnitude is overcome through a considerable distance, and the other in which a considerable resistance is overcome through a comparatively small distance. (S. & A. Hons. Mach. Const. Exam., 1895.)

6. Prove by a skeleton sketch and mathematical investigation the proper direction of rotation of a brake-wheel (with respect to its strap and lever connections) in the case of a winch or crane when the load is being lowered.

7. A strap, bearing on a brake wheel 2 feet in diameter, and tightened by a lever, is used to hold the load on a winch. The shaft to which the brake wheel is keyed also carries a pinion of 10 teeth, gearing with a wheel of 54 teeth on a second shaft. This second shaft has a pinion of 9 teeth gearing with another wheel of 50 teeth on the drum shaft. The diameter of the drum is 12 inches, the length of the handle of the lever is 30 inches, and of the short end 3 inches. If one end of the strap, which subtends an angle of 300° at the centre of the wheel, be fixed, and the other attached to the short end of the lever, find the greatest load on the rope wound on the drum that could be supported by a force of 45 lbs. applied at the end of the lever handle. Take $\mu = 0{\cdot}1$. *Ans.* 18,630 lbs.

8. If a weight of 16,100 lbs., attached to the rope in the last question, is descending with a velocity of 300 feet per minute, find how far it will go after the brake is put on before coming to rest. The kinetic energy of the wheels may be neglected. *Ans.* 4 inches.

9. Explain the use, construction, and position of brake wheels in telegraph cable steamers.

10. Sketch and explain Lord Kelvin's deep-sea sounding machine, including a side elevation and plan of his differential rope-brake for the same.

11. Describe a method of obtaining the brake horse-power of an engine, and state the advantages to buyer and seller of adopting this method over that of nominal or indicated horse-power. An engine is making 150 revolutions per minute, the diameter of the brake pulley being 4 feet, and the pull on the brake 50 lbs., what is the B.H.P.? *Ans.* 2·85.

12. Sketch, and describe with an index to parts, some good form of absorption friction dynamometer. The pulley on the crank shaft to which

the brake is fitted is 3 feet in diameter, and makes 100 revolutions per minute. When the engine is at work, a Salter's balance, fixed at a point 21 inches from the axis of the shaft, registers 200 lbs. Find the brake horse-power of the engine. Prove the formula you use. *Ans.* 6·6 H.P.

13. Describe the ordinary friction dynamometer. If the shaft of an engine being tested makes 20 revolutions per minute, and the weight supported be 200 lbs., the point at which it is supported being 3 feet from the axis of the dynamometer, find the horse-power of the engine. *Ans.* 2·28 H.P.

14. In a friction brake dynamometer a weight of 93 lbs. is hung at a distance of 31¼ inches from the centre of the wheel. The brake wheel is driven by a pulley 5 feet in diameter, on the same axis, which carries a belt from the flywheel of an engine and makes 200 revolutions per minute. Explain the theory of the apparatus and find the horse-power exerted by the engine. *Ans.* 9·3 H.P.

15. State and prove wherein you consider "Appold's Compensating Lever Brake Dynamometer" defective. What precautions or alterations in this apparatus should be given effect to, in order to obtain accurate results with it?

16. Sketch and describe the rope-brake dynamometer, and state its advantages over other forms of absorption dynamometers for ascertaining the B.H.P. of an engine. What benefits are claimed in certain cases for the use of two spring balances instead of a weight and a spring balance with this apparatus? A flywheel is 10 feet diameter and rotates at 100 revolutions per minute, whilst the mean dead load is 1,000 lbs., and the back pull 100 lbs. Find the B.H.P. Supposing that the mechanical efficiency of the engine is 80 per cent., what would be the corresponding I.H P.? *Ans.* 85·7 B.H.P.; 107 I.H.P.

17. Explain the epicyclic train form of transmission dynamometer, and prove the formula for the same.

18. Explain by a sketch and index to parts, a transmission power dynamometer by which the difference in the tensions of the two sides of the driving belt is measured. Explain the advantages of this instrument over the absorption dynamometer.

19. Explain and illustrate a form of spring transmission dynamometer and coupling.

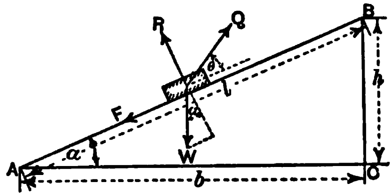
20. Explain and illustrate a form of hydraulic transmission dynamometer.

21. Explain and illustrate a tension dynamometer as used in the paying out or picking up of submarine cables, and indicate by sketches where this apparatus is placed on board a submarine cable steamer. What are the most advantageous conditions for the employment of such a dynamometer? Prove the formula for graduating the scale.

LECTURE IX.

CONTENTS. — Inclined Plane — Examples I., II., and III. — The Double Inclined Plane — Examples IV. and V. — Screws — Efficiency of Screws — Maximum Efficiency of Screws — Non-Reversibility of Ordinary Screws and Nuts — Tension in Bolts due to Screwing up — Example VI. — Questions.

Inclined Plane.—We now proceed to determine the relation between Q and W in the inclined plane when friction is taken into account. The most general case occurs, when the direction of Q makes a given angle θ, with the inclined plane A B. We shall, therefore, consider this case first, since all other particular cases can be easily deduced therefrom.



GENERAL CASE OF THE INCLINED PLANE.

Let R = Reaction perpendicular to the plane.
 „ μ = Coefficient of friction.
 „ F = μ R = Friction between body and plane.

By the "*Principle of Work*," we get:—

Work done by Q = *Work done on* W + *Work done against* F.

Suppose the body to be dragged along the plane a distance A B = l.

Then:—

Work done by Q = $Q \cos\theta \times AB = Q\, l \cos\theta$.
Work done on W = $W \times BC = W h$.
Work done against F = $F \times AB = \mu R l$.

$$\therefore \qquad Q\, l \cos\theta = W h + \mu R l.$$

We must now eliminate R. The simplest way to effect this, is to consider the equilibrium of the forces acting on the body when Q is just about to draw the body up the plane.

Resolving the forces at right angles to A B, we get:—

$$R + Q \sin\theta = W \cos\alpha.$$

$$\therefore \qquad R = W \cos\alpha - Q \sin\theta.$$

By substituting this value of R in the above equation, we get :—

$$Q\,l \cos\theta = W\,h + \mu\,W\,l \cos\alpha - \mu\,Q\,l \sin\theta.$$

$$\therefore \quad Q\,l(\cos\theta + \mu\sin\theta) = W(h + \mu\,l\cos\alpha).$$

Or,
$$\frac{Q}{W} = \frac{h + \mu\,l\cos\alpha}{l(\cos\theta + \mu\sin\theta)} \quad \ldots\ldots \quad (\mathrm{I})$$

This equation can be put into a more convenient form, thus :—

$$\frac{Q}{W} = \frac{\frac{h}{l} + \mu\cos\alpha}{\cos\theta + \mu\sin\theta}.$$

But, $\frac{h}{l} = \sin\alpha$; and $\mu = \tan\varphi = \frac{\sin\varphi}{\cos\varphi}$; where φ = angle of friction.

$$\therefore \quad \frac{Q}{W} = \frac{\sin\alpha\cos\varphi + \sin\varphi\cos\alpha}{\cos\theta\cos\varphi + \sin\theta\sin\varphi}$$

i.e.,
$$\frac{Q}{W} = \frac{\sin(\alpha + \varphi)}{\cos(\theta - \varphi)} \quad \ldots\ldots \quad (\mathrm{II})$$

From this *general* equation the results for any particular case can be deduced.

Case I.—*Suppose the plane to be smooth.* Then $\varphi = 0$ and $Q = P$, the theoretical force required.

$$\therefore \quad \frac{P}{W} = \frac{\sin\alpha}{\cos\theta} \quad \ldots\ldots \quad (\mathrm{II}_a)$$

Case II.—*Suppose* Q *acts parallel to* A B, then $\theta = 0$.

$$\therefore \quad \left.\begin{aligned} \frac{Q}{W} &= \frac{\sin(\alpha + \varphi)}{\cos\varphi} \\ \text{,,} &= \sin\alpha + \mu\cos\alpha \end{aligned}\right\} \quad \ldots\ldots \quad (\mathrm{II}_b)$$

$$\text{,,} = \frac{h + \mu\,b}{l} \quad \ldots\ldots \quad (\mathrm{II}_c)^*$$

$$\therefore \quad Q\,l = W\,h + \mu\,.\,W\,b \quad \ldots\ldots \quad (\mathrm{II}_d)^*$$

Or stated in words :—

The work done in raising a body up a rough inclined plane is equal to the work done in lifting it vertically through the height of the plane, together with the work done in dragging it along the base supposed to be of the same roughness as the plane itself.

* These are two important results which we shall frequently refer to in what follows.

In this case, *when* Q is *parallel to* A B, we get:—

$$\text{Actual Advantage} = \frac{W}{Q} = \frac{\cos\varphi}{\sin(\alpha+\varphi)} \quad . \quad . \quad . \quad . \quad \text{(III)}$$

Or, from (II_c),

$$\text{Actual Advantage} = \frac{l}{h+\mu b} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(IV)}.$$

$$\text{Efficiency} \; . \; . \; . \; . \; = \frac{W h}{Q l} = \frac{W}{Q}\sin\alpha = \frac{\sin\alpha\cos\varphi}{\sin(\alpha+\varphi)} \quad \text{(V)}$$

Or, from (II_c),

$$\text{Efficiency} \; . \; . \; . \; . \; = \frac{h l}{h l + \mu b l} = \frac{h}{h+\mu b} \quad . \quad . \quad . \quad \text{(VI)}$$

Case III.—*Suppose* Q *acts parallel to* A C. Then $\theta = -\alpha$.

$$\therefore \qquad \frac{Q}{W} = \frac{\sin(\alpha+\varphi)}{\cos\{-(\alpha+\varphi)\}} \text{ [equation (II)].}$$

$$\text{i.e.,} \qquad \frac{Q}{W} = \tan(\alpha+\varphi) \; . \quad . \quad . \quad . \quad . \quad . \quad . \quad (II_d).*$$

An application of this case will be met with when we come to treat of screws.

We shall now prove the following:—

Proposition.—**For a given inclination, α, of the plane and angle of friction, φ, the effort Q will have its least value when $\theta = \varphi$; *i.e.*, when the direction of Q makes an angle with the inclined plane equal to the angle of friction.**

$$\text{For,} \qquad Q = W\frac{\sin(\alpha+\varphi)}{\cos(\theta-\varphi)} \text{ [equation (II)].}$$

Now, Q will be a *minimum* when the fraction $\dfrac{\sin(\alpha+\varphi)}{\cos(\theta-\varphi)}$ is a *minimum*. But in this fraction the numerator, $\sin(\alpha+\varphi)$, is a constant quantity, since α and φ are supposed to be given.

$\therefore$ Q will be a *minimum* when $\dfrac{1}{\cos(\theta-\varphi)}$ is a *minimum*.

i.e., Q ,, ,, ,, $\cos(\theta-\varphi)$,, *maximum*.

* The student should be able to prove the results given in the above three cases independently of the general case considered in the text. (See the author's *Elementary Manual of Applied Mechanics*.)

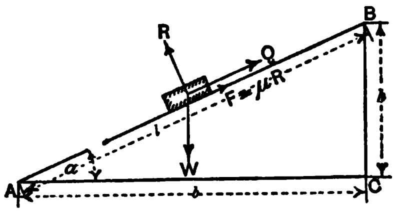
Now, the *maximum* value of a cosine is unity, and this only occurs when the angle is zero.

Hence, Q will be a *minimum* when $(\theta - \varphi) = O$.

i.e., Q " " " $\theta = \varphi$.

This proves the proposition.

In what has preceded we have supposed the effort Q just able to move the body up the plane. We shall now consider the



BODY JUST SLIDING DOWN THE INCLINED PLANE.

cases where Q just prevents the body from sliding down the plane; or, when Q is employed to draw the body downwards.

Case IV.—*When* Q *is parallel to* A B *and prevents the body from sliding down the plane.*

In this case, by resolving along the plane, we get:—

$$Q + F = W \sin \alpha.$$

$$\therefore \quad Q = W \sin \alpha - \mu R.$$

Resolving the forces at right angles to the plane, we get:—

$$R = W \cos \alpha.$$

$$\therefore \quad Q = W \sin \alpha - \mu W \cos \alpha.$$

$$\text{Or,} \quad Q = W (\sin \alpha - \mu \cos \alpha) \quad . \quad . \quad . \quad . \quad (\text{VII})$$

$$\text{i.e.,} \quad Q = W \frac{h - \mu b}{l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\text{VIII})$$

From equation (VII) it will be evident that the body will have no tendency to slide down of its own accord if:—

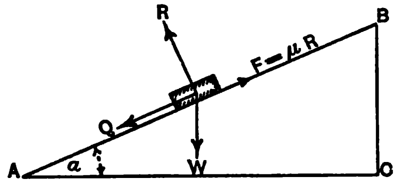
$$\sin \alpha \leqq \mu \cos \alpha.$$

i.e.—if, $\tan \alpha \leqq \mu$.

i.e.—if, $\alpha \leqq \varphi$ (the angle of friction

When α is just slightly greater than ϕ the body would begin to slide down of its own accord, if not prevented by the force, Q. This affords a means of determining the coefficient of friction between two bodies, as explained in Lecture V.

If α is less than ϕ, then an effort, Q, must be applied to the body to drag it down the plane. Then, we get :—

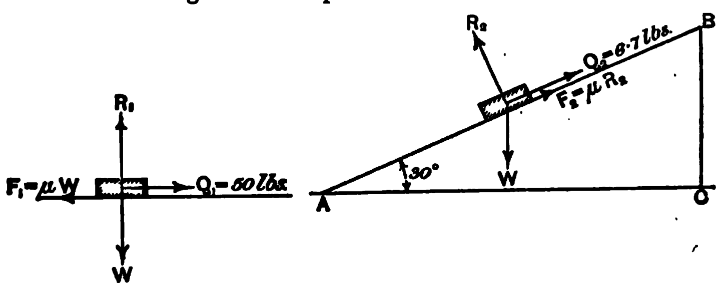


Force Required to Pull the Body Down the Plane.

$$Q + W \sin\alpha = F = \mu R = \mu W \cos\alpha,$$

$$\therefore \qquad \left.\begin{aligned} Q &= W(\mu\cos\alpha - \sin\alpha) \\ \text{Or,} \qquad Q &= W\,\frac{\mu b - h}{l} \end{aligned}\right\} \quad \ldots\ldots \quad \text{(IX)}$$

Example I.—A horizontal force of 50 lbs. is just required to move a weight of W lbs. on a rough horizontal plane. If the plane be now inclined at an angle of 30°, a force of 6·7 lbs. acting parallel to the plane is required to keep the weight from sliding down. Determine the weight, W, and the coefficient of friction between the weight and the plane.



To Determine the Coefficient of Friction.

Answer.—On the horizontal plane, we have :—

$$F_1 = \mu W = Q_1,$$

$$\therefore \qquad \mu = \frac{50}{W} \quad \ldots\ldots\ldots\ldots \quad (1)$$

Since the body is just kept from sliding down the inclined plane, both Q_2 and F_2 will act up the plane. Then:—

$$Q_2 + \mu R_2 = W \sin 30°,$$

$$\therefore \quad Q_2 = W (\sin 30° - \mu \cos 30°),$$

$$\therefore \quad 6{\cdot}7 = W\left(\tfrac{1}{2} - \mu \times \frac{\sqrt{3}}{2}\right).$$

Substituting the value of μ given in equation (1), we get:—

$$6{\cdot}7 = \frac{W}{2}\left(1 - \frac{50 \times \sqrt{3}}{W}\right).$$

$$\therefore \quad 13{\cdot}4 = W - 50\sqrt{3}.$$

$$\therefore \quad W = 13{\cdot}4 + 50 \times 1{\cdot}732 = 100 \text{ lbs.}$$

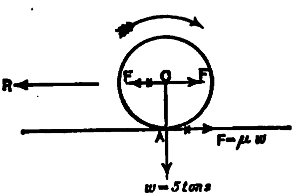
From equation (1), we get:—

$$\mu = \frac{50}{100} = {\cdot}5.$$

Example II.—Suppose a locomotive weighs 30 tons, and that the share of this weight borne by the driving wheels is 10 tons. Then, if the coefficient of friction between the wheels and the rails be ·2, what load will the engine draw on the level if the required coefficient of traction be 10 lbs. per ton of train load? What load will this engine draw at the same rate up an incline of 1 in 20?

Answer.—(1) *On the level line.*

Let the circle represent one of the driving wheels of the locomotive, and let the wheel turn in the direction shown by



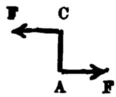
Question on Traction and Friction.

the arrow. Since there are two driving wheels the weight, w, on each will be 5 tons.

Let F = Friction between each wheel and its rail.
„ μ = Coefficient of friction between wheel and rail = ·2.

Then, $F = \mu w = {\cdot}2 \times (5 \times 2{,}240) = 2{,}240$ lbs., and acts in the direction A F.

At C, the centre of the wheel, introduce two opposite forces, F, F, each equal and parallel to the force F at A. This will not affect the equilibrium of the system. After this has been done, it will be evident that the forces standing thus :—



form a *couple*, the moment of which is F × A C. This is the couple resisting the rotation of the wheel about its centre, C, and, therefore, must be equal in moment but opposite in sign, to the couple due to the force on the crank-pin as caused by the steam pressure on the piston. In the meantime, we are concerned only with the remaining force, F, acting to the right at C. This force tends to pull the centre C, and, therefore, the whole train to the right. If R be the resistance offered by the train, then, since there are two driving wheels :—

$$R = 2\,F = 2 \times 2{,}240 = 4{,}480 \text{ lbs.}$$

Let W_1 = Total weight of engine and train in tons.

Then, since the traction is 10 lbs. per ton, we get :—

$$R = 10\,W_1,$$

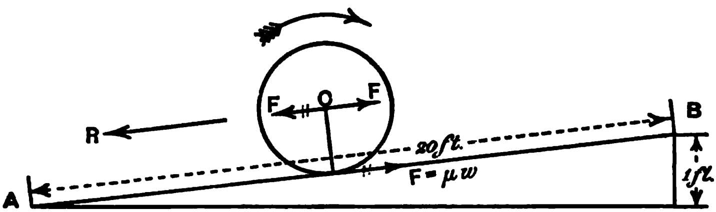
$\therefore$ $$10\,W_1 = 4{,}480,$$

i.e., $$W_1 = 448 \text{ tons.}$$

The engine will, therefore, be able to draw a load of (448 – 30), or 418 tons without fear of the driving wheels slipping on the rails.

(2) *On the gradient.*

Since the inclination of the rails is small (1 in 20), we may assume the pressure or reaction between the wheel and rail to be still = w tons.



QUESTION ON TRACTION AND FRICTION.

Hence, F will be the same as before, viz., 2,240 lbs.

By reasoning as in the previous case, we get :—

$$R = 2\,F = 4{,}480 \text{ lbs.}$$

Let, W_2 = Total weight in tons of engine and train on incline.

Now, suppose the train to move from A to B, a distance of 20 feet. Then, by the *Principle of Work*, we get :—

$$\left.\begin{array}{r}\textit{Total work done on in-}\\ \textit{cline from A to B}\end{array}\right\} = \left\{\begin{array}{l}\textit{Work done against traction from}\\ \quad \textit{A to B.}\\ + \textit{Work done against gravity from}\\ \quad \textit{A to B.}\end{array}\right.$$

But, *Total work done* $= R \times 20 = 4{,}480 \times 20$ (ft.-lbs.)

Work done against traction $= (10 \times W_2) \times 20 = 200\, W_2$ ft.-lbs.

" " *gravity* $= (W_2 \times 2{,}240) \times 1 = 2{,}240\, W_2$ "

$\therefore \quad 4{,}480 \times 20 = 200\, W_2 + 2{,}240\, W_2,$

i.e., $$W_2 = \frac{4{,}480 \times 20}{2{,}440} = 36{\cdot}72 \text{ tons.}$$

Thus, the engine will only be able to draw a load of (36·72—30), or 6·72 tons up an incline of 1 in 20. Any load beyond this would cause a greater resistance than is provided for by the friction between the driving wheels and the rails.

EXAMPLE III.—What must be the effective horse-power of a locomotive engine which moves at a steady speed of 40 miles an hour on a level line, the resistance being estimated at 20 lbs. per ton, and the weight of the engine and train being 200 tons? If the engine continue to exert the same power when ascending a gradient of 1 in 100, what would be the speed?

ANSWER.—(1) *On the level line.*

Total resistance overcome $= 200 \times 20 = 4{,}000$ lbs.

Speed of train $= 40$ miles per hour.

" " $= \dfrac{40 \times 5{,}280}{60} = 3{,}520$ ft. per minute.

$\therefore$ *Work done per minute* $= 4{,}000 \times 3{,}520$ (ft.-lbs.)

$$\therefore \quad \text{H.P.} = \frac{4{,}000 \times 3{,}520}{33{,}000} = 426{\cdot}6.$$

(2) *On the gradient.*

$$\text{Here, } \left.\begin{array}{r}\textit{Total work done}\\ \textit{per minute}\end{array}\right\} = \left\{\begin{array}{l}\textit{Work done against friction per}\\ \quad \textit{minute.}\\ + \textit{Work done against gravity per}\\ \quad \textit{minute.}\end{array}\right.$$

Let v = Speed of train on gradient in feet per minute.

As before, *Total work done* $= 4{,}000 \times 3{,}520$ (ft.-lbs. per minute).

Work done against friction $= (200 \times 20)\, v$ " "

Work done against gravity $= (200 \times 2{,}240) \times \dfrac{v}{100}$ " "

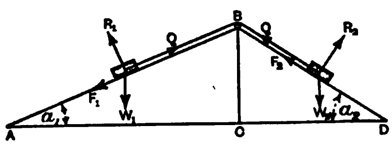
$$\therefore \qquad 4{,}000 \times 3{,}520 = 200 \times 20 \times v + 200 \times 2{,}240 \times \frac{v}{100}$$

$$\text{,,} \qquad \text{,,} \qquad = 40 \times 212 \times v.$$

$$\therefore \qquad v = \frac{4{,}000 \times 3{,}520}{40 \times 212} = 1660{\cdot}37 \text{ ft. per min.}$$

$$\text{Or,} \qquad V = \frac{60\,v}{5{,}280} = 18{\cdot}87 \text{ miles per hour.}$$

The Double Inclined Plane.—Sometimes the double inclined plane is used, when a descending load is employed to draw up another load by means of a rope passing over a fixed drum or pulley at the summit of the incline. To understand the principle of this arrangement we shall suppose two inclined planes placed back to back, as shown in the accompanying figure.



Double Inclined Plane.

Let W_1 be the ascending and W_2 the descending load.

The usual letters which we have hitherto employed with the suffixes 1 and 2 refer respectively to the inclined planes A B C and D B C.

When W_2 is just sufficient to overcome W_1, and neglecting the friction due to the pulley at B, we get for the plane A B C :—

$$Q = W_1 (\sin \alpha_1 + \mu_1 \cos \alpha_1), \text{ [from equation (II}_b\text{)]}$$

And, for the plane D B C,

$$Q = W_2 (\sin \alpha_2 - \mu_2 \cos \alpha_2), \text{ [from equation (VII)]}$$

$$\therefore \qquad W_1 (\sin \alpha_1 + \mu_1 \cos \alpha_1) = W_2 (\sin \alpha_2 - \mu_2 \cos \alpha_2).$$

$$\text{Or,} \qquad \frac{W_1}{W_2} = \frac{\sin \alpha_2 - \mu_2 \cos \alpha_2}{\sin \alpha_1 + \mu_1 \cos \alpha_1} \quad . \quad . \quad . \quad \text{(X)}$$

Putting $\frac{h}{l_1} = \sin \alpha_1$; $\frac{b_1}{l_1} = \cos \alpha_1$; and so on, the last equation may be written thus :—

$$\frac{W_1}{W_2} = \frac{l_1}{l_2}\left(\frac{h - \mu_2 b_2}{h + \mu_1 b_1}\right) \quad . \quad . \quad . \quad . \quad (\text{X}_a)$$

If $\mu_1 = \mu_2 = \mu$, we get :—

$$\frac{W_1}{W_2} = \frac{l_1}{l_2}\left(\frac{h - \mu b_2}{h + \mu b_1}\right) \quad . \quad . \quad . \quad . \quad (\text{X}_b)$$

In practice, we seldom find two planes arranged as shown above. One plane only is used, and the trucks run on parallel lines of rails, being connected by a rope or chain which passes round a pulley or drum at the top of the plane. In this case, $\alpha_1 = \alpha_2 = \alpha$; $l_1 = l_2 = l$; $b_1 = b_2 = b$; and the above equations take the simple forms:—

$$\frac{W_1}{W_2} = \frac{\sin\alpha - \mu\cos\alpha}{\sin\alpha + \mu\cos\alpha} \quad . \quad . \quad . \quad (XI)$$

Or, dividing by cos α,
$$\frac{W_1}{W_2} = \frac{\tan\alpha - \mu}{\tan\alpha + \mu} \quad . \quad . \quad . \quad . \quad . \quad (XI_a)$$

Or,
$$\frac{W_1}{W_2} = \frac{h - \mu b}{h + \mu b} \quad . \quad . \quad . \quad . \quad . \quad . \quad (XI_b)$$

This determines the relation between W_1 and W_2, *when* W_2 *is just able to draw up* W_1. If W_2 be greater than that obtained from equation (XI), the motion will be accelerated. To obtain a uniform motion for given loads, W_1, W_2, we must either adjust the inclination of the plane, or provide the drum with a friction brake when W_2 is greater than necessary, or with the assistance of an engine when W_2 is less than required.

(1) *To determine the requisite inclination of the plane when* W_2 *is required to draw up* W_1.

From equation (XI_a), we get:—

$$W_1 \tan\alpha + \mu W_1 = W_2 \tan\alpha - \mu W_2,$$

$$\therefore \quad \mu(W_2 + W_1) = (W_2 - W_1)\tan\alpha,$$

$$\therefore \quad \tan\alpha = \mu\frac{W_2 + W_1}{W_2 - W_1}.$$

(2) *To determine the friction couple, which must be applied to the pulley or drum at the top of a double inclined plane, in order to obtain a uniform motion when* W_2 *is too great.*

This problem is very similar to the case of a driving belt transmitting motion in machinery. The tension in the two parts of the rope or chain is not *now* the same throughout its length, being greater on the side of the descending load W_2.

Let Q_1, Q_2, denote the tension in the two parts of the rope or chain on the driven and driving sides respectively.

Then,
$$Q_2 > Q_1.$$

The following figure represents a side elevation and plan of the double inclined plane with its pulley and loads, W_1, W_2.

Let M = Friction couple required to be applied to the brake wheel, B W, by the brake handle, B H.
,, r = Radius of drums.

Taking moments about the centre of this wheel, we get :—

$$M + Q_1 r = Q_2 r,$$

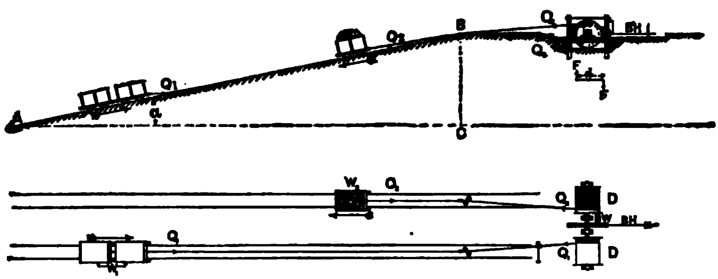
$$\therefore \qquad M = (Q_2 - Q_1)\, r, \quad \dots \dots \dots \dots \quad (1)$$

$$\text{But,} \qquad Q_2 = W_2 (\sin\alpha - \mu\cos\alpha),$$

$$\text{And,} \qquad Q_1 = W_1 (\sin\alpha + \mu\cos\alpha),$$

$$\therefore \qquad M = \left\{ (W_2 - W_1)\sin\alpha - \mu (W_2 + W_1)\cos\alpha \right\} r \quad \text{(XII)}$$

$$\text{Or,} \qquad M = \frac{r}{l}\left\{ h\,(W_2 - W_1) - \mu\, b\,(W_2 + W_1) \right\}. \quad (\text{XII}_a)$$



PRACTICAL EXAMPLE OF THE DOUBLE INCLINED PLANE.

(3) *If* W_2 *is not sufficient to overcome* W_1, *then a moment*, M, *must be applied to the pulley or drum*, D, *to assist it.*

In this case, by taking moments about the centre of the pulley as before, we get :—

$$M + Q_2 r = Q_1 r,$$

$$\therefore \qquad M = (Q_1 - Q_2)\, r,$$

$$\text{Hence,} \qquad M = \left\{ (W_1 - W_2)\sin\alpha + \mu (W_1 + W_2)\cos\alpha \right\} r \quad \text{(XIII)}$$

$$\text{Or,} \qquad M = \frac{r}{l}\left\{ h\,(W_1 - W_2) + \mu\, b\,(W_1 + W_2) \right\}. \quad . \quad (\text{XIII}_a)$$

EXAMPLE IV.—In a double inclined plane having a rise of 1 in 20, the loaded and empty trucks run on parallel rails and are connected by a rope which passes over a pulley, 6 feet in diameter, at the top of the plane. Find the greatest number of empty trucks which a descending loaded one is capable of drawing up; having given weight of each empty truck, 5 cwts., weight of material in loaded truck, 20 cwts., the coefficient of traction on the level being taken at 20 lbs. per ton. Again, if 5 loaded trucks going down pull up an equal number of empty ones, what must be the mean frictional resistance on the circumference of a brake wheel, 3 feet in diameter, fitted on the pulley at the top of the incline, so that the whole may be kept moving uniformly?

ANSWER.—Let w = Weight of empty truck = 5 cwts.
" W = Weight of material in loaded truck = 20 cwts.

Since the coefficient of traction on the level is 20 lbs. per ton,

$$\therefore \qquad \mu = \frac{20}{2{,}240} = \frac{1}{112}.$$

Again, since the inclination of the plane is small, we may assume:—

That, $$\cos a = 1 \text{ and } \sin a = \frac{1}{20}.$$

Now, let there be n empty trucks drawn up by a descending loaded one.

Then, according to the previous notation:—

$$W_1 = n\,w = 5\,n \text{ cwts.}$$

$$W_2 = W + w = 25 \text{ cwts.}$$

$$\frac{W_1}{W_2} = \frac{\sin a - \mu \cos a}{\sin a + \mu \cos a} \text{ [equation (XI)].}$$

$$\therefore \qquad \frac{5\,n}{25} = \frac{\frac{1}{20} - \frac{1}{112} \times 1}{\frac{1}{20} + \frac{1}{112} \times 1}.$$

$$\therefore \qquad \frac{n}{5} = \frac{23}{33},$$

$$\therefore \qquad n = \frac{115}{33} = 3 \text{ fully.}$$

Or, the greatest number of empty trucks that can be drawn up by 1 descending loaded truck is 3.

Next, with 5 loaded trucks going down and 5 empty ones coming up, we have :—

$$W_1 = 5\,w = 25 \text{ cwts.}; \; W_2 = 5\,(W + w) = 125 \text{ cwts.}$$

and R = Radius of drum = 3 ft.

The friction moment to be applied to the brake wheel at the top of the plane is by equation (XII) :—

$$M = \left\{ (W_2 - W_1) \sin \alpha - \mu (W_2 + W_1) \cos \alpha \right\} R.$$

Substituting the above values, we get :—

$$M = \left\{ (125 - 25) \times \frac{1}{20} - \frac{1}{112} \times (125 + 25) \times 1 \right\} 3 \text{ (ft.-cwts.)}$$

$$,, = \frac{410 \times 3}{112} \text{ ft.-cwts.}$$

i.e., $M = 1{,}230$ ft.-lbs.

Let F = Mean frictional resistance applied at circumference of brake wheel.

,, r_b = Radius of brake wheel = $1\frac{1}{2}$ feet.

Then, *Friction couple* $= F \times r_b = M.$

$$F \times 1\tfrac{1}{2} = 410 \times 3.$$

$$\therefore \quad F = \frac{410 \times 3}{1\frac{1}{2}} = 820 \text{ lbs.}$$

Example V.—In the latter part of Example IV., suppose the operations to be reversed, so that the five loaded trucks are to be hauled up the plane by means of an engine situated at the top of the plane, the engine being assisted by the descending five empty trucks. Find the tensions in the two parts of the hauling rope, and the H.P. of the engine; given the length of inclined plane, 1 mile and the time taken to complete the run, five minutes.

Answer.—From last example we get the following data :—

W_1 = Weight of five empty trucks = 25 cwts.,
W_2 = ,, ,, loaded ,, = 125 ,,

$$\mu = \frac{1}{112}, \; \sin \alpha = \frac{1}{20}, \; \cos \alpha = 1, \text{ approximately.}$$

Let Q_1 = Tension in that part of rope attached to W_1,
,, Q_2 = Tension ,, ,, ,, W_2.

Then, since W_1 is let down the plane, we get :—

$$Q_1 = W_1 (\sin \alpha - \mu \cos \alpha), \text{ [equation (VII)]}$$

$$\therefore \quad Q_1 = 25 \left(\frac{1}{20} - \frac{1}{112} \times 1\right) \text{ cwts.} = \textbf{115 lbs.}$$

Also, since W_2 is pulled up the plane, we get :—

$$Q_2 = W_2 (\sin \alpha + \mu \cos \alpha), \text{ [equation (II}_b\text{)]}$$

$$\therefore \quad Q_2 = 125 \left(\frac{1}{20} + \frac{1}{112} \times 1\right) \text{ cwts.} = \textbf{825 lbs.}$$

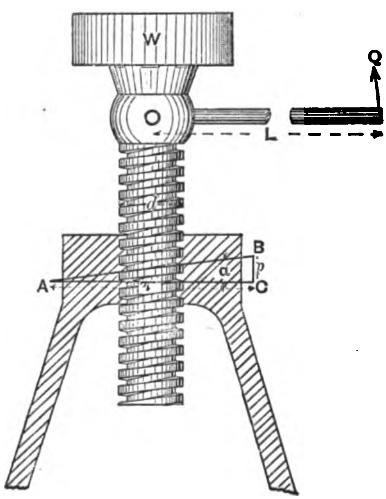
Let l = Length of incline = 5,280 feet.
,, t = Time taken to traverse it = 5 minutes.

Then, *Work done by engine per minute* = Work done per minute *by* rope in pulling up full trucks *minus* work done per minute *on* rope by descending trucks.

$$= \frac{l}{t} \times Q_2 - \frac{l}{t} \times Q_1 = \frac{l}{t}(Q_2 - Q_1).$$

$$\therefore \text{ H.P. of engine} = \frac{\frac{l}{t}(Q_2 - Q_1)}{33{,}000} = \frac{5{,}280 \times 710}{5 \times 33{,}000} = \textbf{22·72.}$$

Screws.—The various forms of screw threads, their development, characteristics, and manufacture, have been fully described and illustrated in our *Elementary Manual of Applied Mechanics*, and, therefore, need not be further considered here. In what follows we shall content ourselves by determining the *Advantage* and *Efficiency* of the ordinary screw arrangement. Take the case of the square threaded screw working in its nut, and suppose the pressure, due to the load, W, to be uniformly distributed along the bearing surface of the thread. Since the pitch angle, α, is everywhere the same, it will be sufficient to take a single point on the screw thread,



THE SCREW.

and consider the whole load, W, concentrated at this point. We have then the case of an inclined plane, A B C, as shown by the accompanying figure, which represents an ideal helix or screw line, traced on a cylinder, and a development of one complete turn of this line.

Let p = Pitch of screw thread,
" d = Mean diameter of cylinder of bolt,
" Q = Turning effort applied to lever or spanner,
" L = Leverage of Q measured from the axis of the bolt,
" μ = Coefficient of friction between nut and screw.
" H = Force acting along A C due to the effort, Q.

Then, by the "Principle of Moments," we get:—

$$\mathrm{Q} \times \mathrm{L} = \mathrm{H} \times \frac{d}{2}$$

Or,
$$\mathrm{Q} = \mathrm{H} \times \frac{d}{2\,\mathrm{L}}.$$

And from equation (II.) in this Lecture:—

$$\mathrm{H} = \mathrm{W} \tan(\alpha + \varphi).$$

$$\therefore \quad \mathrm{Q} = \mathrm{W}\frac{d}{2\,\mathrm{L}} \tan(\alpha + \varphi).$$

Or,
$$\frac{\mathrm{Q}}{\mathrm{W}} = \frac{d}{2\,\mathrm{L}} \tan(\alpha + \varphi). \quad . \quad . \quad . \quad . \quad . \quad (\mathrm{XIV})$$

$$\therefore \quad \frac{\mathrm{Q}}{\mathrm{W}} = \frac{d}{2\,\mathrm{L}}\left(\frac{\tan\alpha + \tan\varphi}{1 - \tan\alpha\tan\varphi}\right).$$

But, from the figure,

$$\tan\alpha = \frac{\mathrm{B\,C}}{\mathrm{A\,C}} = \frac{p}{\pi d}, \text{ and } \tan\varphi = \mu.$$

$$\therefore \quad \frac{\mathrm{Q}}{\mathrm{W}} = \frac{d}{2\,\mathrm{L}}\left(\frac{p + \mu\pi d}{\pi d - \mu p}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (\mathrm{XV})$$

Hence,

$$\text{Actual Advantage} = \frac{\mathrm{W}}{\mathrm{Q}} = \frac{2\,\mathrm{L}}{d}\left(\frac{\pi d - \mu p}{p + \mu\pi d}\right). \quad . \quad . \quad . \quad (\mathrm{XVI})$$

Efficiency of Screw.—Suppose the weight to be raised through a distance equal to the pitch, p. Then Q will have moved through a distance equal to 2π L. Hence:—

$$\text{Efficiency} = \frac{W \times p}{Q \times 2 \pi L},$$

$$,, \quad = \frac{2 L}{d} \left(\frac{1}{\tan (\alpha + \varphi)} \times \frac{p}{2 \pi L} \right)$$

$$,, \quad = \frac{1}{\tan (\alpha + \varphi)} \frac{p}{\pi d},$$

i.e., $$\text{Efficiency} = \frac{\tan \alpha}{\tan (\alpha + \varphi)} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (\text{XVII})$$

Maximum Efficiency of Screw.—We can now find what value of α will give the *greatest efficiency*. Clearly the efficiency will be a *maximum* when $\dfrac{\tan \alpha}{\tan (\alpha + \varphi)}$ is a *maximum*.

But, $$\frac{\tan \alpha}{\tan (\alpha + \varphi)} = \frac{\sin \alpha \cos (\alpha + \varphi)}{\cos \alpha \sin (\alpha + \varphi)},$$

$$,, \quad = \frac{\sin (2 \alpha + \varphi) - \sin \varphi}{\sin (2 \alpha + \varphi) + \sin \varphi},$$

$$,, \quad = 1 - \frac{2 \sin \varphi}{\sin (2 \alpha + \varphi) + \sin \varphi}.$$

From this it is clear that the efficiency will be a *maximum* when $\dfrac{2 \sin \varphi}{\sin (2 \alpha + \varphi) + \sin \varphi}$ is a *minimum*.

i.e., when $\dfrac{1}{\sin (2 \alpha + \varphi) + \sin \varphi}$ is a *minimum*.

i.e., when $\sin (2 \alpha + \varphi)$ is a *maximum*.

But the greatest value for the sine of an angle is unity, and this occurs when the angle is 90°.

The efficiency will, therefore, be a maximum when :—

$$2 \alpha + \varphi = 90^\circ.$$

Or, $$\alpha = 45^\circ - \frac{\varphi}{2}.$$

Substituting this value for α in the expression for the efficiency, we get :—

$$\text{Maximum Efficiency} = \frac{\tan\left(45^\circ - \frac{\varphi}{2}\right)}{\tan\left(45^\circ + \frac{\varphi}{2}\right)}$$

$$\text{,,} \quad \text{,,} \quad = \frac{1 - \tan\frac{\varphi}{2}}{1 + \tan\frac{\varphi}{2}} \div \frac{1 + \tan\frac{\varphi}{2}}{1 - \tan\frac{\varphi}{2}},$$

$$\text{,,} \quad \text{,,} \quad = \left(\frac{1 - \tan\frac{1}{2}\varphi}{1 + \tan\frac{1}{2}\varphi}\right)^2.$$

But φ is always a small angle, and we may, therefore, substitute $\frac{1}{2}\mu$ for $\tan\frac{1}{2}\varphi$, so that:—

$$\text{Maximum Efficiency} = \left(\frac{1 - \frac{1}{2}\mu}{1 + \frac{1}{2}\mu}\right)^2 \text{ (approximately).}$$

From this we see that for the maximum efficiency in the case of a screw the best pitch angle is 45° nearly.

Taking the coefficient of friction = ·16, and pitch angle 45°, we get:—

$$\text{Maximum Efficiency} = \left(\frac{1 - \frac{1}{2} \times \cdot 16}{1 + \frac{1}{2} \times \cdot 16}\right)^2 = \cdot 72 \text{ or 72 per cent. nearly.*}$$

If the pitch angle be greater than 45°, it will be possible to reverse the action of the screw, so that a weight, W, on the nut or screw may be able to overcome a small force, Q, on the end of the lever. Instances of this may be met with in some forms of hand drills, and in certain instruments used for domestic purposes. The student will be able, from the general investigations in Lecture VII., to prove that the efficiency of a screw when working in the reversed way is given by the equation.

$$\text{Reversed Efficiency} = \frac{\tan(\alpha - \varphi)}{\tan\alpha}.$$

Non-reversibility of Ordinary Screws and Nuts.—In bolts and most other applications of screws the pitch angle is very much less than 45°, consequently, the efficiency of these screws is often

* In the case of the Sprague-Pratt Electric Elevators of New York an efficiency of over 90 per cent. is claimed for the nut and screw, owing to the introduction of hardened steel friction balls between the screw and nut threads. The author had an opportunity of testing roughly the efficiency of these elevators, and can testify to their excellent design, workmanship, and action.

lower than 20 per cent. In such cases, however, mechanical advantage and non-reversibility are the objects chiefly aimed at, and not high efficiency.

Tension in Bolts due to Screwing Up.—Consider the case of a square-threaded screw.

Let W = Tension in bolt due to screwing up,
,, Q = Force applied at end of spanner,
,, L = Length of spanner,
,, d = Mean diameter of bolt thread,
,, p = Pitch of thread,
,, μ = Coefficient of friction between screw and its nut,
,, μ_1 = ,, ,, nut and its washer.

Then, *Friction between nut and washer* $= \mu_1 W$.

Suppose this friction to act at the circumference of a circle of diameter D; in other words, let D be the diameter of the friction circle between nut and washer.

Then, *Friction moment between nut and washer* $= \mu_1 W \times \frac{D}{2}$.

Hence, by taking moments about the axis of the bolt, we get:—

$$Q \times L = H \times \frac{d}{2} + \mu_1 W \times \frac{D}{2}$$

$$,, \quad = \frac{W}{2}\left\{\frac{p + \mu \pi d}{\pi d - \mu p} \times d + \mu_1 D\right\}$$

[From previous formula for H and equation XV.]

$$\therefore \qquad W = \frac{2 Q L}{\frac{p + \mu \pi d}{\pi d - \mu p} d + \mu_1 D}.$$

The average length of a spanner is $L = 15 d$, and D may be taken at $\frac{4}{3} d$, while $\mu_1 = \mu$ very nearly.

Hence, *Tension in bolt* $= W = \frac{90 (\pi d - \mu p)}{7 \mu \pi d + (3 - 4 \mu^2) p} Q$.

For ordinary sized bolts we may take $\mu = 0{\cdot}15$, and $p = 0{\cdot}16\, d$.

Hence, substituting these values in the last equation, we get:—

$$\textbf{Tension in bolt} = \frac{90 \times 3{\cdot}1176}{3{\cdot}76} Q = 75\, Q, \text{ nearly.}$$

Now, suppose a force of 30 lbs. to be applied at the end of the spanner by a workman, then :—

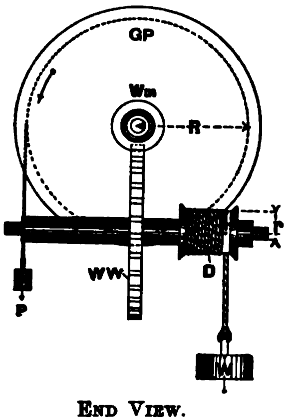
$$\textit{Tension in bolt} = 75 \times 30 = 2{,}250 \text{ lbs.},$$

This tension would be about sufficient to break a wrought-iron bolt $\frac{3}{8}$ inch in diameter, and would seriously injure a bolt $\frac{1}{2}$ inch in diameter.

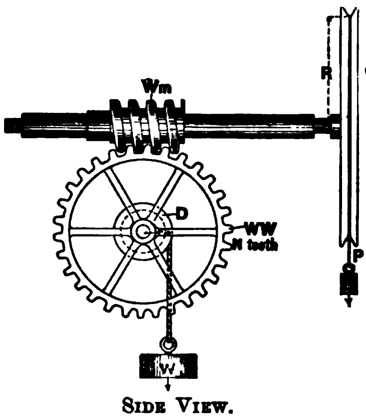
Hence the practical rule :—*That bolts less than $\frac{3}{4}$ inch in diameter should never be employed for joints requiring to be tightly screwed up.*

In estimating the friction of such machines as screw jacks, where the end of the screw terminates in a loose cap supporting the load, the friction between the cap and the part of the screw supporting it must not be neglected. In many cases this friction is about as great as that between the thread of the screw and its nut.

EXAMPLE VI.—Apply the *Principle of Work* to calculate the relation of the effort, P, to the resistance, W, in the following



END VIEW.



SIDE VIEW.

PULLEY, WORM, WORM-WHEEL, AND WINCH DRUM.

INDEX TO PARTS.

GP represents Grooved Pulley.	WW represents Worm-wheel.
Wm ,, Worm or endless screw.	D ,, Drum.

combination :—In a model to show the action of an endless screw and worm-wheel, the pulley which turns the screw is 18 inches in diameter, the screw is double threaded, and the worm-wheel

has 30 teeth. On the axis of the worm-wheel is a drum $4\frac{1}{2}$ inches in diameter round which the cord is coiled. What load, W, hanging on this cord would be supported by a weight, P, of 14 lbs. at the circumference of the pulley, friction being neglected?

ANSWER.—Two views of the essential parts of this combination are given above.

Let R = Radius of pulley = 9 inches.
„ r = „ drum = $2\frac{1}{4}$ „
„ N = Number of teeth on worm-wheel = 30.
„ n = „ threads on endless screw = 2.

If the effort, P, receive a displacement equal to the circumference of the pulley, then the worm-wheel will make $\frac{n}{N}$ of a turn, since every complete turn of the worm displaces n teeth on the worm-wheel. Hence :—

$$\textit{Displacement of } W = \frac{n}{N} \times 2\pi r.$$

But, by the *Principle of Work*, we have :—

$$P \times \textit{its displacement} = W \times \textit{its displacement}.$$

$$\therefore \quad P \times 2\pi R = W \times \frac{n}{N} \times 2\pi r.$$

$$\therefore \quad \frac{P}{W} = \frac{n\,r}{N\,R}.$$

Substituting the values of P, n, N, r and R, we get :—

$$\frac{14}{W} = \frac{2 \times 2\frac{1}{4}}{30 \times 9}.$$

$$\therefore \quad W = 840 \text{ lbs.}$$

Lecture IX.—Questions.

1. State and prove the relation between the weight, W, of a body resting on a rough inclined plane, the reaction, R, from the plane, and the force, Q, necessary to just balance the weight: (1) when the force, Q, acts parallel to the plane, (2) when it acts parallel to the base, (3) when it acts at an angle, θ, to the plane.

2. The resistance of friction along an inclined plane is taken at 150 lbs. for each ton of weight moved. Find the work done in drawing 2 tons up 100 feet of an incline which rises 1 foot in height for 25 in length. *Ans.* 47,920 ft.-lbs.

3. If 150 lbs. per ton is a sufficient tractive force to draw a loaded waggon along a horizontal road, what tractive force per ton will be required to draw the load up an incline 1 in 10? *Ans.* 374 lbs. per ton.

4. What must be the effective horse-power of a locomotive engine which moves at a steady speed of 40 miles per hour on a level rail, the resistance being 15 lbs. per ton, and the weight of the engine and train being 100 tons? If the rails were laid at a gradient of 1 in 100, what additional horse-power would be required? *Ans.* 160 H.P.; 238·93 H.P.

5. A train of 200 tons ascends an incline which has a rise of ·5 foot per cent. (*i.e.*, 5 feet in 1,000), with a uniform speed of 30 miles per hour, what is the effective horse-power of the engine, the friction being 5·5 lbs. to the ton? *Ans.* 267·2 H.P.

6. A train of 330 tons ascends an incline which has a rise of ·2 per cent. (*i.e.*, 2 feet in 1,000), what is the maximum speed in miles per hour with an engine of 120 horse-power, the friction being 8 lbs. to the ton? *Ans.* 10·92 miles per hour.

7. Prove the formula for the relation between the weights, W_1 and W_2, in the double inclined plane, taking friction into account. *Ex.* A double inclined plane is formed by two inclined planes placed back to back so that they have a common summit. Their inclinations to the horizon are 30° and 40° respectively. The weight of the body on the latter plane is 500 lbs.; find the greatest weight which it is capable of drawing up on the other plane, coefficient of friction in both cases being 0·2. *Ans.* 363·9 lbs.

8. A stationary engine at the top of an inclined plane is employed to draw loaded waggons up the plane, and is assisted by an equal number of empty waggons which descend by a parallel line of rails. The total weight of loaded waggons is 35 tons, the weight of the descending empty ones being 12 tons. Find the maximum H.P. developed by the engine, the maximum speed of the waggons being 6 miles per hour; inclination of plane 1 in 15; coefficient of traction on the level being taken at 10 lbs. per ton. Find also the tensions in the two ropes. *Ans.* 62·48 H.P.; 5,577 lbs.; 1,672 lbs.

9. Define the pitch of a screw. In the Whitworth angular screw-thread, what is the angle made by opposite sides of the thread? To what extent is the thread rounded off at the top and bottom? Distinguish between a *single* and a *double-threaded* screw; in what cases would the latter be used? Why are holding down bolts made with angular threads?

10. What is meant by *backlash?* How may backlash be prevented in a screw?

11. Sketch and describe some form of screw-jack, and estimate the relation between the force applied and the resistance overcome, friction being neglected.

12. Find the relation between Q and W in an ordinary screw-jack fitted with a square-threaded screw, taking friction into account. In an ordinary screw-jack the mean diameter of the screw-thread is 4 inches, pitch of screw 1 inch, length of lever measured from axis of screw 4 feet; find the weight raised by an effort of 60 lbs., applied at the end of the lever, the coefficient of friction being taken at 0·1. *Ans.* 3·55 tons.

13. Find an expression for the efficiency of the screw in last question, and state its numerical value for example given. What are the conditions for maximum efficiency? Prove your answer. *Ans.* 44 per cent.

14. In question 12 find the weight raised and the efficiency of the apparatus when the friction between the cylinder of the screw and the loose cap fitted thereon is taken into account. You may take diameter of friction circle for loose cap equal to mean diameter of screw thread, and coefficient of friction same as before. *Ans.* 1·57 tons; 19·4 per cent.

15. State the principle of work, and apply it to calculate the relation of the force, P, to the resistance, W, in the following combination:—*Ex.* A worm-wheel having 16 teeth forms the nut of a screw of $\frac{1}{2}$-inch pitch; an endless screw, actuated by a lever handle of 14 inches in length, works in the worm-wheel. Find the pressure exerted by the screw when a force of 20 lbs. is applied to the end of the lever handle. *Ans.* 25 tons.

16. Explain the mechanical advantage resulting from the employment of an endless screw and worm-wheel. The lever handle which turns an endless screw is 14 inches long, the worm-wheel has 32 teeth, and a weight, W, hangs by a cord from a drum of 6 inches in diameter, whose axis coincides with that of the worm-wheel. If a pressure, P, be applied to the lever handle, find the ratio of P to W for equilibrium. *Ans.* 3 : 448.

17. Sketch and describe the screw lifting jack as fitted with screw and worm-wheel gear. The handle being 15 inches long, the pitch of the screw $1\frac{1}{4}$ inches, and the worm-wheel having 15 teeth, find the force on the handle for raising 5 tons (friction is neglected). (S. & A. Adv. Exam., 1889.) *Ans.* 9·9 lbs.

18. A common screw-jack, with a lever 16 inches in length, has a worm-wheel of 20 teeth, and a screw of $1\frac{1}{4}$ inches pitch. Sketch the arrangement and calculate the weight lifted by the application of a constant pressure of 30 lbs. at the end of the handle, friction being neglected. (S. & A. Adv. Exam., 1892.) *Ans.* 48,255 lbs.

19. The table of a drilling machine is raised by a worm and wheel in combination with a rack and pinion. Sketch the arrangement, and find what weight would be balanced on the table if a pressure of 12 lbs. were applied to the end of the handle, which is 12 inches long, the worm being single-threaded, while the worm-wheel has 30 teeth, and the pitch circle of the rack-pinion is 4 inches in diameter. Suppose the table and accessories to weigh 500 lbs., and that 45 per cent. of the work applied is lost by friction. (S. & A. Adv. Exam., 1892.) *Ans.* When the table is on the point of moving downwards then 3,427 lbs. is the maximum weight that can be balanced. When the table is on the point of being raised then 688 lbs. is the minimum weight which produces a balance.

LECTURE X.

CONTENTS.—Frictional Resistances and Efficiencies of Machines in General—Example I.—Application to the Steam Engine—Efficiency of a Reversible Machine—Example II.—Questions.

Frictional Resistances and Efficiencies of Machines in General.—In Lecture VII. we showed how to calculate the work lost in friction in the cases of plane and cylindrical surfaces. As these are the principal kinds of rubbing surfaces in machinery, we might, if we knew the exact pressures between the various surfaces, calculate the total frictional losses and thus find the efficiency of the machine. But there are other losses of work during its transmission, such as the bending of ropes, belts, chains, &c., which it is almost impossible to calculate with exactness. Even if we could calculate all these various losses, the task would be a most tedious and unprofitable one. Hence, in finding the efficiency of any machine, we have recourse to direct experiment on the machine as a whole, the results of which furnish us with data from which we can determine the exact efficiency. In general, however, a machine will not have the same efficiency when working under different loads, owing to the fact that the frictional and other resistances are not proportional to the efforts and loads. To make this clearer, suppose we take the case of a steam engine. Here the friction between the piston and its cylinder, the piston-rod, valve rods, &c., and their stuffing boxes, as well as the friction at the journals due to the weights of the flywheel and shaft, are, severally, constant in amount, whether the engine be developing full power or running light. Hence, in all machines there is a certain proportion of the frictional and other resistances constant, whatever be the magnitude of the effort and the load. This (as just explained in the case of the steam engine) is due to forces between the parts of the machine itself, such as the weights and inertia of the moving parts, or the resistances offered to the bending and stretching of parts, and which have little or no connection with the effort exerted or the load overcome. From these facts we see that the lost work will be proportionally less, and the useful work proportionally more the greater the total work expended. In other words, *the efficiency of a machine will, in general, be higher the greater the load.* This statement, however, is not true for hydraulic and other machines where fluid resistances occur, or where the speed of the machinery is *very* great. As will

be seen later on, fluid resistances increase very rapidly with the speed. In high-speed machinery the effects of the inertia of the moving parts introduce other serious losses which must not be ignored when calculating their efficiency.

The "Principle of Work," when applied to a machine, has already been written in the form :—

Total work expended = *Useful work done* + *Lost work.*

Or, $$W_T = W_U + W_L \quad \text{[see eqn. (I.), Lect. IV.]}$$

The last term, W_L, is made up of two distinct parts—one part depending on W_T and W_U, and a second part which is constant, and, therefore, independent of W_T and W_U. Hence, we may put :—

$$W_L = \mu_1 W_T + \mu_2 W_U + C.$$

Where μ_1 μ_2 are numerical coefficients, and $\mu_1 W_T$, $\mu_2 W_U$ are the frictional resistance due to the effort and load applied; while C represents an amount of lost work which is constant for the same machine.

$$\therefore \qquad W_T = W_U + \mu_1 W_T + \mu_2 W_U + C.$$

Or, $$(1 - \mu_1) W_T = (1 + \mu_2) W_U + C. \quad . \quad . \quad . \quad . \quad . \quad \text{(I)}$$

This is a *general* equation for the "Principle of Work" as applied to machines. In most machines the coefficient, μ_1, which depends on the effort, Q, must necessarily be very small, and may sometimes be neglected.

Dividing both sides of the equation (I) by $(1 - \mu_1)$ we get :—

$$W_T = \frac{1 + \mu_2}{1 - \mu_1} W_U + \frac{C}{1 - \mu_1},$$

$$\text{,,} \quad = \left(1 + \frac{\mu_1 + \mu_2}{1 - \mu_1}\right) W_U + F.$$

Or, $$W_T = (1 + f) W_U + F. \quad . \quad . \quad . \quad . \quad \text{(II)}$$

Where $f\left(= \dfrac{\mu_1 + \mu_2}{1 - \mu_1}\right)$ and $F\left(= \dfrac{C}{1 - \mu_1}\right)$ are new constants derived from the old ones, as shown.

For some purposes, it is more convenient to write equation (II) in the following forms :—

$$W_T = k\, W_U + F. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(III)}$$

Or, $$Q\,x = k\, W\,y + F. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(IV)}$$

Where (as in Lecture IV.) x and y are respectively the displacements of the effort Q and the load W, in a given time.

Suppose the machine to run light. Then $W = 0$, and Q_0 is the effort required to drive the machine.

From equation (IV), we get :—

$$Q_0 x = F.$$

That is, F *represents the work done in driving the machine unloaded.*

Dividing both sides of this equation by x, we get :—

$$Q_0 = \frac{F}{x}.$$

This is the effort required to drive the machine light.

Substituting the above value for F, in equation (IV), we get :—

$$Q x = k\ W y + Q_0 x.$$

Or,
$$Q = k\ W \frac{y}{x} + Q_0$$

$\therefore$
$$Q = k\ W \frac{v}{V} + Q_0. \quad \ldots \quad (V)$$

This is a *general* equation connecting the effort Q and the load W for any machine. Since the velocity ratio, $\frac{V}{v}$, is constant for the same machine, we might write the last equation in this convenient form :—

$$Q = K\ W + Q_0. \quad \ldots \quad (VI)$$

Where, $K = k \Big/ \frac{V}{v}$, and can be found by experiment for any machine.

Example I.—In an ordinary block and tackle having three sheaves in the upper and two in the lower block, it is found by experiment that a force of 11 lbs. is required to lift a weight of 40 lbs., and a force of $24\frac{1}{2}$ lbs. to lift a weight of 100 lbs. Find a general expression for the relation between Q and W in this arrangement, and the weight which could be raised by a force of 56 lbs. Find, also, the efficiency of the machine in all three cases, and the actual mechanical advantage.

Answer.—The general relation between Q and W must be of the form :—

$$Q = K\ W + Q_0. \quad \ldots \quad (1)$$

From the results of the first experiment, we get:—

$$11 = 40\,K + Q_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

And from the results of the second experiment, we get:—

$$24\tfrac{1}{2} = 100\,K + Q_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

(3)–(2) $\qquad 13\tfrac{1}{2} = 60\,K,$

$$\therefore \qquad K = \frac{27}{120} = \frac{9}{40}.$$

Substituting this value of K in equation (2), we get:—

$$11 = \frac{9}{40} \times 40 + Q_0,$$

$$\therefore \qquad Q_0 = 2 \text{ lbs.}$$

Or, the effort required to drive the machine light is 2 lbs.

Substituting the values of K and Q_0 in equation (1), we get:—

$$Q = \frac{9}{40}\,W + 2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

which is the general formula required.

To find the weight which could be lifted by an effort of 56 lbs., we substitute this value in equation (4), and get:—

$$56 = \frac{9}{40}\,W + 2,$$

$$\therefore \qquad W = \frac{54 \times 40}{9} = 240 \text{ lbs.}$$

The efficiency of the machine in any case is found from the usual formula, viz.:—

$$\textit{Efficiency} = \frac{\text{Useful work done}}{\text{Total work expended}} = \frac{W\,y}{Q\,x} = \frac{W}{Q}\,\frac{v}{V}.$$

Since there are five ropes supporting the weight, W, in this system of block and tackle, it is clear that:—

$$\frac{v}{V} = \frac{1}{5}.$$

$$\therefore \qquad \text{Efficiency} = \frac{1}{5}\,\frac{W}{Q}.$$

When raising the weight of 40 lbs., we get :—

$$\text{Efficiency} = \frac{1}{5} \times \frac{40}{11} = \cdot 727, \text{ or } 72\cdot 7 \text{ per cent.} \quad (1)$$

When raising the weight of 100 lbs., we get :—

$$\text{Efficiency} = \frac{1}{5} \times \frac{100}{24\cdot 5} = \cdot 816, \text{ or } 81\cdot 6 \text{ per cent.} \quad (2)$$

When raising the weight of 240 lbs., we get :—

$$\text{Efficiency} = \frac{1}{5} \times \frac{240}{56} = \cdot 857, \text{ or } 85\cdot 7 \text{ per cent.} \quad (3)$$

The student should notice how the efficiency increases as the load, W, is increased.

$$\left.\begin{array}{l}\textit{The actual advantage}\\ \textit{when raising } 40 \textit{ lbs.}\end{array}\right\} = \frac{W}{Q} = \frac{40}{11} = 3\cdot 63.$$

By multiplying the numbers expressing the efficiencies by 5 (the number of ropes attached to the lower block), we get the actual mechanical advantage in each case.

Application to the Steam Engine.—Since the above reasoning is applicable to *all* machines, when the frictional resistances are not greatly influenced by speed, &c., we may here show its application to the steam engine.

Let p_m = Mean pressure on piston in lbs. per square inch.
,, p_u = Mean pressure per square inch (being part of p_m) required to overcome the useful load, W.
,, p_o = Mean pressure per square inch required to drive the engine when unloaded, or simply the "friction pressure" per square inch.

Since, $\frac{V}{v}$ or the velocity ratio does not enter into this case, and all the pressures are considered as acting on the same piston, we get from equation (V) :—

$$p_m = k\, p_u + p_o\,.$$

By experiment it is found that the constant p_o, called the "Friction Pressure," has a value between 1 and $1\frac{1}{2}$ lbs. per square inch, in ordinary land engines, and about 2 lbs. per square inch in marine engines. The value of k is about 1·15, but varies with both size and speed of engine. In large or high-speed engines, k is often less than 1·15, though it can never be less than 1.

Efficiency of a Reversible Machine.—The student will have noticed, from the Table of Efficiencies in Lecture II., the great difference in the efficiencies of such machines as the ordinary block and tackle, and the Weston's pulley block. In the examples worked out at the end of Lecture IV., it was proved that the efficiency of the former machine may be as high as 75 per cent., while the efficiency of the latter never reaches 50 per cent., and seldom exceeds 40 per cent. He also knows that when the efficiency of any machine is less than 50 per cent. it will not reverse, even if the hauling force or effort be withdrawn. Hence the difference in the working of the two machines just mentioned. The "block and tackle" is, under ordinary conditions, a *reversible* machine (*i.e.*, the load at the lower block is capable of overcoming a smaller load at the hauling part of the rope), while the Weston's pulley block will not reverse even when the hauling force is entirely withdrawn. To lower the load with a Weston's block a force has to be applied to the opposite part of the hauling chain from that at which the effort had to be applied when raising the load.

The screw, wedge, and worm-wheel arrangements are, generally speaking, examples of non-reversible machines. In fact, their usefulness depends to a large extent on this condition.

We can now show that the efficiency of a machine, when working reversed, is not the same as when working in the usual way. Further, if the efficiency of any machine be less than ·5 or 50 per cent., it is not reversible.

We have seen that in any direct working machine the "Principle of Work" takes the form :—

$$(1 - \mu_1)\, W_T = (1 + \mu_2)\, W_U + C. \text{ [equation (I)]}$$

Or, $$(1 - \mu_1)\, Q\, x = (1 + \mu_2)\, W\, y + C \quad . \quad . \quad . \quad . \quad (1)$$

where the coefficients, μ_1 and μ_2, have the meanings already assigned to them, and C represents a quantity of work absorbed in the machine, but which is independent of both Q and W.

Now, suppose we gradually diminish the effort, Q, until the machine reverses. When this takes place, let the new value of Q be denoted by w, so that the new load is w, and the original load, W, becomes the new effort. The above relation being still approximately true, we only require to substitute the new values for the new effort and load. At the same time it must be observed that the coefficients, μ_1 and μ_2, are taken along with their proper terms, $w\, x$ and $W\, y$; *i.e.*, $\mu_1\, w\, x$ is the lost work due to new load, w, while $\mu_2\, W\, y$ is lost work due to new effort or original load, W.

Then :—

$$(1 - \mu_2)\, W\, y = (1 + \mu_1)\, w\, x + C. \quad . \quad . \quad . \quad . \quad (2)$$

Now subtracting equation (2) from (1), in order to eliminate C, we get :—

$$(1 - \mu_1)\, Q\, x - (1 - \mu_2)\, W\, y = (1 + \mu_2)\, W\, y - (1 + \mu_1)\, w\, x$$

$$\therefore \qquad (1 + \mu_1)\, w\, x = 2\, W\, y - (1 - \mu_1)\, Q\, x.$$

Dividing both sides by $(1 + \mu_1)$ and $W\, y$, we get :—

$$\frac{w\, x}{W\, y} = \frac{2}{1 + \mu_1} - \left(\frac{1 - \mu_1}{1 + \mu_1}\right) \frac{Q\, x}{W\, y}.$$

$$\textit{Efficiency when reversed} = \frac{\text{Useful work done in raising } w}{\text{Total work expended by } W}.$$

$$\text{,,} \qquad = \frac{w\, x}{W\, y},$$

$$\text{,,} \qquad = \frac{2}{1 + \mu_1} - \left(\frac{1 - \mu_1}{1 + \mu_1}\right) \frac{Q\, x}{W\, y}.$$

But $\frac{W\, y}{Q\, x}$ = the *original efficiency* of the machine, or efficiency when working in the usual manner.

$$\therefore \quad \left.\begin{array}{c}\text{Efficiency of Machine}\\ \text{when Reversed}\end{array}\right\} = \frac{2}{1 + \mu_1} - \left(\frac{1 - \mu_1}{1 + \mu_1}\right) \times \frac{1}{\eta} \quad . \quad \text{(VII)}$$

where η denotes the original efficiency of the machine.

It is clear that the machine will not reverse unless the above efficiency be greater than 0—

i.e., unless .. . $\dfrac{2}{1 + \mu_1} - \left(\dfrac{1 - \mu_1}{1 + \mu_1}\right) \times \dfrac{1}{\eta} > 0.$

i.e., unless . . . . $\dfrac{2}{1 + \mu_1} > \left(\dfrac{1 - \mu_1}{1 + \mu_1}\right) \times \dfrac{1}{\eta}.$

i.e., unless . . . . $\eta > \frac{1}{2}(1 - \mu_1).$

Consequently, the machine will not reverse until the original efficiency, η, be greater than $\frac{1}{2}(1 - \mu_1)$, and, if it be less than this, it will not reverse even if the original hauling force, Q, be entirely withdrawn. For, if $\eta = \frac{1}{2}(1 - \mu_1)$, the efficiency of the machine when reversed would vanish (as may be seen by substituting this value in equation (VII)). If $\eta < \frac{1}{2}(1 - \mu_1)$, the efficiency would be negative, which is absurd.

We have already stated that in most machines the fraction, μ_1, is *very small*, and may be neglected. Hence we get the important statement that

A machine will not reverse even when the hauling force is entirely withdrawn, if the efficiency is less than $\frac{1}{2}$ or 50 per cent.

EXAMPLE II.—Apply the "Principle of Work" to calculate the relation between P and W in the screw lifting jack, fitted with screw and worm-wheel gear. The handle which works the jack has a radius of 14 inches, pitch of screw 1 inch, number of teeth on worm-wheel 20, and the worm is double threaded; find the force which must be applied at the end of the handle in order to raise a weight of 4 tons, friction being neglected. If the actual force required to raise this weight be 40 lbs., what is the efficiency of the apparatus?

ANSWER. — Let L = length of handle, p = pitch of screw, N = number of teeth on worm-wheel, n = number of threads on worm.

(1) Suppose the handle to make one complete turn. Then, since there are n threads on the worm, and N teeth on the worm-wheel, it is clear, that for one turn of the handle, the worm-wheel, which forms the nut of the screw, will have made $\frac{n}{N}$ part of a complete turn. Hence the weight, W, will have been raised through a height $\frac{n}{N} \times p$.

$\therefore$ By the Principle of Work, we get :—

$$P \times \text{its displacement} = W \times \text{its displacement.}$$

$$\therefore \qquad P \times 2\pi L = W \times \frac{n}{N} \times p,$$

$$\therefore \qquad \frac{P}{W} = \frac{np}{2\pi L N}.$$

(2) In the example $p = 1''$, $L = 14''$, $n = 2$, $N = 20$, $W = 4 \times 2{,}240$ lbs.

$$\therefore \qquad P = 4 \times 2{,}240 \times \frac{2 \times 1}{2 \times \frac{22}{7} \times 14 \times 20} \text{ lbs.,}$$

$$\text{,,} = 10{\cdot}18 \text{ lbs.}$$

(3) $$\textit{Efficiency} = \frac{P}{Q} = \frac{10{\cdot}18}{40} = {\cdot}2545, \text{ or } 25{\cdot}45 \text{ per cent.}$$

LECTURE X.—QUESTIONS.

1. Explain why the force necessary to drive a machine does not vary in exact proportion with the load.

2. With a pair of three-sheaved blocks it is found by experiment that a weight of 40 lbs. can be raised by a force of 10 lbs., and a weight of 200 lbs. by a force of 40 lbs. Find the general relation between P and W, and also the efficiency when raising 100 lbs. *Ans.* $P = \frac{3}{16} W + \frac{5}{2}$; ·784 or 78·4 per cent.

3. With a screw jack it is found that a force of 47·5 lbs. must be applied at the end of the handle to lift 1·5 tons, and a force of 85 lbs. to lift 3 tons. Find what force will be required to raise 2 tons. *Ans.* 60 lbs.

4. If the length of the handle in the above example be 2 feet and the pitch of the screw ¾ inch, find the efficiency in each case. *Ans.* 35·3, 39·5, and 37·3 per cent.

5. Find the "*friction pressure*" of a steam engine which requires a mean effective pressure of 24 lbs. per square inch to drive it at full load, 20 lbs. being taken up in overcoming the load. At three-quarters load a mean effective pressure of 18·5 lbs. is required, of which 15 lbs. is similarly taken up. *Ans.* 2 lbs. per square inch.

6. It is found that a force of 2 lbs. must be applied to the handle of a crane in order to wind up the rope when no weight is attached and of 80 lbs. when lifting a weight of 10 cwts. If the velocity ratio be 20, find the efficiency in this last case and also when the force at the handles is lessened so as just to allow the weight to descend. *Ans.* 70 per cent.; 57 per cent.

PART II.—GEARING.

LECTURE XI.

CONTENTS.—Definition of Gearing—Train of Wheels—Pitch Surface—Pitch Circle—Definitions of Pitch Surface, Pitch Line or Pitch Circle, Pitch Point—Sizes of Spur and Bevel Wheels—Velocity-Ratio of Two Wheels in Gear—Angular Velocity-Ratio—Definition of Angular Velocity—Velocity-Ratio of a Train of Wheels—Definition of Value of Train—Example I.—Intermediate or Idle Wheel—Marlborough Wheel—Change Wheels for Screw Cutting Lathes—Example II.—Force-Ratio and Power Transmitted by Gearing—Examples III. and IV.—Questions.

Definition of Gearing.—The term *gearing* is applied generally to any arrangement of wheel-work or link-work, for transmitting motion and power from one place to another. Engineers, however, restrict the term to denote any combination of *wheels* used for the transmission of motion and power from one shaft to another.

When the wheels are so arranged that they are capable of communicating motion from one to the other, they are said to be *in gear;* otherwise they are said to be *out of gear.*

Train of Wheels.—When a number of wheels are employed in transmitting motion and power from one place to another, it is usual to so arrange the wheels that each shaft, except the first and last, shall carry two wheels of different sizes; the smaller of these is made to gear with the larger one on the shaft next in order. Such an arrangement is termed a **Train of Wheels.**

In any train of wheels, that wheel which causes motion is termed the **Driver**, and that which receives the motion is called the **Follower.** Usually, however, the terms driver and follower are applied to *any contiguous* pair of wheels in the train.

The connection between a driver and its follower may be made in either of three ways :—

I. By *rolling contact* at their surfaces, as in *toothless wheels* or *friction gearing.*

II. By *sliding contact* of their surfaces, as in *toothed gearing.*

III. By *belts, ropes,* or *chains.*

The first two methods are adopted when the shafts to be connected are close together. In such cases the wheels are in actual contact with each other. The third method is adopted when the

shafts are so far apart that it becomes impracticable or inconvenient to use friction or toothed wheels. However, circumstances other than the mere distance apart of the shafts often determine which method of connection should be adopted. The advantages and disadvantages attending each method will appear in what follows.

In this Lecture we shall deal with the velocity- and force-ratios communicated by wheels in gear, particularly with friction and toothed gearing, leaving their theory of construction and design to subsequent Lectures. We begin by defining a few of the general terms employed.

Pitch Surface—Pitch Circle.—Motion may be communicated from one shaft to another by rolling contact between the surfaces of bodies rigidly fixed to the shafts. The shafts to be connected may be (1) parallel, (2) intersecting, or (3) neither parallel nor intersecting. The third case will not come under our notice in this text-book. The velocity-ratio transmitted may require to be *constant* or *variable*. It is only for very special machinery that a variable velocity-ratio is required, and we shall, therefore, not consider it here. We have, therefore, to consider the case of a constant velocity-ratio between two parallel or intersecting shafts. The rigid bodies fixed to the shafts, and through which the motion has to be transmitted, must be cylindrical for parallel shafts, and conical for intersecting shafts. These bodies we shall now call wheels, being **spur** or **bevel**, according as they are cylindrical or conical—*i.e.*, according as they are used to connect parallel or intersecting shafts.

The surfaces of the wheels, which, by their rolling contact, communicate the required velocity-ratio, are called **Pitch Surfaces.** In ordinary friction gearing these pitch surfaces have a real physical existence, but in toothed gearing they have no such existence. However, for constructive and other purposes, it is necessary to imagine such surfaces as also existing in their case. Hence, we have the following:—

DEFINITION.—**The Pitch Surface of a Toothed Wheel is an ideal surface (intermediate between the crests of the teeth and the bottoms of the spaces), which, by rolling contact with the pitch surface of another wheel would communicate the same motion that the toothed wheels communicate.**

DEFINITION.—**The Pitch Line or Pitch Circle is a section of the pitch surface perpendicular to it and to the axis of the shaft.**

In the case of cylindrical pitch surfaces, the surface of section is a *plane* perpendicular to the axis of the shaft; while for conical pitch surfaces it is a *sphere* having its centre at the apex of the cone.

Definition.—**The Pitch Point of a pair of wheels in gear is the point of contact of their pitch lines or circles.**

In bevel gearing only frusta of the conical surfaces are employed. Equal bevel wheels having the angle at the apex of their pitch cones equal to a right angle are called **Mitre Wheels.** Mitre wheels are used when two shafts at right angles to each other have to rotate with equal speeds.

Sizes of Spur and Bevel Wheels.—The size of a spur wheel is measured by the diameter of its pitch circle. With bevel wheels, however, the pitch circle is of variable diameter. For some purposes, the size of the bevel wheel is measured at the larger end of the conical frustum, while for other purposes it is measured at a pitch circle half way between the larger and smaller ends of the frustum. For mere convenience in stating the relative sizes of bevel wheels, the first of these methods is adopted; but for calculations relating to power transmitted, &c., we require to adopt the second method. In applying the following results of this Lecture to bevel gearing, it must, therefore, be remembered that the diameter, pitch circle velocity, &c., refer to the pitch circle *half way* between the larger and smaller ends of the frustum.

Velocity-Ratio of Two Wheels in Gear.—When toothed gearing has properly formed teeth, and the wheels are accurately in gear, then the velocity-ratio transmitted is kinematically identical with that obtained by the rolling of their pitch surfaces in contact without slipping. Hence, in what follows, we shall consider the velocity-ratio of the pitch circles only.

Let A and B be the centres of two wheels in gear.
" D_1, D_2 = Diameters of pitch circles A and B respectively.
" N_1, N_2 = Number of revolutions of wheels A and B " in unit time.
" n_1, n_2 = Number of teeth on " " "

Then, since no slipping takes place, we must have :—

Circumferential Velocity of A = *Circumferential Velocity of* B.

$$\therefore \qquad \pi D_1 N_1 = \pi D_2 N_2 .$$

Or,
$$\frac{N_2}{N_1} = \frac{D_1}{D_2}$$

Also,
$$\frac{n_1}{n_2} = \frac{D_1}{D_2}$$

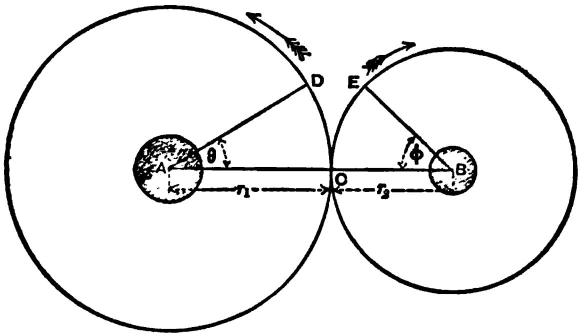
$$\therefore \qquad \frac{N_2}{N_1} = \frac{D_1}{D_2} = \frac{n_1}{n_2} . \quad \ldots \ldots \quad \text{(I.)}$$

Consequently, *The speeds of two wheels in gear, are inversely as their diameters or number of teeth.*

Angular Velocity-Ratio.—Sometimes we require to know the *angular* velocity-ratio of two wheels in gear in terms of their diameters or number of teeth. This we now proceed to determine, but, in the first place, we give the following :—

Definition.—**The Angular Velocity of a rotating body is the circular measure of the angle described in unit time by any line in that body.**

Let ω_1, ω_2 = Angular Velocity of wheels A and B respectively.



Velocity-Ratio of Two Circular Discs.

During a small interval of time let the radii, A C, B C, be displaced through the angles θ and φ into the positions A D, B E respectively.

Then, $$\frac{\omega_1}{\omega_2} = \frac{\theta}{\varphi}.$$

But, $$\text{Arc } CD = \text{Arc } CE.$$

i.e., $$r_1 \theta = r_2 \varphi.$$

Or, $$\frac{r_1}{r_2} = \frac{\varphi}{\theta}.$$

$\therefore$ $$\frac{\omega_1}{\omega_2} = \frac{r_2}{r_1} = \frac{D_2}{D_1}.$$

Hence, $$\frac{\omega_1}{\omega_2} = \frac{N_1}{N_2} = \frac{D_2}{D_1} = \frac{n_2}{n_1}. \quad . \quad . \quad . \quad . \quad \text{(II)}$$

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Consequently, *the angular velocities of two wheels in gear, are inversely as their diameters or number of teeth.*

It may be useful here to state the relation between the angular velocity of a wheel and the linear velocity of a point on its pitch circle.

Let V = Linear velocity of a point on the pitch circle.
„ ω = Angular velocity of wheel.
„ r = Radius of pitch circle.

Since ω is the angle described in unit time by any radius, then :—

$$\left.\begin{array}{l}\textit{Arc described in unit time by a}\\ \quad\textit{point at distance, } r\textit{, from axis,}\end{array}\right\} = \omega r.$$

But the length of this arc represents the velocity, V.

Hence, $$V = \omega r.$$

Again, since $$V = 2 \pi r N.$$

$$\therefore \quad 2 \pi r N = \omega r.$$

Or, $$\omega = 2 \pi N.$$

Velocity-Ratio of a Train of Wheels.—In questions relating to trains of wheels there is a certain advantage in denoting the radii, diameters, or number of teeth on the various wheels by single letters, such as A, B, C, . . . , which letters also may indicate the wheels themselves.*

Definition.—The ratio of the number of revolutions of the last wheel in a train to the number of revolutions of the first wheel in the same time is called the Value of the Train.

Thus, denoting the value or velocity-ratio of the train by e, we get :—

$$e = \frac{\text{Revolutions of last wheel in any time}}{\text{Revolutions of first wheel in the same time}}.$$

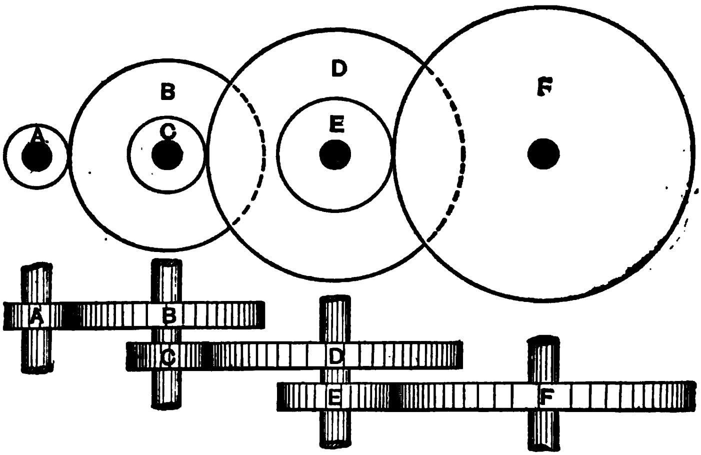
Let N_1, N_l denote the number of revolutions made by *first* and *last* wheels in a train in a given time; then :—

$$e = \frac{N_l}{N_1}\text{; or, } N_l = e\,N_1. \quad . \quad . \quad . \quad . \quad . \quad \text{(III)}$$

* In the *Elementary Manual on Applied Mechanics*, we denoted the drivers of a train of wheels by D_1, D_2, D_3, &c., and the followers by F_1, F_2, F_3, &c. (see Lecture XII. of the same). Students may adopt either method of notation.

The figure shows a train of wheels, A, B, C, D, E, and F, whereof A may be called the first wheel, or driver, and F the last wheel, or follower. On each of the intermediate shafts, B C, D E, there are two wheels of different sizes, the smaller wheel of each shaft gearing with the larger one on the following shaft. From what has been said at the beginning of this Lecture, we may consider each of the pairs of wheels, A, B; C, D; and E, F, as driver and follower with respect to each other. The first and last shafts, A and F, each carry one wheel only.

Let N_1, N_2, N_3, N_4 denote the speeds or number of revolutions



VELOCITY-RATIO OF A TRAIN OF WHEELS.

per minute of the first, second, third, and fourth shafts respectively :—

Then, $$\frac{N_2}{N_1} = \frac{A}{B}; \quad \frac{N_3}{N_2} = \frac{C}{D}; \quad \frac{N_4}{N_3} = \frac{E}{F}.$$

Multiplying together the corresponding sides of these three equations, we get :—

$$\frac{N_2}{N_1} \times \frac{N_3}{N_2} \times \frac{N_4}{N_3} = \frac{A}{B} \times \frac{C}{D} \times \frac{E}{F},$$

$$\therefore \qquad \frac{N_4}{N_1} = \frac{A \times C \times E}{B \times D \times F}.$$

i.e., The velocity-ratio $e = \dfrac{A \times C \times E}{B \times D \times F}$. . . . . (IV)

This is true, however many wheels are in the train, and thus we have the following rule :—

$$\text{Value of a Train} = \frac{\left\{\begin{array}{l}\text{Product of radii, diameters, or number}\\ \quad\text{of teeth of all the drivers}\end{array}\right.}{\left\{\begin{array}{l}\text{Product of radii, diameters, or number}\\ \quad\text{of teeth of all the followers}\end{array}\right.}.$$

EXAMPLE I.—The table of a planing machine has to be moved backward and forward at the rate of 12 feet per minute by a rack and pinion arrangement underneath it. The gearing consists of three shafts; the first carrying the pinion which gears with the rack; the last carrying the pulley on which a belt works. The pinion gearing with the rack has 12 teeth of $1\frac{1}{4}$ inches pitch. On the same shaft as this pinion is a wheel of 40 teeth. This wheel gears with a pinion of 15 teeth on the second shaft. A wheel of 30 teeth on this shaft gears with a pinion of 12 teeth on the last shaft. Find the number of revolutions of the last shaft per minute.

ANSWER.—Let A, B, C, D denote the numbers of teeth on the wheels gearing together, where :—

$$A = 40, \ B = 15, \ C = 30, \ D = 12,$$

Then,
$$e = \frac{A \times C}{B \times D} = \frac{40 \times 30}{15 \times 12} = \frac{20}{3}.$$

Since the pitch of the teeth on rack and pinion is $1\frac{1}{4}$ inch, and the speed of the rack is 12 feet per minute :—

$$\therefore \left.\begin{array}{r}\textit{Revolutions of first}\\ \textit{shaft per minute}\end{array}\right\} = \frac{\textit{Speed of table}}{\textit{Circumference of pinion}},$$

$$\text{,,} \quad \text{,,} \quad = \frac{12 \times 12 \textit{ inches per minute}}{12 \times 1\frac{1}{4} \textit{ inches}},$$

$$\text{,,} \quad \text{,,} \quad = \frac{48}{5} = 9\tfrac{3}{5}.$$

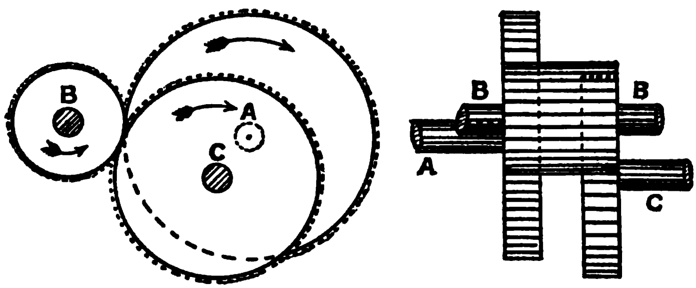
$$\therefore \left.\begin{array}{r}\textbf{Revolutions of last}\\ \textbf{shaft per minute}\end{array}\right\} = e\ N_A = \frac{20}{3} \times 9\tfrac{3}{5} = 64.$$

Intermediate or Idle Wheel.—Sometimes a wheel carried on a separate axle is interposed between two other wheels, or introduced into a train of wheels, for the purpose of *changing the relative directions of rotations* of the first and last wheel. Such an intermediate wheel is called an *idle* wheel, because it does not affect the

numerical value of the train, but only its *sign*. That an idle wheel has the effect just stated may easily be proved by considering the velocity-ratio of a pair of wheels connected through an intermediate one.

Instances of the use of idle wheels are very common in gearing, but the two following will serve as examples :—

I. **Marlborough Wheel.**—When two parallel shafts, A and C, are so close together that they cannot be conveniently connected in the ordinary way, a broad wheel, B, called a *Marlborough wheel*, may be introduced as shown. This wheel has the effect of causing the shafts, A and C, to rotate in the same direction, but in no way does it affect the velocity-ratio which would be obtained by the direct gearing of the wheels A and C. There are other methods whereby the same object could be attained as with a



MARLBOROUGH WHEEL.

Marlborough wheel arrangement; but with these we are not at present concerned.

II. **Change Wheels for Screw Cutting Lathes.***—In screw cutting lathes, a train of wheels, called change wheels, is interposed between the back end of the lathe spindle and the leading screw, for the purpose of transferring motion to the saddle, and determining that the cutting tool shall be moved through a definite pitch for each rotation of the cylinder to be turned or screwed. Every turn of the leading screw moves the saddle and cutting tool through a distance equal to its pitch, and, consequently, if the bar to be screwed turn at the same rate as the leading screw, the pitch of the screw cut upon it will be the same as that of the leading screw. If it move faster than the leading screw, the pitch will be less; and if slower, the pitch will be correspondingly greater. It therefore follows as a matter of

* See the author's *Manual on Applied Mechanics*, Lecture XVI., for description, &c., of a screw cutting lathe.

course, that if we fit wheels on the lathe spindle and on the leading screw of the same diameter, or having the same number of teeth, the screw being cut will have the same pitch as the leading screw. If we fix a small pinion, or one with few teeth, on the lathe spindle and a wheel of large diameter, or many teeth, on the leading screw, the pitch of the screw to be cut will be small, compared with that of the leading screw. The leading screw is generally right-handed, in which case the screw to be cut will be right-handed or left-handed, according as its direction of rotation is the same as, or different from, that of the leading screw. In the former case, there must be at least one intermediate axis between the lathe mandril and the leading screw. If the wheels on the lathe mandril and on the end of the leading screw are of the proper size for the necessary velocity-ratio, then the intermediate axis must carry an idle wheel. Sometimes the wheels required to give the proper velocity-ratio and relative direction of rotation cannot be correctly adjusted without the interposition of more wheels in the train, when it may be necessary to introduce one or more idle wheels.

Let p_c = Pitch of screw to be cut, in inches.
„ p_l = Pitch of parent or leading screw, in inches.
„ e = Effect or value of train of change wheels.

Then,

$$\frac{\textit{Pitch of screw to be cut}}{\textit{Pitch of leading screw}} = \frac{\textit{Speed of leading screw}}{\textit{Speed of lathe mandril}}.$$

$$\therefore \qquad \frac{p_c}{p_l} = e. \qquad \text{(V)}$$

The problem, then, consists in finding a train of wheels which shall have its value, $e = \frac{p_c}{p_l}$.

EXAMPLE II.—The leading screw of a lathe is $\frac{1}{2}$ inch pitch and right-handed. The set of change wheels belonging to the lathe consists of the following:—20, 25, 30, 35, 40, 45, 50, 60, 70, 80, 90, 100, 110, and 120 teeth respectively. Devise suitable trains to cut (1) a right-handed screw of 8 threads to the inch, and (2) a left-handed screw of 12 threads to the inch.

ANSWER.—(1) The number of threads on the screw to be cut being 8 per inch, its pitch is, therefore, $p_c = \frac{1}{8}$ inch.

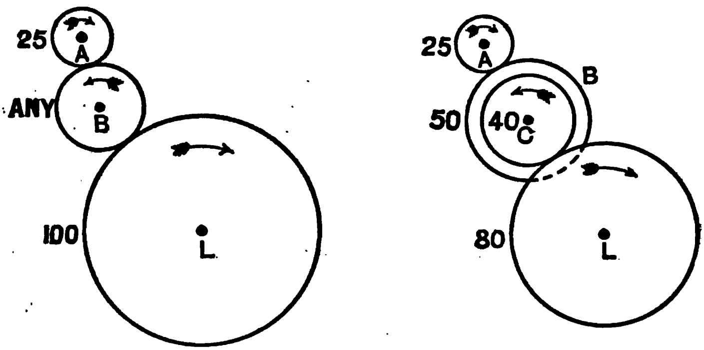
Hence, $$e = \frac{p_c}{p_l} = \frac{\frac{1}{8}}{\frac{1}{2}} = \frac{1}{4}.$$

Both screws being right-handed, they must rotate in the same direction. Hence the train of change wheels must have an *odd* number of axes. Let there be *three* axes in the train.

(*a*) The sizes of the first and last wheels in the train may be such that the required velocity-ratio could be obtained by those wheels in direct gear. In this case the intermediate axis must carry an idle wheel of *any* convenient size, such as B, in the left-hand figure.

$$\therefore \qquad \frac{A}{L} = e = \tfrac{1}{4} = \frac{20}{80}; \text{ or, } = \frac{25}{100}; \text{ or, } = \frac{30}{120}.$$

(*b*) The sizes of the first and last wheels need not be such as



CHANGE WHEELS FOR CUTTING A RIGHT-HANDED SCREW.

give the required velocity-ratio by their direct gear. In this case the intermediate axis must carry two wheels of different sizes as in the right-hand figure.

The following train will answer the purpose :—

$$\frac{A \times C}{B \times L} = e = \tfrac{1}{4} = \frac{25 \times 40}{50 \times 80}.$$

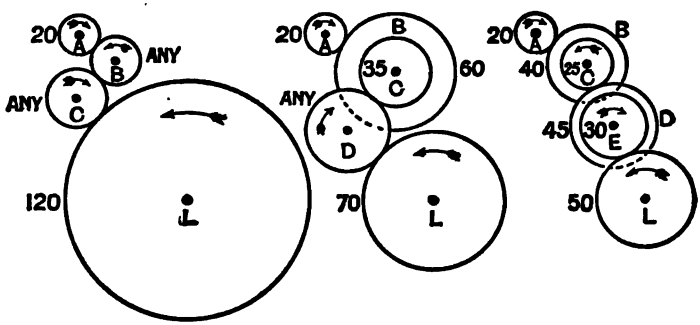
(2) The screw to be cut being left-handed, the leading screw must rotate in the opposite direction from that of the lathe mandril; hence the train must consist of an *even* number of axes. Let there be *four* axes in the train.

(*a*) Let A and L be of such sizes that they would transmit the proper velocity-ratio if geared directly. Then the two intermediate axes must each carry an idle wheel as in the left-hand figure.

Here, $$e = \frac{p_c}{p_l} = \frac{\frac{1}{12}}{\frac{1}{2}} = \tfrac{1}{6}.$$

$$\therefore \qquad \frac{A}{L} = e = \tfrac{1}{6} = \frac{20}{120}.$$

(*b*) The wheels A and L, not being of the above sizes, we may arrange the train so that *one* of the intermediate axes carries an idle wheel, or that there are no idle wheels in the train. These



Change Wheels for Cutting a left-handed Screw.

arrangements are shown by the middle and right-hand figures. The trains may be as follows :—

With one idle wheel, $$\frac{A \times C}{B \times L} = e = \tfrac{1}{6} = \frac{20 \times 35}{60 \times 70}.$$

Or, With no idle wheel, $$\frac{A \times C \times E}{B \times D \times L} = e = \tfrac{1}{6} = \frac{20 \times 25 \times 30}{40 \times 45 \times 50}.$$

Force-Ratio and Power Transmitted by Gearing.—For certain purposes, as in hoisting and similar machinery, a small effort, P, moving through a comparatively great distance, may be utilised in overcoming a much greater effort or resistance, W, through a much smaller distance. For other purposes, where speed is the ultimate desideratum, the converse of the above would be adopted; *i.e.*, a large effort, P, moving slowly, overcomes a smaller resistance, W, moving rapidly. In any case, it is evident from the *Principle of Work*, that what is lost or gained in speed is gained or lost in the resistance overcome. To modify the effort during its transmission to the working point, suitable mechanism has

to be employed. Several forms of mechanism for this purpose have been considered in previous Lectures; consequently, we proceed at once to express the force-ratio obtained by a train of wheels.

Let the effort, P, be applied by hand or by an engine to the end of a lever or crank rigidly fixed to the *first* axle or shaft at A, while the resistance to be overcome is applied at the circumference of a drum or pulley keyed to the *last* axle or shaft, F, in the following train of wheels:—

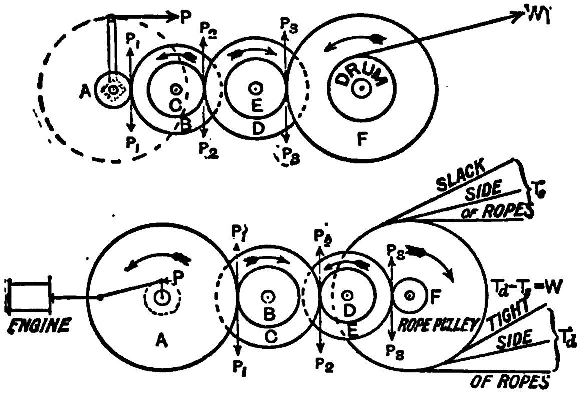
Let P_1, P_2, P_3 = Tangential pressures at points of contact of wheels in gear,

" r_a, r_b, &c. = Radii of wheels denoted by the capitals of these suffix letters,

" R = Length of lever handle or crank,

" r = Radius of drum or pulley,

" e = Value of train.



POWER TRANSMITTED BY GEARING.

Then, by the *Principle of Moments*, we get:—

$$P \times R = P_1 \times r_a \quad \ldots \ldots \quad (1)$$

$$P_1 \times r_b = P_2 \times r_c \quad \ldots \ldots \quad (2)$$

$$P_2 \times r_d = P_3 \times r_e \quad \ldots \ldots \quad (3)$$

$$P_3 \times r_f = W \times r \quad \ldots \ldots \quad (4)$$

Multiplying together the corresponding members of these equations, and cancelling the terms P_1, P_2, and P_3, we get :—

$$P \times R \times r_b \times r_d \times r_f = W \times r \times r_a \times r_c \times r_e.$$

$$\left.\begin{aligned} \therefore \quad & \frac{P}{W} = \frac{r}{R}\left(\frac{r_a \times r_c \times r_e}{r_b \times r_d \times r_f}\right) \\ \text{But,} \quad & e = \frac{r_a \times r_c \times r_e}{r_b \times r_d \times r_f} \quad . \quad . \\ \therefore \quad & \frac{P}{W} = \frac{r}{R} \times e. \quad . \quad . \quad . \end{aligned}\right\} \quad . \quad . \quad \text{(VI)}$$

The results expressed in equation (VI) may be more easily arrived at in the following way :—

By the *Principle of Work* (neglecting friction, &c.), we get :—

$$P \times \textit{its displacement} = W \times \textit{its displacement}.$$

Or,
$$\frac{P}{W} = \frac{\text{W's displacement in a given time}}{\text{P's displacement in the same time}}.$$

Hence,
$$\frac{P}{W} = \frac{\text{Velocity of W}}{\text{Velocity of P}} = \frac{2\,\pi\,r \times N_2}{2\,\pi\,R \times N_1}.$$

Or,
$$\frac{P}{W} = \frac{r}{R} \times e.$$

Equations (1), (2), (3), and (4) may be used in finding the tangential resistances P_1, P_2, P_3.

Neglecting friction, the power transmitted at any stage of the transmission is constant. Thus, the power transmitted from the second to the third shaft is the same as that done at the driving or working ends.

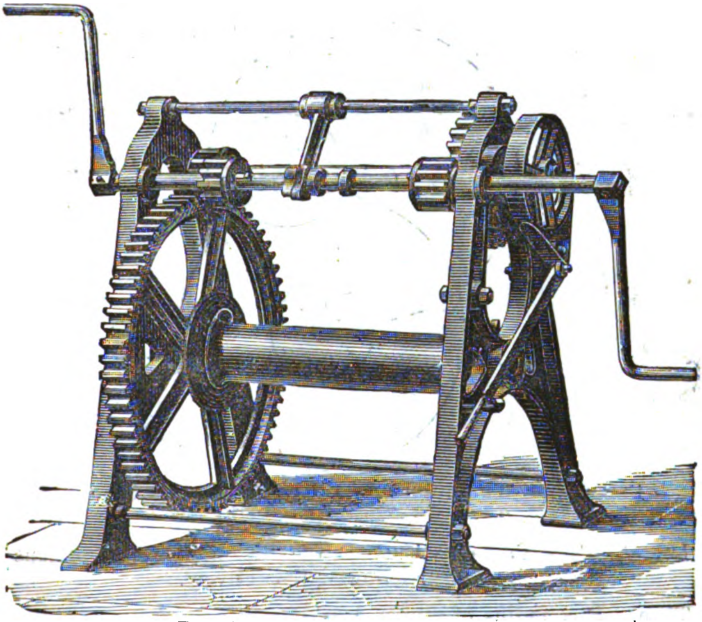
Let H.P. = Horse-power transmitted,

" P = Tangential pressure in lbs. at pitch surface of any given wheel in the train,

" V = Velocity in feet per minute at the pitch surface of the given wheel.

Then, clearly,
$$\text{H.P.} = \frac{P\,V}{33{,}000}. \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(VII)}$$

EXAMPLE III.—The handles of a double-purchase lifting crab are 16 inches long. The pinions have 14 and 20 teeth respectively, and the wheels gearing with these have 84 and 100 teeth respectively. The diameter of the barrel is 12 inches, thickness of rope 1 inch. Find the effort, P, necessary to raise a load of 30 cwts., friction being neglected. If the diameter of the pinion on the first motion shaft be 6 inches, find the pressure between the teeth of each pair of wheels in gear.



DOUBLE-PURCHASE WINCH OR CRAB.

ANSWER.—The accompanying figure shows a general view of a double-purchase lifting crab. For sketches and description of similar lifting machinery we must refer the student to the author's *Manual on Applied Mechanics*, Lecture XIII.

Here R = 16 inches, r = radius of barrel + $\frac{1}{2}$ thickness of rope = $6\frac{1}{2}$ inches.

$$\text{Value of train} = e = \frac{14 \times 20}{84 \times 100} = \frac{1}{30}.$$

$$\therefore \qquad \frac{P}{W} = \frac{r}{R} \times e.$$

Or, $$\frac{P}{30 \times 112} = \frac{6\frac{1}{2}''}{16''} \times \frac{1}{30}.$$

i.e., $$P = 45{\cdot}5 \text{ lbs.}$$

Since there are two handles, the effort exerted on each will be:—

$$\tfrac{1}{2} P = 22{\cdot}75 \text{ lbs.}$$

Next, to find the pressure between the teeth of the wheels.

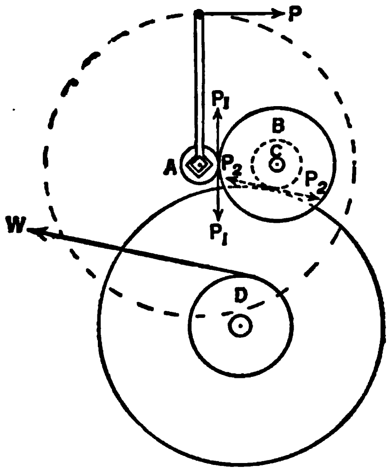


Diagram of Wheel-work of a Double-purchase Crab to Illustrate Example III.

Let P_1, P_2 = Pressures between the first and second pinions and their followers respectively.

Then, $P_1 \times$ radius of pinion A $= P \times R$.

$\therefore$ $$P_1 \times 3 = 45{\cdot}5 \times 16.$$

$\therefore$ $$P_1 = 243 \text{ lbs.}$$

Also, $P_2 \times$ radius of pinion C $= P_1 \times$ radius of wheel B.

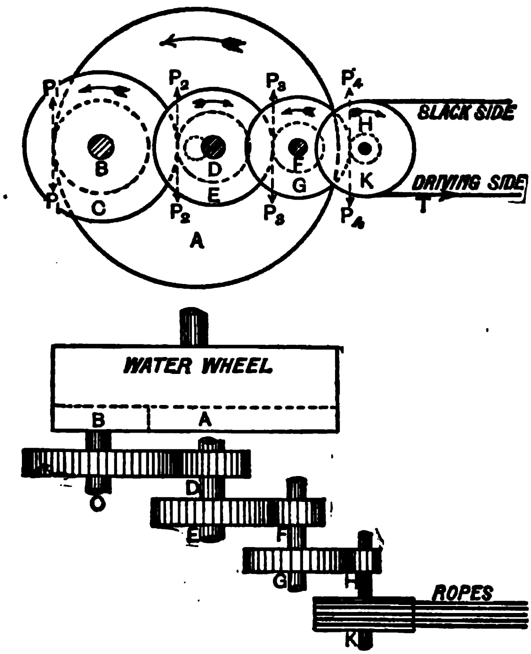
$\therefore$ $$P_2 = P_1 \times \frac{\text{radius of wheel B}}{\text{radius of pinion C}}.$$

Or, $$P_2 = P_1 \times \frac{\text{number of teeth on B}}{\text{number of teeth on C}}.$$

i.e., $$P_2 = 243 \times \frac{84}{20} = 1{,}019 \text{ lbs.}$$

From these results we notice that the wheels should be made stronger as they approach the barrel shaft.

EXAMPLE IV.—A water-wheel making two revolutions per minute is provided with an internal toothed wheel of 24 feet diameter at the pitch circle. This wheel gears with a train of wheels, thus:—Pinions of 8, 6, 4, and 2 feet diameter, and wheels of 12, 10, and 8 feet diameter. The last shaft in the train carries



To Illustrate Example IV.

Power Transmitted from a Water-Wheel by Toothed-Gearing and Ropes.

a rope pulley of 8 feet diameter. If 40 horse-power be transmitted, find the total driving tension in the ropes and the pressure between the teeth of each pair of wheels in the toothed gearing.

ANSWER.—The arrangement is shown by the accompanying sketch.

The value of the train is :—

$$e = \frac{24 \times 12 \times 10 \times 8}{8 \times 6 \times 4 \times 2} = 60.$$

$$\therefore \quad \left.\begin{array}{c}\textit{Circumferential speed}\\ \textit{of rope pulley}\end{array}\right\} = \mathrm{V} = \pi \mathrm{D} \mathrm{N}_l = \pi \mathrm{D} \mathrm{N}_1 e$$

$$\text{,,} \qquad = \frac{22}{7} \times 8 \times 2 \times 60 \text{ ft. per minute.}$$

Let T = Driving tension in ropes in lbs.

Then,

$$\mathrm{H.\,P.} = \frac{\mathrm{T\,V}}{33{,}000},$$

$$\therefore \qquad \mathrm{T} = \frac{40 \times 33{,}000}{\frac{22}{7} \times 8 \times 2 \times 60} = 437 \cdot 5 \text{ lbs.}$$

Let P_1, P_2, &c., denote the tangential pressures at the pitch circles of the various pairs of wheels in gear.

Let r_a, r_b, &c., denote the radii of wheels, A, B, &c., respectively.

Then, since the power developed at pitch circle of wheel A, is equal to the power transmitted by ropes on pulley K, we get :—

$P_1 \times$ *Circumferential speed of* A = T $\times$ *Circumferential speed of* K.

$$\therefore \qquad P_1 \times 2\,\pi\, r_a\, N_1 = T \times 2\,\pi\, r_k\, N_l\,.$$

$$\therefore \qquad P_1 = T \times \frac{r_k}{r_a} \times e.$$

Or,

$$P_1 = 437 \cdot 5 \times \frac{4}{12} \times 60 = 8{,}750 \text{ lbs.}$$

Next,

$$P_2 \times r_c = P_1 \times r_b\,.$$

$$\therefore \qquad P_2 = 8{,}750 \times \frac{4}{6} = 5{,}833 \cdot \dot{3} \text{ lbs.}$$

But,

$$P_3 \times r_e = P_2 \times r_d\,.$$

$$\therefore \qquad P_3 = 5{,}833 \cdot \dot{3} \times \frac{3}{5} = 3{,}500 \text{ lbs.}$$

Lastly,

$$P_4 \times r_g = P_3 \times r_f\,.$$

$$\therefore \qquad P_4 = 3{,}500 \times \frac{2}{4} = 1{,}750 \text{ lbs.}$$

Lecture XI.—Questions.

1. Define the following terms as applied to gearing:—Pitch surface, pitch circle, and pitch point. Illustrate your answers by reference (1) to a spur wheel, (2) to a bevel wheel, and (3) to a rack.

2. Define angular velocity. Given the angular velocity of a body about a given axis, show how you would find the linear velocity of any point in the body.

3. Define the *pitch circle* of a toothed wheel. Prove that when two wheels, whose axes are parallel, gear together, their angular velocities are inversely as the diameters of their pitch circles. Two parallel shafts are at a distance of $4\frac{1}{2}$ feet, and they are to rotate with velocities as the numbers 7 and 11 respectively. Determine the diameters of the pitch circles of a pair of wheels which would give the required motion. *Ans.* $5\frac{1}{2}$ ft. and $3\frac{1}{2}$ ft.

4. Define the term *train of wheels*, and explain how to find the value of a given train. Arrange trains of wheels for the following values of e, no pinion to have less than 12 teeth, and no wheel to have more than 120:— $\frac{2,000}{240}$, $\frac{1,200}{490}$, $\frac{880}{35}$, $\frac{35}{880}$, $\frac{490}{1,200}$, $\frac{240}{2,000}$.

5. What are mitre and idle wheels, and for what purposes are they used? Give instances of the use of both.

6. Explain, by aid of a sketch, the use of a Marlborough wheel. A shaft is divided into two parts, the parts being still in line. Sketch an arrangement of wheels whereby one part of the shaft may drive the other at twice the speed of the first.

7. Describe the operation of cutting a screw in a lathe, showing the wheels required, and how they are placed to cut a right-handed screw with 8 threads to the inch in a lathe whose leading screw is of $\frac{1}{2}$ inch pitch.

8. Explain the use of change wheels in a screw-cutting lathe. It is desired to cut a screw of $\frac{3}{8}$ inch pitch in a lathe with a leading screw of 4 threads to the inch, using 4 wheels. If both screws be right-handed, what wheels would you employ? (S. & A. Adv. Exam., 1887.)

9. The leading screw in a self-acting lathe has a pitch of $\frac{1}{2}$ inch, show an arrangement of change wheels for cutting a screw of $\frac{3}{8}$ inch pitch.

10. You are required to cut a left-handed screw of 5 threads to the inch in a lathe fitted with a right-handed guide-screw of $\frac{1}{2}$ inch pitch. Show clearly by the aid of sketches the change-wheels which you would employ for the purpose, indicating how they would be respectively carried, and the number of teeth in each wheel. (S. & A. Exam., 1891.)

11. The leading screw of a lathe is $\frac{1}{4}$ inch pitch and right-handed. The set of change wheels belonging to the lathe consists of the following:—20, 25, 30, 35, 40, 45, 50, 60, 70, 80, 90, 100, 110, and 120 teeth. From these devise suitable trains to cut the following screws and draw up a table of your results:—4, $4\frac{1}{2}$, 5, $5\frac{1}{2}$, 6, $6\frac{1}{2}$, 7, 8, 9, 10, 12, 14, and 16 threads per inch.

12. A wheel of 40 teeth is driven by a winch handle 14 inches long, and gears with a rack having teeth of 1 inch pitch; apply the principle of work to find the driving pressure exerted on the rack when a force of 50 lbs. is applied at the end of the winch handle. *Ans.* 110 lbs.

13. Sketch, in side elevation, the wheelwork of an ordinary 5-ton lifting crane. In doing so, it will be sufficient to represent the wheels by their pitch circles. If the weight raised moves through 1 inch when the driving

handle moves through 40 inches, find the weight which could be raised by 60 lbs. applied at the end of the lever handle. *Ans.* 2,400 lbs.

14. In a model of a lifting crab, the circumference of the circle described by the end of the winch handle is 43 inches, and the circumference of the drum which raises the weight is 14·9 inches. The wheelwork gives an advantage of 8 to 1, and it is found by trial that a force of 3·1 lbs. on the winch handle just suffices to raise a weight of 56 lbs. hanging on a cord wound upon the drum. What proportion of the power exerted is lost in this model? *Ans.* 21·76 per cent.

15. Sketch two views of a treble-purchase lifting crab. In doing so it will be sufficient to represent the wheels in side elevation by their pitch circles. Apply the Principle of Work, *or* the Principle of Moments to determine the force-ratio, P : W, and the pressure between the teeth of each pair of wheels in gear. *Ex.* In a treble-purchase lifting crab the handles are 16 inches long; diameter of drum 16 inches, thickness of rope $2\frac{1}{4}$ inches. The wheelwork consists of the following:—Pinions 6, 6, and 8 inches diameter; wheels 24, 30, and 36 inches diameter. Supposing two men to work at each handle, each man exerting a force of 30 lbs., find the weight which could be raised, and the pressure between the teeth of each pair of wheels in gear. Allowing 30 per cent. for friction, find the actual weight which could be raised by the four men. *Ans.* (1) 8·34 tons nearly, 640 lbs., 2,560 lbs., 9,600 lbs.; (2) 5·84 tons nearly.

16. A water-wheel making two and a-half revolutions per minute is provided with an internal toothed wheel of 20 feet diameter at the pitch circle. This wheel gears with a train of wheels, thus:—Pinions of 6, 4, and 2 feet in diameter, and wheels of 10 and 9 feet in diameter respectively. On the last shaft a pulley 10 feet in diameter is keyed, on the rim of which there are four ropes. The horse-power transmitted by the ropes is 20. You are required to find the pull on each rope, the pressure between the teeth of each pair of wheels gearing together, and the revolutions per minute made by the pulley. *Ans.* 56 lbs., 4,200 lbs., 2,520 lbs., 1,120 lbs., 93·75 revolutions.

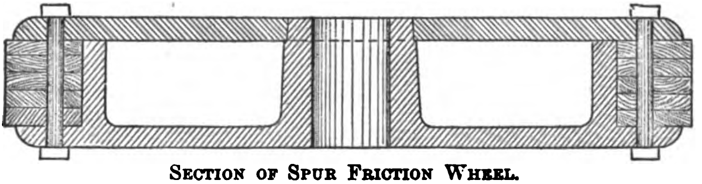
LECTURE XII.

CONTENTS.—Friction Gearing—Power Transmitted by Ordinary Friction Gearing—Examples I. and II.—Robertson's Friction or Wedge Gearing—Power Transmitted by Wedge Gearing—Questions.

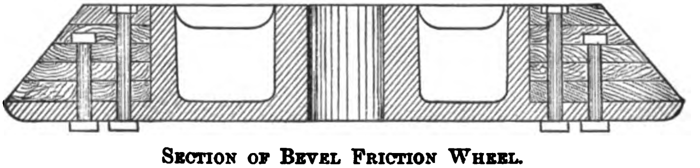
Friction Gearing.—Friction gearing is that form of gearing wherein the wheels in contact are driven, the one by the other, by reason of the friction between their pitch surfaces. The wheels require to be pressed together in a direction normal to their pitch surfaces at the line of contact, with a force sufficient to give a frictional resistance greater than the tangential resistance to motion. The wheels may be spur or bevel, according as the shafts are parallel or intersecting.

In order to insure sufficient frictional resistance and smooth working, it is usual to face one wheel of the pair with some compressible material, such as wood, leather, india-rubber, compressed paper, &c. When slipping takes place between the wheels, "flats" are soon formed on the face of the *follower* (this being the wheel which lags behind the other), while the face of the driver gets equally worn all round. For this reason the driver, and not the follower, is faced with the softer material.

The usual forms of rims suitable for spur and bevel friction wheels are shown by the accompanying figures.



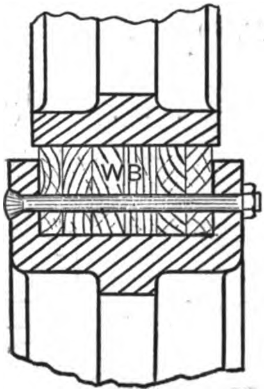
SECTION OF SPUR FRICTION WHEEL.



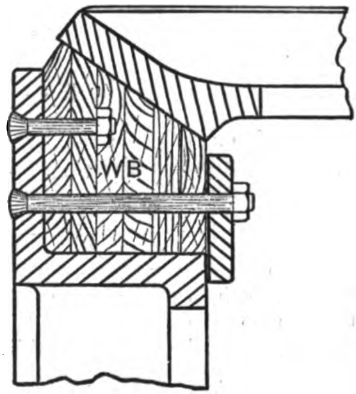
SECTION OF BEVEL FRICTION WHEEL.

The rim of the driver is faced with wood or leather, the different

layers of which are nailed or glued together and then held in position by bolts as shown. When wood is used the grain should lie in a direction tangential to the working surfaces, the wear being



Sectional View of Rim for Spur Friction Wheel.



Sectional View of Rim for Bevel Friction Wheel.

then more uniform all over. The rim of the follower is of cast iron turned in a lathe.

Friction gearing of this kind is more employed in America than in this country, being often applied for driving saw-mills, &c.

Power Transmitted by Ordinary Friction Gearing.—We shall now proceed to calculate the necessary pressure to be applied to the wheels in order to transmit a given power.

(1) *By Spur Friction Gearing.*

Let P = Pressure between wheels at pitch line in *pounds.*

„ T = Tangential resistance at pitch line in *pounds.*

„ V = Circumferential velocity of wheels in *feet per minute.*

„ μ = Coefficient of friction for surfaces in contact.

Then, *Work transmitted* = T V ft.-lbs. per minute.

$$\therefore \qquad \text{H.P.} = \frac{\text{T V}}{33{,}000} \quad \ldots \ldots \ldots \quad \text{(I)}$$

If there is to be no slipping, we must have :—

$$T \leqq \mu P.$$

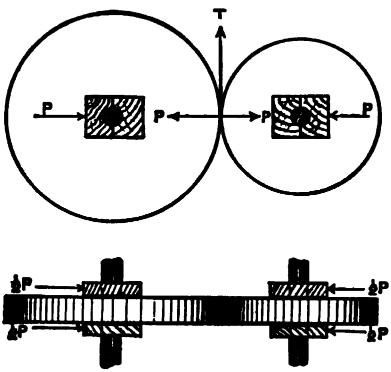
Or,
$$P \geqq \frac{T}{\mu}.$$

But,
$$T = \frac{\text{H.P.} \times 33{,}000}{V}.$$

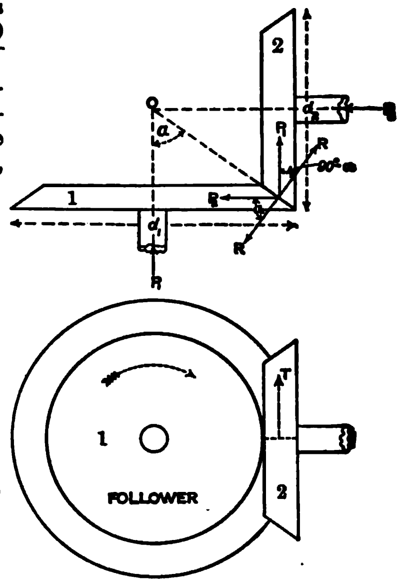
$\therefore$
$$P \geqq \frac{\text{H.P.} \times 33{,}000}{\mu V} \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(II)}$$

Equation (II) gives the least pressure between the wheels in order to transmit a given power.

The pressure, P, should always be greater than $\frac{\text{H.P.} \times 33{,}000}{\mu V}$ so as to provide against contingencies, such as oil or water getting on to the surfaces of the wheels. If P be less than this, then slipping must take place.



Spur Friction Gearing.



Bevel Friction Wheels.

(2) *By Bevel Friction Gearing.*

Let P_1, P_2 = Thrusts along shafts 1 and 2 respectively.
" d_1, d_2 = Mean diameters of wheels 1 and 2 respectively.
" R = Normal reaction between pitch cones.
" T = Tangential resistance at pitch line.
" 2α = Angle of pitch cone 1.

Then, $90° - \alpha$ = Half-angle of pitch cone 2.

Since the pressures along the axes of the shafts are P_1 and P_2, it follows that these are also the pressures at the pitch line, and act as indicated by the figure.

The normal pressure between the surfaces is :—

$$P_1 = R\cos(90^\circ - \alpha) = R\sin\alpha.$$

Or, $$P_2 = R\cos\alpha.$$

But, $$\tan\alpha = \frac{\frac{1}{2}d_1}{\frac{1}{2}d_2} = \frac{d_1}{d_2}.$$

$\therefore$ $$\sin\alpha = \frac{\tan\alpha}{\sqrt{1+\tan^2\alpha}} = \frac{d_1}{\sqrt{d_1^2+d_2^2}}.$$

And, $$\cos\alpha = \frac{1}{\sqrt{1+\tan^2\alpha}} = \frac{d_2}{\sqrt{d_1^2+d_2^2}}.$$

$\therefore$ $$R = P_1\frac{\sqrt{d_1^2+d_2^2}}{d_1}.$$

Or, $$R = P_2\frac{\sqrt{d_1^2+d_2^2}}{d_2}.$$

But, $$T \leqq \mu R.$$

Or, $$R \geqq \frac{T}{\mu}.$$

And, as before, $$T = \frac{\text{H.P.} \times 33{,}000}{V}.$$

$\therefore$ $$R \geqq \frac{\text{H.P.} \times 33{,}000}{\mu V}.$$

Consequently, $$P_1 \geqq \frac{\text{H.P. } 33{,}000}{\mu V} \cdot \frac{d_1}{\sqrt{d_1^2+d_2^2}} \quad . \quad . \quad . \quad . \quad \text{(III)}$$

And, $$P_2 \geqq \frac{\text{H.P. } 33{,}000}{\mu V} \cdot \frac{d_2}{\sqrt{d_1^2+d_2^2}} \quad . \quad . \quad . \quad . \quad \text{(IV)}$$

Equations (III) and (IV) determine the least values of the axial thrusts in order to insure sufficient frictional resistance for transmitting a given power.

The following values of μ may be taken *:—

For metal on metal, . . .	μ = ·15 to ·20.
For wood on metal, . . .	μ = ·25 to ·30.
For millboard on metal, . . .	μ = ·20.

* Unwin's *Machine Design*, part I., p. 283.

Taking the greatest and least of the above values for μ, we see that in spur friction gearing, the smallest value of P must lie between $\frac{T}{\cdot 3}$ and $\frac{T}{\cdot 15}$, *i.e.*, between $3\frac{1}{3}$ T and $6\frac{2}{3}$ T.

In practice, the width of the face of friction wheels is about the same as that of a single leather belt which is required to transmit the same power. The tangential force may be taken at from 15 to 30 lbs. per inch of width when the face of the driver is lined with wood. In the case of a wheel with millboard face, the tangential force transmitted was observed, by Prof. Unwin, to be as great as 80 lbs. per inch of width.*

It will be apparent that friction gearing of the above kind is unsuited for transmitting great power. The constancy of velocity-ratio between the wheels cannot be relied upon. Gearing of this kind is only used (1) when the power to be transmitted is small; (2) when the speed is so high that toothed gearing would be noisy; (3) when the wheels require to be frequently put into or out of gear.†

Example I.—In a spur friction gearing the driving wheel is faced with wood, and gears with a metal wheel $3\frac{1}{2}$ feet in diameter. The latter makes 200 revolutions per minute, and transmits 10 H.P. Find the tangential resistance at the circumferences of the wheels, and the necessary thrust to be applied to the bearings of the shafts, taking $\mu = \cdot 25$.

Answer.—Using the same notation as in the text, we have,

$$\text{H.P.} = 10; \quad V = \pi d n = \frac{22}{7} \times 3\tfrac{1}{2} \times 200 = 2{,}200 \text{ ft. per minute.}$$

$$\therefore \qquad T = \frac{\text{H.P.} \times 33{,}000}{V} = \frac{10 \times 33{,}000}{2{,}200} = 150 \text{ lbs.}$$

Also,
$$P = \frac{T}{\mu} = \frac{150}{\cdot 25} = 600 \text{ lbs.}$$

Example II.—5 H.P. has to be transmitted through a pair of bevel friction wheels. The diameters of the wheels are 2 feet and $1\frac{1}{2}$ feet respectively. The circumferential speed of the wheels is 1,000 feet per minute. Find the normal and tangential pressures at the surface of contact of the two wheels, and the axial thrusts to be applied to each shaft, taking $\mu = \cdot 2$.

Answer.—Here $d_1 = 2$ ft., $d_2 = 1\frac{1}{2}$ ft., V = 1,000 ft. per min.

* Unwin's *Machine Design*, p. 284. † *Ibid.*, p. 281.

Then :—

$$\text{Normal pressure} = R = \frac{\text{H.P.} \times 33{,}000}{\mu V},$$

$$\text{,,} \qquad \text{,,} = \frac{5 \times 33{,}000}{\cdot 2 \times 1{,}000} = 825 \text{ lbs.}$$

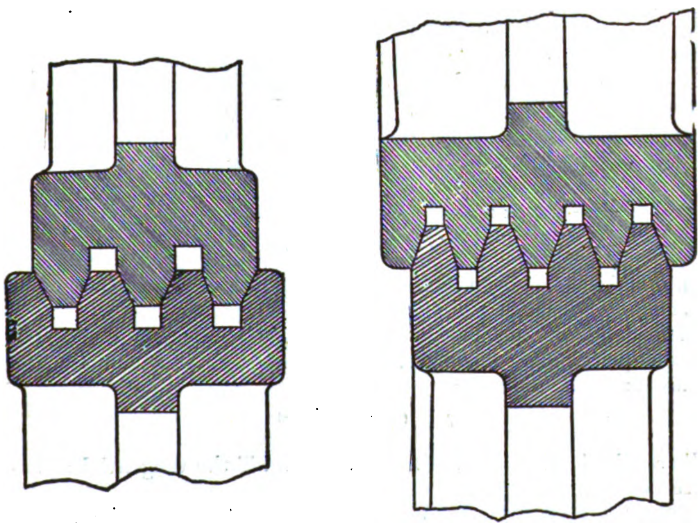
$$\text{Tangenital pressure} = T = \mu R = \cdot 2 \times 825 = 165 \text{ lbs.}$$

Also,
$$P_1 = R \frac{d_1}{\sqrt{d_1^2 + d_2^2}},$$

$\therefore$
$$P_1 = \frac{825 \times 2}{\sqrt{2^2 + (1\frac{1}{2})^2}} = 660 \text{ lbs.}$$

Similarly,
$$P_2 = \frac{825 \times 1\frac{1}{2}}{\sqrt{2^2 + (1\frac{1}{2})^2}} = 495 \text{ lbs.}$$

Robertson's Friction or Wedge Gearing.—One objection to the friction gearing just described is the great pressure which is brought on the bearings, due to the force with which the wheels



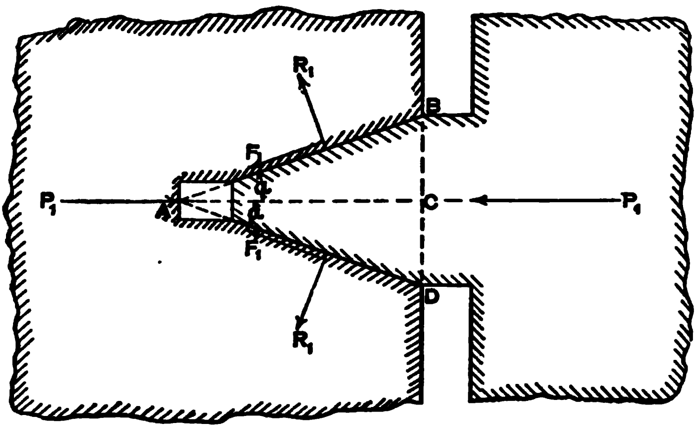
ROBERTSON'S WEDGE GEARING.

require to be pressed together in order to secure sufficient frictional resistance at their surfaces. To overcome this and the previous objections to the ordinary friction gearing, the wheels are now

made of cast iron with parallel wedge-shaped projections round their rims. The projections of the one wheel fit into the wedge-shaped grooves on the other; and by this means the friction is greatly increased.

Sections of the rims of such wheels are shown by the accompanying figures, and need no further explanation.

There is, however, one serious objection to such wheels—viz., the grinding action, and consequent excessive wear, while working. This is due to the sliding contact between the sides of the projections and those of the grooves. On this account the wheels sometimes work with great noise. This difficulty can be overcome



WEDGE GEARING.

to a certain extent by making the depth of the surfaces in contact as small as possible.

The depth of the acting surface (*i.e.*, the distance which the wheels penetrate each other when in gear) is given by the formula :—

$$t = 0{\cdot}025\sqrt{\mathrm{T}}.*$$

Where t is the depth of acting surface, and T the tangential resistance between the wheels.

Power Transmitted by Wedge Gearing.—Consider the action of one of the wedge-shaped projections in its groove. When the wheels rotate, the action is similar to that of a wedge thrust into the groove by a horizontal force.

* Unwin's *Machine Design*, part I., p. 286.

Let P_1 = Horizontal force on projection considered.
„ R_1 = Total normal reaction on each side of groove.
„ F_1 = Total friction between projection and each side of groove.
„ 2α = Angle of wedge.

Then, $P_1 = 2R_1 \sin\alpha + 2F_1 \cos\alpha$.

But, $F_1 = \mu R_1$.

$\therefore$ $P_1 = 2R_1(\sin\alpha + \mu\cos\alpha)$.

Then, for all the wedges, if P denote the total force pushing the wheels together, and R the total reaction, $P = \Sigma P_1$, $R = \Sigma . 2R_1$, and we get:—

$$P = R(\sin\alpha + \mu\cos\alpha).$$

$\therefore$ as before, $T \overline{\overline{<}} \mu R$.

Or, $R \overline{\overline{>}} \dfrac{T}{\mu}$.

But, $T = \dfrac{\text{H.P.} \times 33{,}000}{V}$.

$\therefore$ $R \overline{\overline{>}} \dfrac{\text{H.P.} \times 33{,}000}{\mu V}$.

$$\therefore \quad P \overline{\overline{>}} \frac{\text{H.P.} \times 33{,}000}{V} \cdot \frac{\sin\alpha + \mu\cos\alpha}{\mu} \quad . \quad (\text{V})$$

Equation (V) determines the least pressure with which the wheels must be forced together in order to transmit a given power.

From this equation it is seen that the number of grooves or projections has no effect on the power transmitted. The number of grooves may be anything we please, but generally there are no fewer than two nor more than ten. The pitch of the grooves may vary from $\frac{1}{8}$ inch to $1\frac{3}{4}$ inches.

Usually the groove angle 2α is 40°. Hence, taking $\mu = \cdot 15$, we get:—

$$P \overline{\overline{>}} T\frac{\sin\alpha + \mu\cos\alpha}{\mu},$$

$$,, \overline{\overline{>}} T\frac{\sin 20^\circ + \cdot 15\cos 20^\circ}{\cdot 15},$$

$$,, \overline{\overline{>}} T\frac{\cdot 342 + \cdot 15 \times \cdot 94}{\cdot 15},$$

$$,, \overline{\overline{>}} 3\cdot 22\,T.$$

i.e., P must be at least $3\frac{1}{4}$ times T. In practice we may take $P = 4T$.

Lecture XII.—Questions.

1. Define friction gearing. State the advantages and disadvantages of such a gearing, and mention under what circumstances it is likely to be employed in preference to other kinds of gearing.

2. Describe, with sketches, the construction of the wheels used for friction gearing. State your reasons why, in ordinary friction gearing, one of the wheels only is faced with a softer material than the other, and say which wheel it is.

3. In a spur friction gearing, the driving wheel is faced with wood and gears with a metal wheel of $2\frac{1}{2}$ feet in diameter. The circumferential speed is 2,000 feet per minute. The force pressing the wheels together is 550 lbs. Taking $\mu = \frac{1}{3}$, find the maximum H.P. which can be transmitted. Sketch the arrangement, showing a method of engaging and disengaging the wheels. *Ans.* 11·1 H.P.

4. The diameters of a pair of bevel friction wheels are $1\frac{1}{2}$ feet and 4 feet respectively. The larger wheel makes 200 revolutions per minute, and transmits 10 H.P. Find the normal and tangential pressures at the pitch surfaces, and the axial thrusts on each shaft, $\mu = \cdot 25$. Prove the formula which you employ. *Ans.* 525 lbs.; 131·25 lbs.; 184 lbs.; 491 lbs.

5. Describe, with sketches, Robertson's wedge gearing, and deduce a formula for the pressure between the wheels for a given H.P.

LECTURE XIII.

CONTENTS.—Constancy of the Velocity-Ratio of Toothed Gearing—Proportions of Teeth of Wheels—Clearance—Arc of Action—Relation between Length of Arc of Action and Pitch of Teeth—Clock and Watch Wheels —Primary Conditions for Correct Working of Toothed Wheels—Curves which satisfy the above Conditions—Particular Cases—(I.) When the Cycloid is a Straight Line—(II.) When the Epicycloid is the Involute of the Base Circle—(III.) When the Hypocycloid is a Straight Line—(IV.) When the Hypocycloid is a Point—Cycloidal Teeth—Gee's Patent Toothed Gearing—Exact Method of Drawing the Curves for Cycloidal Teeth—Practical Method of Drawing the Curves for Cycloidal Teeth—Application of Preceding Principles to the Case of a Rack and its Pinion—Particular Forms of Teeth as Dependent upon Changes in the Sizes of the Generating Circles Employed—First Particular Case—When the Hypocycloid is a Straight Line—Rack having Teeth with Radial Flanks—Practical Method of Drawing the Involute Curves for the Faces of the Teeth on the Pinion—Second Particular Case—When the Hypocycloid is a Point—Pin Wheels—Pins are always placed on the Follower—Rack and Pinion—Disadvantage of Pin Wheels—Questions.

Constancy of the Velocity-Ratio of Toothed Gearing.*—In nearly every case of the transmission of motion by friction or belt gearing, slipping takes place to a greater or less extent, and hence these methods of transmitting motion are unsuited where an exact or constant velocity-ratio is desired. In such a case it is best to employ toothed gearing. But, to insure a constant velocity-ratio and smooth working with toothed gearing, the teeth of the wheels must be carefully constructed, and of such shapes that, when gearing together, certain geometrical conditions are fulfilled. In this Lecture we shall endeavour to explain the principles according to which all properly constructed teeth of wheels are made and act.

In the first place, we shall give some further definitions and general explanations relating to toothed gearing.

DEFINITION.—**The pitch of the teeth is the distance from the centre of one tooth to the centre of the next tooth, measured along the pitch line.**

* The student may refer to the following books on gearing:—

Practical Treatise on Gearing, by Browne & Sharpe, printed by J. W. Pratt & Son, New York.

Odontics, by Geo. B. Grant, published by the Lexington Gear Works, Lexington, Mass.

Handbook on the Teeth of Gears, by Geo. B. Grant, Boston, Mass.

Bevel-Gears, by John W. Newall & Co , Manchester.

A paper on "Setting out the Curves of Wheel Teeth," by W. J. Last, in *Proc. Inst. Civil Engineers*, vol. lxxxix., p. 335.

Elements of Machine Design, by Prof. Unwin, published by Longmans, Green & Co.

Let d = Diameter of the pitch circle *in inches.*
„ p = Pitch of teeth *in inches.*
„ n = Number of teeth on wheel.

$$\left.\begin{aligned} &\text{Then,} & np &= \pi d \\ &\text{Or,} & p &= \frac{\pi d}{n} \\ &\text{And,} & d &= \frac{np}{\pi} \end{aligned}\right\} \quad \ldots \ldots \quad (\mathrm{I})$$

The pitch, as measured in this way, is called the **Circular Pitch**, and is the one chiefly used by engineers. But another and more convenient method of measuring the pitch is sometimes adopted, and is called the **Diametral Pitch**. According to this method the pitch of the teeth is stated as a fraction of the diameter of the pitch circle.

Thus, if the diameter of the pitch circle be 40 inches, and the number of teeth on the circumference be 120, then :—

$$\textit{Diametral Pitch} = \frac{40}{120} = \tfrac{1}{3} \text{ inch.}$$

In this case, the size of the wheel would be spoken of as one of 3 teeth per inch of pitch circle diameter.

Let p_d = Diametral pitch of teeth *in inches.*

Then, using the same notation as in the case of circular pitch:—

$$\left.\begin{aligned} & & p_d &= \frac{d}{n} \\ &\text{Or,} & d &= n\,p_d \end{aligned}\right\} \quad \ldots \ldots \quad (\mathrm{II})$$

The student will notice that equations (II) are much simpler than equations (I), from the fact, that it is easier to treat of the sub-division of a straight line than that of a circle. It is for this reason, that several engineers, especially American, advocate the adoption of this method as being more convenient for stating the sizes of wheels. In this country the circumferential pitch is the one chiefly used, except in the case of small wheels (such as the change wheels for a lathe) whose sizes are often stated in terms of their number of teeth per inch in diameter.

The relation between the circular and diametral pitches can be stated thus :—

From (I), $\qquad d = \dfrac{np}{\pi},$

„ (II), $\qquad d = n\,p_d,$

∴ $\qquad p_d = \dfrac{p}{\pi}.$

Or, $\qquad p_d : p = 1 : \pi. \quad \ldots \ldots \quad (\mathrm{III})$

In this book the term *pitch* must always be understood as meaning the *circular pitch*, unless otherwise expressly stated.

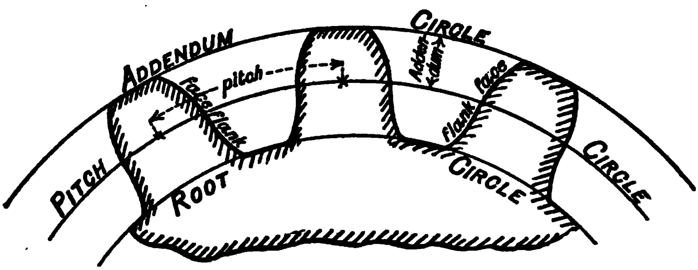
DEFINITIONS.—The Face of a tooth is that part of its acting surface which lies between the pitch surface and the crest of the tooth.

The Flank of a tooth is that part of the acting surface which lies between the pitch surface and the bottom of the spaces between the teeth.

The Addendum Circle is that imaginary circle which touches the crests of all the teeth on the wheel.

The Root Circle is that imaginary circle which touches the bottoms of the spaces between the teeth.

The Addendum of a tooth is the length of the tooth projecting beyond the pitch surface; or, it is that part of the tooth lying between the pitch surface and the addendum surface.



ILLUSTRATING THE DEFINITIONS OF TERMS.

The above terms will be understood from the accompanying figure.

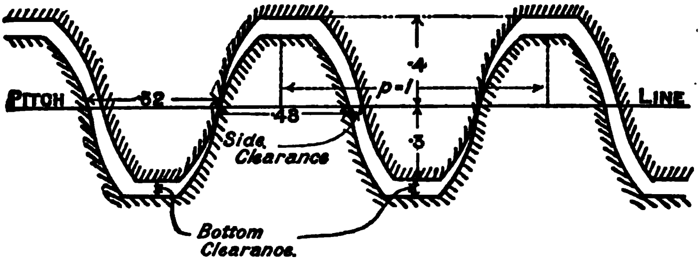
Proportions of Teeth of Wheels.—The proportions for the teeth of wheels vary slightly with different makers, but the following rules, as given by Prof. Unwin, represent good average practice :—

Let p = *Pitch of teeth in inches.*

Then,	*Height of tooth above pitch line,* .	$= \cdot 3\, p$,
	Depth of tooth below pitch line, .	$= \cdot 4\, p$,
	Thickness of tooth (measured along pitch line), ...	$= \cdot 48\, p$,
	Width of space between the teeth, .	$= \cdot 52\, p$,
	Width of face of tooth, ...	$=$ 2 p to 3 p.

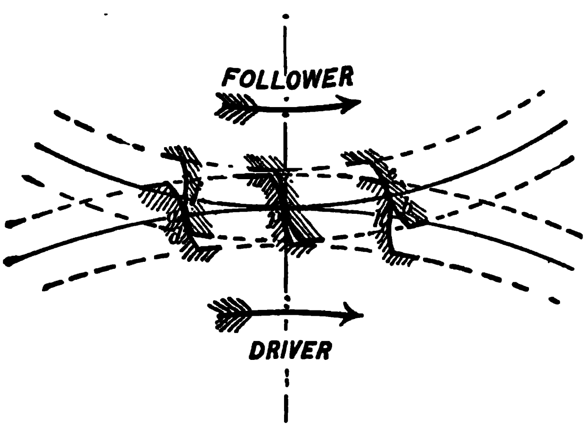
Clearance.—These proportions, as shown in the following figure, give a *side clearance* of $(\cdot 52 - \cdot 48)\, p = \cdot 04\, p$, and a *bottom clearance* of $(\cdot 4 - \cdot 3)\, p = \cdot 1\, p$.

Arc of Action.—Consider the action of a pair of teeth gearing together, from the instant at which contact begins to the instant at which contact terminates. During the first part of the action the *flank* of the tooth on the *driver* is in contact with the *face*



PROPORTIONS OF TEETH OF WHEELS.

of the tooth on the *follower*. This continues until the pitch point is reached, at which instant the line of contact of the teeth coincides with the line of contact of the pitch surfaces of the wheels. After passing the pitch point, *p*, the *face* of the tooth on the *driver* continues in contact with the *flank* of the tooth on the *follower*, until contact ceases. Action *begins* at *a*, the point of the tooth on the *follower*, and *terminates* at *b*, the point of the tooth on the *driver*.



ILLUSTRATING THE TERMS ARCS OF APPROACH AND RECESS.

Arc of Approach = Arc $a\,p$, or Arc $f\,p$.
Arc of Recess = Arc $p\,h$, or Arc $p\,b$.

DEFINITION.—**The arc of either of the pitch circles over which there is contact between a pair of teeth is called the Arc of Action.**

Thus, either arc *a p h*, or arc *f p b*, is called the arc of action. The arc of action is divided at the pitch point into two parts, called respectively the **Arc of Approach** and the **Arc of Recess.** By an inspection of the above figure it will be seen that the length of the arc of approach depends on the *addendum* of the teeth on the *follower;* whilst the length of the arc of recess depends on the *addendum* of the teeth on the *driver*. To increase or diminish the arc of contact, the *addendum* of the teeth must be increased or diminished, as will be shown further on.

Relation between Length of Arc of Action and Pitch of Teeth.—To insure continuous action between a pair of toothed wheels, there must, at any instant, be at least one pair of teeth in gear. Moreover, contact between one pair of teeth must not terminate before the succeeding pair comes into operation. This condition is insured by making the arc of action greater than the pitch of the teeth. In most cases, especially with heavy gearing, two or three teeth are in gear at once. *Hence, the usual rule is to make the arc of action from three to four times the pitch of the teeth.*

Clock and Watch Wheels.—In wheelwork, such as in clocks or watches, where friction is most injurious, the teeth of the wheels are usually so designed that the driving teeth have no flanks, and the driven teeth no faces. Contact, in such cases, occurs during the period of recess only, and then the arc of recess must be at least equal to the pitch. The reason for this is, that the friction, due to the sliding of the teeth on each other during action, is said to be greater during the period of approach than that during the period of recess.*

Primary Conditions for Correct Working of Toothed Wheels.—Having explained some general principles relating to toothed gearing, we shall now proceed to consider the necessary conditions to be fulfilled in order that such gearing may work correctly.

The two necessary conditions are :—

I. The radii of the pitch surfaces must be such that by rolling together they give the desired velocity-ratio.

Let R_A, R_B = Radii of pitch circles of wheels A and B.
,, ω_A, ω_B = Angular velocities ,, ,,

Then, as a first condition we must have :—

$$R_A : R_B = \omega_B : \omega_A.$$

* Friction between the teeth of wheels will be considered in Lecture XVI.

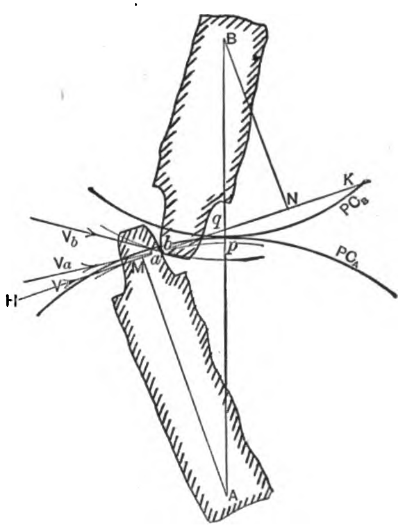
Or, the radii of the pitch circles must be inversely as the angular velocities of the wheels.

II. The shape of the teeth of the wheels must be such, that the motion resulting from their mutual action shall be the same as that obtained by the rolling action of the pitch surfaces.

Let A and B be the centres of the pitch circles of two wheels working together. A B the *line of centres*, and p the pitch point.

For clearness we have represented only one tooth on each wheel. The teeth are in contact at the point $a\,b$; a being the point of contact on tooth A, and b the point of contact on tooth B.

Let H K, the common normal to the curves of the teeth at their point of contact, intersect the line of centres at q. From A and B draw the perpendiculars, A M and B N, upon H K.



ILLUSTRATING PRIMARY CONDITIONS FOR CORRECT WORKING OF WHEEL TEETH.

With centres A and B draw the circles passing through the point $a\,b$. Then, at any instant the point a is moving along the tangent to the circle passing through a, and having its centre at A; *i.e.*, the point a is moving in a direction at right angles to A a with a velocity $v_a = \omega_A \times \text{A}\,a$. Similarly, the point b is moving in a direction at right angles to B b with a velocity $v_b = \omega_B \times \text{B}\,b$. But though the points a and b are thus moving in different directions and with different velocities, yet their *component velocities along the common normal*, H K, must be equal, otherwise the teeth would either separate from or penetrate each other. For an indefinitely small movement of the wheels the only relative motion of a and b is in a direction perpendicular to H K.

Let v denote the equal component velocities of v_a and v_b along H K. Then, with reference to wheel A :—

$$v = \omega_A \times AM$$

Similarly, $$v = \omega_B \times BN$$

$\therefore$ $$\omega_A \times AM = \omega_B \times BN,$$

$\therefore$ $$\omega_A : \omega_B = BN : AM,$$

i.e., $$R_B : R_A = BN : AM,$$

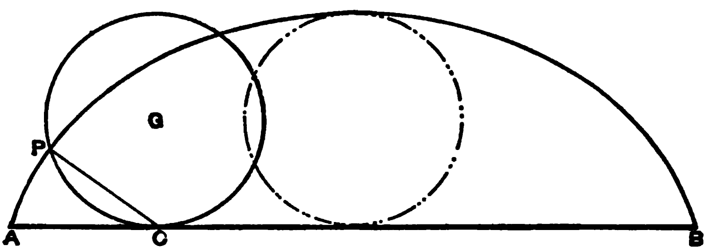
Or, $$Bp : Ap = Bq : Aq \quad \text{[By similar triangles]}$$

This can only be true when q coincides with p. Hence, the condition to be fulfilled by the curves forming the teeth of wheels is :—

The common normal to the curves at the point of contact of a pair of teeth must always pass through the pitch point.

Curves which Satisfy the above Condition.—It now remains to describe curves which shall fulfil this condition. The problem of determining the proper shape of teeth admits of many solutions. Any shape can be given to the teeth of one of a pair of wheels gearing together, so long as a corresponding shape be given to the teeth of the other wheel, to fulfil the above condition.

Two principal curves have been used by engineers for describing the teeth of wheels. These are the **cycloid** and the **involute.*** We shall now explain the nature of these curves, and then show that



THE CYCLOID AND HOW IT IS DESCRIBED.

teeth formed according to them satisfy the above condition for correct working.

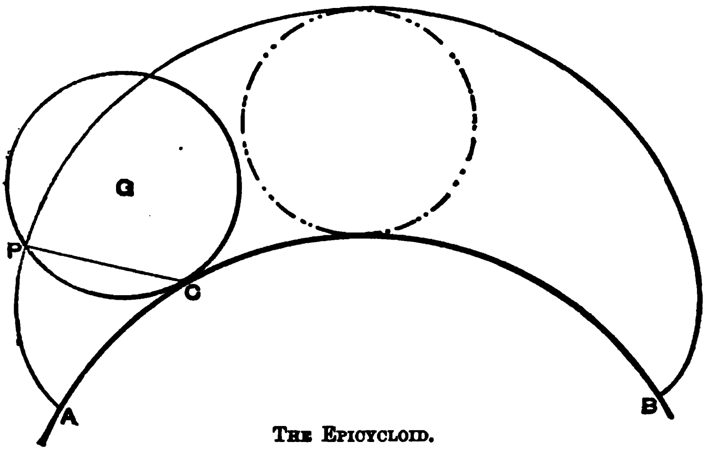
DEFINITION.—**A cycloid is a curve traced out by a point on the circumference of a circle which rolls along a straight line.**

The form of this curve will be understood from the accompanying figure. The rolling circle, G, is called the *generating circle*. The point, P, which traces out the curve is called the *tracing point*. The line, A B, on which the circle rolls is called the *base line*.

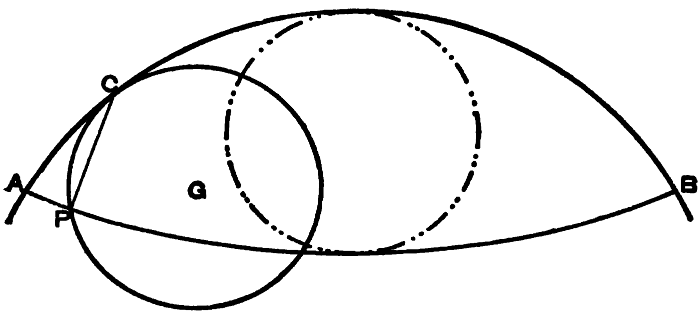
* *Cycloid* is derived from the Greek word κύκλος, signifying a *ring* or a *circle; Epicycloid* from ἐπί, signifying *upon* and κύκλος; and *Hypocycloid* from ὑπό, signifying *under* and κύκλος. *Involute* is derived from the Latin words *in*, signifying *upon*, and *volvo*, to *roll.*

The length of A B is equal to the circumference of the generating circle. For any position, G, of the generating circle, we have A C = arc P C of the circle.

When the base line is a *circle* the curve traced out by the tracing point is called an **Epicycloid** or a **Hypocycloid** according as



THE EPICYCLOID.



THE HYPOCYCLOID.

A C B is the Base circle, P the Tracing point, G the Generating circle.

the generating circle rolls on the *convex* or *concave* side of the base circle. We may define these curves separately, as follows :—

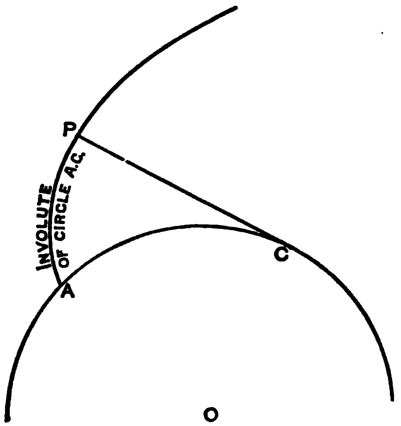
DEFINITION.—**An Epicycloid is a curve traced out by a point on the circumference of a circle which rolls on the Convex arc of another circle.**

Definition.—A Hypocycloid is a curve traced out by a point on the circumference of a circle which rolls on the Concave arc of another circle.

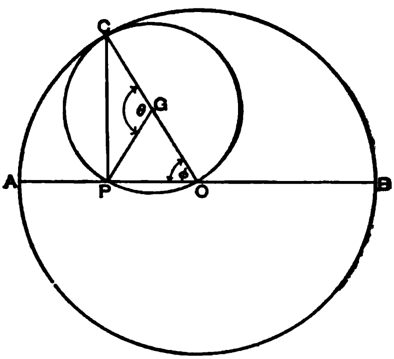
The forms of these curves will be understood from the accompanying figures.

Particular Cases—I. When the Cycloid is a Straight Line.—Suppose the radius of the generating circle to become infinitely great; then, clearly, the *cycloid* would become a *straight line perpendicular to the base line*, A B. An application of this is to be found in the case of racks having teeth with straight flanks.

II. When the Epicycloid is the Involute of the Base Circle.—If the radius of the generating circle of the *epicycloid* be infinitely great, then the arc, P C, becomes a straight line, and the epicycloid,



Involute Curve.
A C the Base circle, P the Tracing point.



Hypocycloid a Straight Line.

A P, is now termed the **Involute of the Base Circle** or simply an **Involute.** An *involute of a circle* is the curve traced out by a point on the free end of a stretched string when the string is being unwound from the circle. The form of this curve will be understood from the left-hand figure.

III. When the Hypocycloid is a Straight Line.—If the diameter of the generating circle be equal to *half* the diameter of the circle inside which it rolls, then the *hypocycloid* traced out by the tracing point will be a straight line, and this straight line will be a diameter of the base circle. This case is shown by the figure on the right.

Let the tracing point be at A at the beginning of its motion, and at P for any position of the generating circle. Join O A, G P, and O C. O C passes through G, the centre of the generating circle.

Let $\angle C G P = \theta$, $\angle C O A = \varphi$.

Then, arc P C = arc A C, by definition.

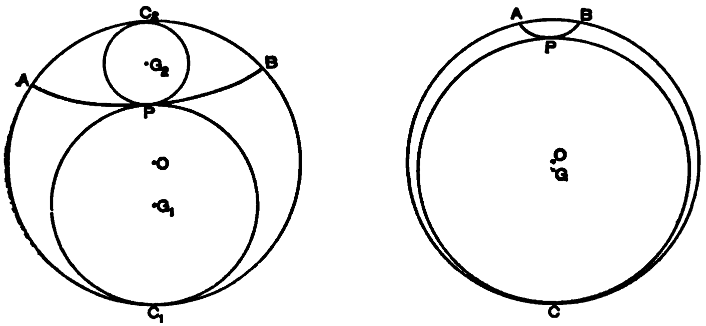
$\therefore$ $G C \times \theta = O C \times \varphi$.

But, $G C = \frac{1}{2} O C$, by hypothesis.

$\therefore$ $\theta = 2\varphi$.

Now θ is an angle at the centre of the circle G, and φ is an angle at the circumference of the same circle. But, since $\theta = 2\varphi$, it follows (converse of Euc. III., 20) that these two angles must stand on the same arc, P C, of the circle G. Therefore, P lies on the line O A. This being true for any position of G, we conclude that P moves along the straight line A B, which is a diameter of the base circle.

IV. **When the Hypocycloid is a Point.**—By a reference to the following left-hand figure, it appears that the same hypocycloid can be traced out by either of the generating circles, G_1 or G_2, when the diameters of these circles are such, that their sum is equal to the diameter of the circle inside which they roll.

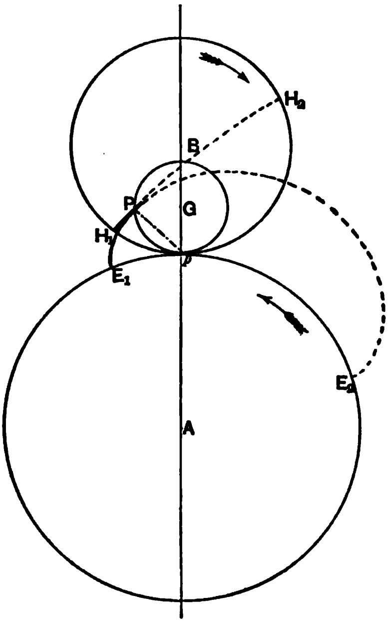


THE HYPOCYCLOID DEGENERATING TO A POINT.

Now, if the circle G_1 goes on increasing in size, the hypocycloid, A P B, becomes more and more convex towards the centre, O, until, ultimately, when G_1 becomes nearly equal in size to the base circle, the hypocycloid is a small half-loop, as shown by the right-hand figure. The same thing takes place as the circle G_2 decreases in size. Hence, when the generating circle is equal in size to the base circle, the hypocycloid degenerates into a point.

The curves of the cycloid class have many important geometrical properties, some of which are familiar to students of higher Dynamics, but the only property with which we are concerned here is that one relating to the *normal* at any point of the curve.

Referring to the above figures, let C be the point of contact of the generating circle and base line; P the tracing point. Then, at any particular instant during the rolling of the circle G, the tracing point, P, will be moving, as it were, in a circle whose radius is C P, and having C as its centre. In other words, C is the *instantaneous centre* of motion. C P is, therefore, the radius of curvature of the curve at the point P. Hence, the *normal* to the curve at the point P is in the direction P C. It is this property of the cycloidal and involute curves which fit them so well for the teeth of wheels.



ILLUSTRATING FULFILMENT OF PRIMARY CONDITIONS BY CYCLOIDAL TEETH.

Cycloidal Teeth.—Let $E_1 p E_2$, $H_1 p H_2$ be the pitch circles of two wheels gearing together, p being the pitch point. Let G be the *fixed* centre of a third circle touching the other two circles at the pitch point, p. Let P be a tracing point on the circumference of this circle.

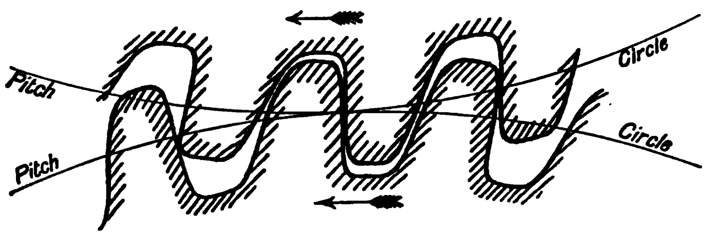
At the beginning of motion, let P, E_1, and H_1 all coincide at the point, p. Let the three circles now roll in contact with each other in the directions indicated. The point, P, will then describe simultaneously the epicycloid, E_1 P E_2, outside the pitch circle, A, and the hypocycloid, H_1 P H_2, inside the pitch circle, B. Since both curves are traced out by the *same* generating circle and tracing point, it follows, from what has already been said, that the common normal at their point of contact, P, always passes through the pitch point, p. But this is the very condition which we have

been seeking to fulfil, and we now see that teeth of the cycloidal class satisfy this condition for correct working. The part, $E_1 P$, of the epicycloid may represent the curve for the *face* of a tooth on the wheel, A, and the part, $H_1 P$, of the hypocycloid the curve for the *flank* of a tooth on the wheel, B. Hence, if the *faces* of the teeth on the one wheel, and the *flanks* of the teeth on the other be described by the *same generating circle*, the two wheels will work correctly together.

The student should observe that the action between a pair of teeth, however perfectly formed, is not wholly due to the *rolling* of one tooth on the other. An inspection of the previous figure will make this quite clear. Thus, at the beginning of the motion described, when P coincides with p, E_1 coincides with H_1. When the motion is such that P is brought into the position shown on the figure, the length of epicycloid described is $E_1 P$, and that of the hypocycloid, $H_1 P$. These arcs are not equal in length, $E_1 P$ being greater than $H_1 P$. Therefore, the amount of *sliding* is $E_1 P - H_1 P$. Hence, the action between a pair of teeth in contact is partly sliding and partly rolling.

It is not necessary that the same generating circle be employed for describing both faces or both flanks of the teeth on the same wheel, but it is very advantageous to have the teeth so described, especially if the wheels require to run in either direction.

Gee's Toothed Gearing.—When the wheels require to run in one direction only, the posterior or unacting surfaces of the teeth may be given any shape whatever. A form of toothed

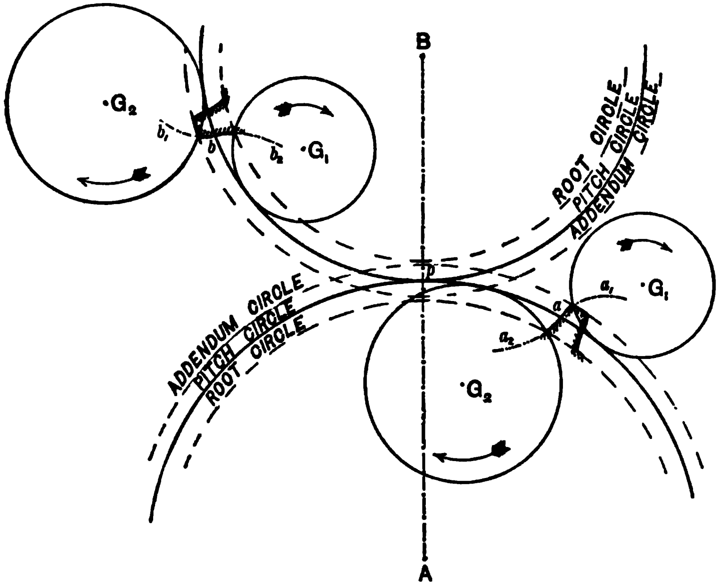


GEE'S TOOTHED GEARING.

gearing, known as Gee's patent gearing, has lately been introduced, and is said to be 35 per cent. stronger than the ordinary form. The driving surfaces of the teeth are of the usual form, but the other surfaces are more inclined, as shown by the accompanying figure. This causes the roots of the teeth to be much thicker than with ordinary teeth, and hence the increase of strength.

Exact Method of Drawing the Curves for Cycloidal Teeth.—The method of describing the curves for the teeth of wheels will

now be easily understood. Let G_1 represent the generating circle used for describing the *faces* of the teeth on wheel A and the *flanks* of the teeth on B; G_2 the generating circle used for describing the faces of the teeth on B and the flanks of the teeth on A. Draw the addendum and root circles and divide the pitch circles into as many equal arcs as there will be teeth on the wheels. On each side of these points of division set off equal distances to represent *half* the thickness of a tooth as measured along the pitch circle.



ILLUSTRATING METHOD OF SETTING OUT CURVES FOR TEETH OF WHEELS.

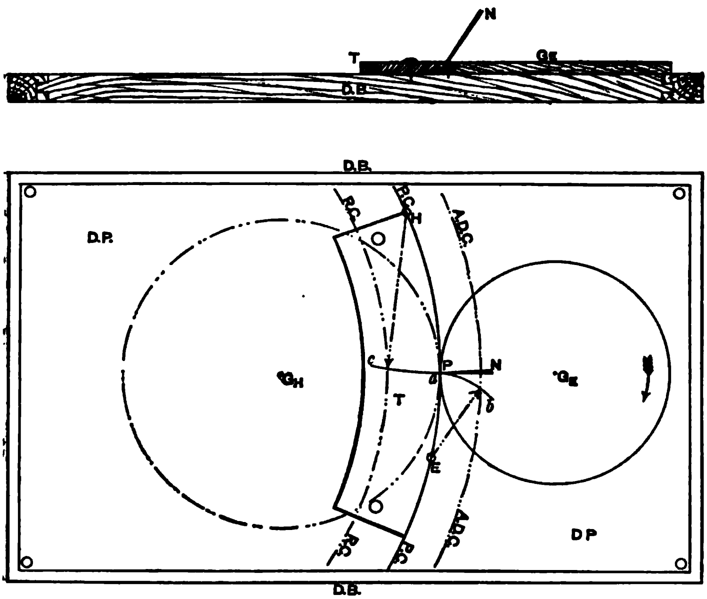
Let a, be a point on pitch circle of wheel A, and b, a point on pitch circle B, from which the curves for a tooth on each wheel have to be set out.

By placing the generating circles, G_1, G_2, in contact with the pitch circles at these points, and then tracing out the parts of the epicycloids and hypocycloids between the pitch circles and addendum and root circles as shown, the curves for a tooth on each wheel may be thus described. This process may be repeated for all the teeth on both wheels, and we thus obtain a complete representation of a pair of spur wheels having cycloidal teeth.

Practical Method of Drawing the Curves for Cycloidal Teeth.—The above method of setting out the curves for the teeth of

wheels, although mathematically exact and apparently quite simple, is found to be rather tedious in practice, and consequently in working drawings we always find the true curves represented approximately by arcs of circles. The following method of obtaining curves for the teeth of wheels is very often used in practice :—

Make a wooden template, T, having a thickness of about $\frac{1}{8}$ inch, and of such a shape that its outer and inner edges are each arcs of a circle having a radius equal to that of the pitch circle of the wheel upon which the teeth have to be described. Make also template segments of the generating circles, G_E, G_H, and pass a



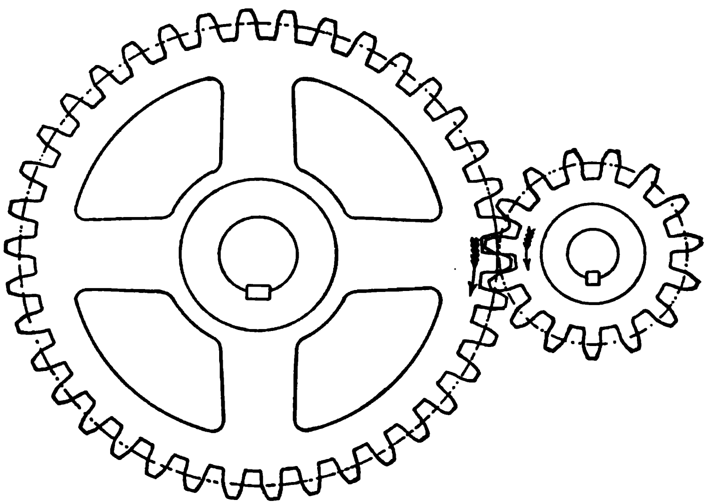
Practical Method of Setting out Curves for Teeth of Wheels.

Index to Parts.

D B	represents	Drawing board.	N	represents	Needle.
D P	,,	Drawing paper.	P	,,	Tracing point of needle.
T	,,	Template.	E	,,	Centre for circular arc approximately coinciding with epicycloidal arc, *a b*.
P C	,,	Pitch circle.			
A D C	,,	Addendum circle.			
R C	,,	Root circle.			
G_E	,,	Generating circle for faces of teeth.	H	,,	Centre for circular arc approximately coinciding with hypocycloidal arc, *a c*.
G_H	,,	Generating circle for flanks of teeth.			

small pencil or needle, N P, through each, so that the point, P, coincides with the outer edge, as shown in the elevation.

Fix a sheet of drawing paper on a drawing board, D B. On this paper draw an arc of a circle having a radius equal to that of the pitch circle of the wheel. By means of nails attach the pitch circle template, T, to the drawing board in such a position that its *convex* arc coincides with the arc of the pitch circle drawn on the paper. Now take the generating circle, G_E, and bring it in contact with the convex edge of the pitch circle template, so that the point, P, coincides with the point, *a*. Roll G_E along the template, T, in the direction indicated, when the point, P, will describe an arc of an epicycloid, *a b*. The arc of the epicycloid intercepted between the pitch circle and the addendum circle represents the *face* of a tooth. Now shift the pitch circle template, T, so that its *concave* edge coincides with the pitch circle drawn on the paper. By placing the generating circle, G_H, in contact

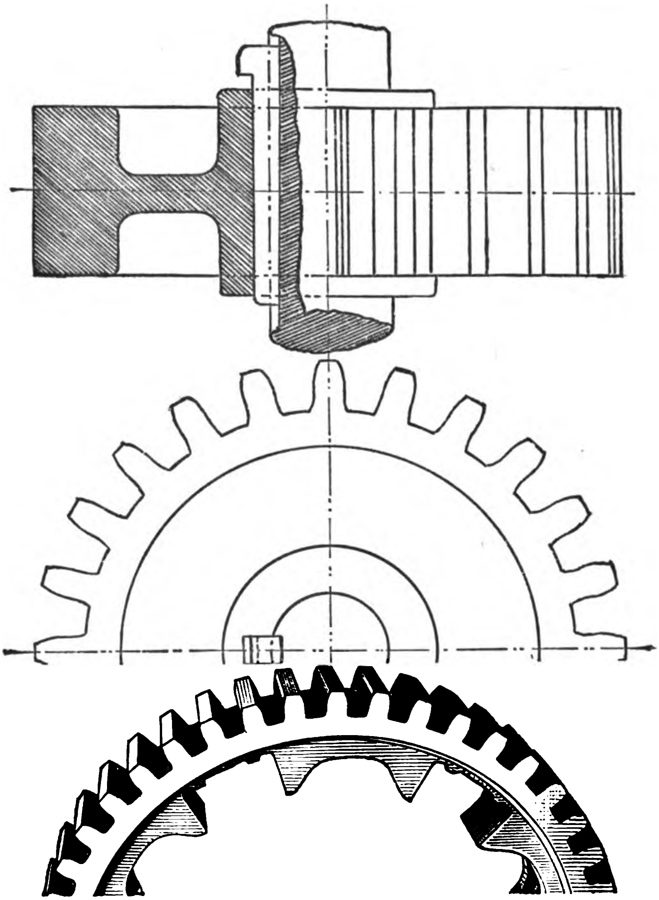


Spur Wheel and Pinion with Cycloidal Teeth.

with the concave edge of T, and having the tracing point or pencil, P, coinciding with *a*, the hypocycloid, *a c*, can then be traced in the same manner. The arc of the hypocycloid intercepted between the pitch circle and the root circle will represent the *flank* of a tooth on the wheel.

Having obtained these curves, it remains to find, by trial, the radii and centres, E, H, of arcs of circles which approximately

coincide with the epicycloidal and hypocycloidal arcs respectively. These being found, approximate curves can readily be drawn to represent the faces and flanks of the teeth. This method is



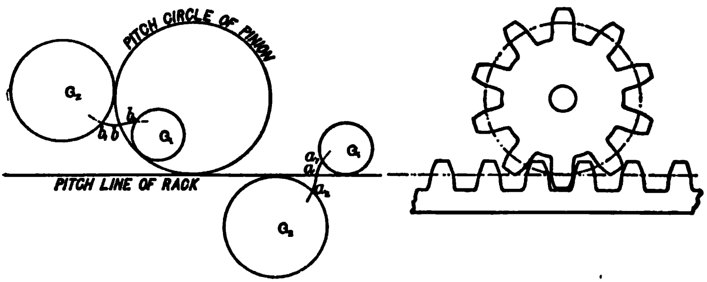
SPUR WHEELS WITH CYCLOIDAL TEETH.

often used by patternmakers when setting out the curves for the teeth of wheels. For ordinary methods of representing on drawings the curves for the teeth of wheels, the student must consult works

on Machine Drawing. The student must be reminded, however, that true cycloidal curves can never be accurately represented by arcs of circles, however many of these may be employed in completing the drawing.

The previous figures represent spur wheels with cycloidal teeth.

Application of Preceding Principles to the Case of a Rack and its Pinion.—For our present purpose, we may consider a rack as being simply a toothed wheel having a pitch circle of infinite radius; and it therefore follows, that the preceding principles are applicable to it.



RACK AND PINION WITH CYCLOIDAL TEETH.

The figure on the right-hand side shows a rack and its pinion, the pitch lines being shown by dotted lines. The figure on the left-hand side shows the application of the preceding principles in obtaining the curves for the teeth on both rack and pinion. The generating circle, G_1, is represented describing the face of a tooth on the rack and the flank of a tooth on the pinion, while the circle, G_2, is employed in describing the flank of a tooth on the rack and the face of a tooth on the pinion. The curves forming the faces and flanks of the teeth on the rack will thus be arcs of cycloids.

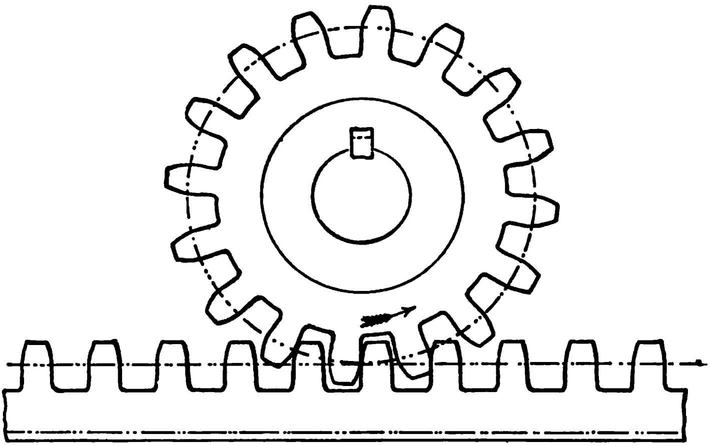
Particular Forms of Teeth as Dependent upon Changes in the Sizes of the Generating Circles Employed.—In what has preceded, we have been chiefly concerned with a discussion of the general character and shape of teeth of the cycloidal class, and we now go on to consider a few particular cases as dependent upon the sizes of the generating circles employed.

First Particular Case—When the Hypocycloid is a Straight Line.—We have already shown, that when the diameter of the generating circle is half that of the circle inside which it rolls, the hypocycloid traced out by the tracing point is a diameter of the base or pitch circle. This being the case it is easy to see, that if

the flanks of the teeth of a pair of wheels be described by generating circles, whose diameters are half that of the corresponding pitch circles, such flanks will be straight or *radial.*

The method of setting out such teeth may be briefly stated thus:—

Let A and B be the centres of the pitch circles. Take generating circles, G_A, G_B, *having diameters respectively equal to half those of*



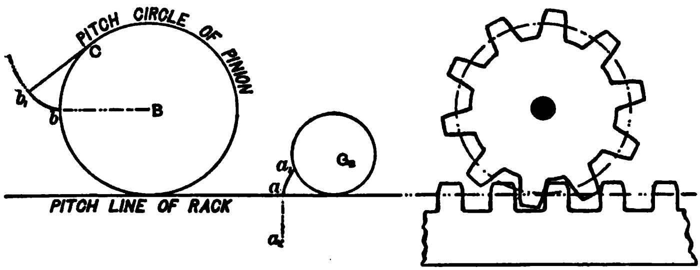
RACK AND PINION.

the pitch circles A *and* B By rolling G_A on the convex side of pitch circle, B, an epicycloid, $b_1 b_2$, will be traced out. This curve determines the form of the faces of the teeth on B. Similarly, by rolling G_B on the convex side of pitch circle, A, the epicycloid, $a_1 a_2$, will be obtained, which will determine the form of the faces of the teeth on A.

The hypocycloids corresponding to these generating circles are straight lines or radii of the pitch circles. Hence, to complete the curves for the acting surfaces of the teeth, it is only necessary to draw the radii from the points a_1 . . . . ., b_1 . . . . ., &c.

Teeth with radial flanks are thinner at the roots than at the pitch circle, and if the wheel is small and has few teeth, it is not difficult to see that such teeth may exhibit comparative weakness at the roots, the very place where they should be strongest. With ordinary sized wheels, this need not present any serious obstacle, since the thickness at the roots may be increased by simply putting in fillets between the straight flanks and the root circle.

Rack having Teeth with Radial Flanks.—In carrying out the above idea for the case of a rack and pinion, we notice that the generating circle to be used in describing the faces of the teeth on the pinion must have a diameter *equal to that of the radius of the pitch "circle" of the rack.* But since this latter "circle" has an infinite diameter, it follows that the diameter of the generating circle just referred to must also be infinite. Now, we have already shown that the epicycloid traced out by a generating circle of infinite diameter is an *involute of the base or pitch circle,* outside which this generating circle is supposed to roll. Hence, *the faces of the teeth on the pinion must be involutes of its own pitch circle.* The faces of the teeth on the rack are *cycloids* described by a generating circle, having a diameter equal to the radius of the pinion.



PINION HAVING TEETH WITH INVOLUTE FACES AND RADIAL FLANKS.
RACK HAVING TEETH WITH CYCLOIDAL FACES AND STRAIGHT FLANKS.

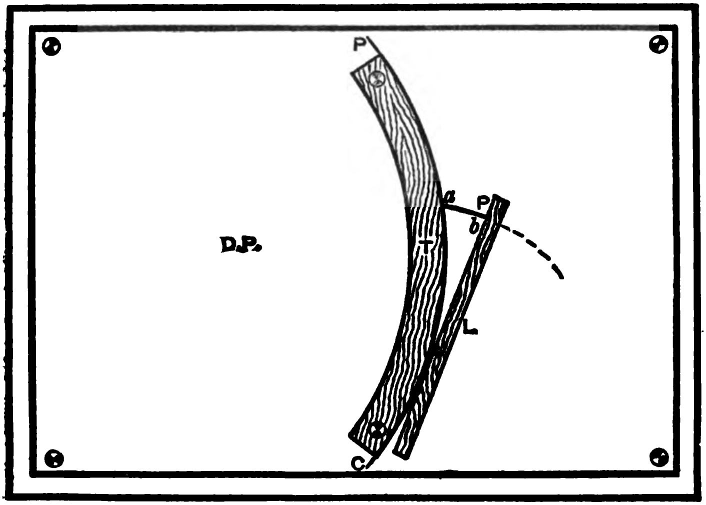
The principle of construction for this case will be understood from the left-hand figure above. The complete teeth are represented by the right-hand figure.

It must be carefully borne in mind, that the form of the acting surfaces of the teeth on the one wheel, determines the necessary form of the acting surfaces of the teeth on the other wheel in gear with it. In the case of cycloidal teeth, the only necessary condition to be observed in their construction, in order to insure correct working is, that the same generating circle be used in describing the faces of the teeth on the one wheel as that used in describing the flanks of the teeth in the other. This condition is clearly fulfilled in the two particular cases just considered.

Practical Method of Drawing Involute Curves for the Faces of Teeth on a Pinion.—The following is a simple practical method of drawing the involute curves for the faces of the teeth on the pinion, gearing with a rack having teeth with radial flanks.

D B is a drawing board, having a sheet of drawing paper, D P, fixed to it. Draw on the paper, full size, an arc, P C, of the

pitch circle of the pinion. Having made a template, T, of wood with a convex edge, struck with a radius equal to that of the pitch circle of the pinion, fix it to the board with this edge coinciding with P C, as shown by the figure. Next take a lath, L, of wood, having one of its edges perfectly straight, and carrying a small



PRACTICAL METHOD OF DRAWING INVOLUTE OF PITCH CIRCLE.

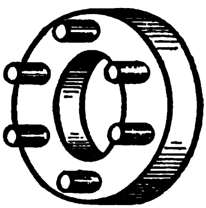
INDEX TO PARTS.

D B	represents	Drawing board.	T	represents	Wooden template.
D P	,,	Drawing paper.	L	,,	Wooden lath.
P C	,,	Pitch circle of wheel.	P	,,	Tracing point.

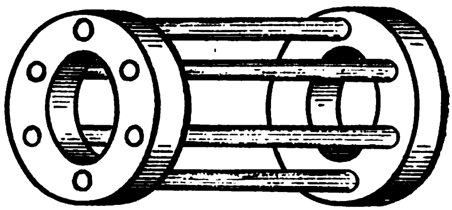
pencil or needle, P, projecting from the straight edge. Let the straight edge of L be placed against the convex edge of T, and let the point P coincide with the point *a*, from which the curve must start. Now allow the lath to *roll* on the edge of T, so that the straight edge of L will always form a tangent to the pitch circle, care being taken not to allow any slipping during the process. By this means, the point P will describe a curve which will be an involute of the pitch circle, P C. An arc of a circle can now be drawn, which will approximately coincide with the involute arc so found, and then the curves for the faces of the teeth may be set out.

Second Particular Case—When the Hypocycloid is a Point—Pin Wheels.—We have already shown that the hypocycloid degenerates to a point when the diameter of the generating circle is

equal to that of the base circle inside which it rolls. Hence, if the diameters of the generating circles be taken equal in size to the respective pitch circles of a pair of wheels intended to gear together, it is clear, that the teeth on both wheels will possess the peculiar property of having *no flanks*. In this particular case, the teeth on *one* of the wheels must, theoretically, be mere points. In practice the teeth must have some magnitude, and consequently we find pins instead of mere points. A wheel of this description would be called a *pin wheel*, and consists of a series of pins projecting from the face of a circular disc, as shown by the following figures. When the pins are fixed between two discs we then obtain what is called a *lantern wheel;* a form of wheel now rarely used, except in clock and watch mechanism.



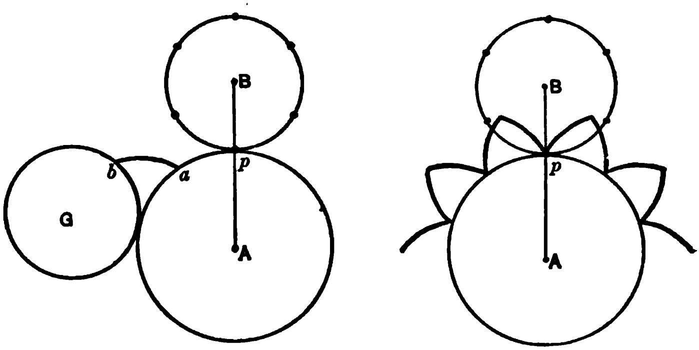
PIN WHEEL.



LANTERN WHEEL.

The problem now before us is, given a pin or lantern wheel, to describe the teeth on another wheel which shall work accurately with it.

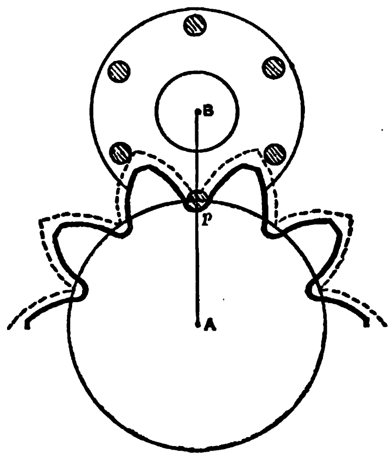
We shall first suppose the pins to have no diameter, in other words, to be mere points.



METHOD OF DRAWING TEETH ON A WHEEL TO GEAR WITH A PIN WHEEL.

Let A be the centre of the pitch circle upon which the required teeth have to be described, and B the centre of the pin wheel. The generating circle, G, used for describing the faces of the teeth on A, must be of the same size as the pitch circle, B. The shape of the complete teeth on A is shown by the figure on the right-hand side, from which it will be seen that the teeth have no flanks.

To complete the problem we must modify the above figure on the right-hand side to suit the actual case when pins of definite diameter are substituted for the points on the wheel B. Having fixed upon the diameter of the pins, draw circles to represent these round the pitch circle, B, as shown. At the points of intersection of the dotted epicycloids with the pitch circle, A, draw the small arcs inward (with a radius equal to that of the pins), to represent the recesses into which the pins enter when approaching the pitch point, *p*. Then draw curves from the ends of these small arcs parallel to the dotted epicycloids as indicated, the distance between the parallel curves being half the diameter of the pins.



TEETH TO GEAR WITH A PIN WHEEL.

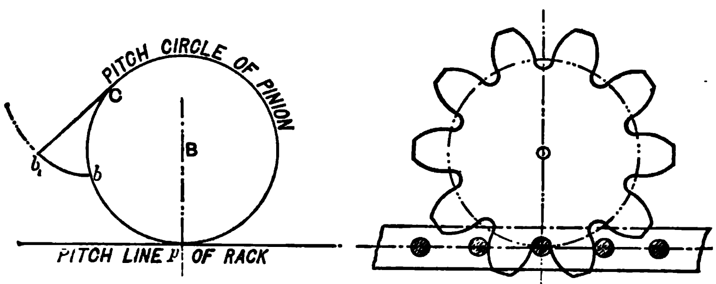
It should be noticed, however, that the parallel curves so drawn are very approximately epicycloids traced by a generating circle equal in size to the one used in describing the dotted curves. Hence, it is only necessary to draw from the ends of the small circular arcs, epicycloids with a generating circle equal in diameter to that of the pitch circle, B, and these will represent the working faces of the teeth on A.

Pins are always placed on the Follower.—When one of a pair of wheels in gear has pins instead of teeth, it is the practice to *place the pins on that wheel which is to be the follower.* The reason for this will be apparent when we remember what has been said regarding the friction between the teeth during the arcs of approach and recess. The friction during the arc of approach is said to be greater than that during recess, and if this be the case, it follows, that the arc of approach should be as small as possible.

Now, in the arrangement just considered, wherein the teeth have no flanks, it is clear that there will be no arc of approach or no arc of recess according as the pin wheel is the follower or the driver. If the pin wheel be the follower, the whole of the action between the teeth on A and the pins on B will occur *after* the line of centres—*i.e.*, during the arc of recess. If B becomes the driver, then the whole of the action takes place during the arc of approach. This latter arrangement should therefore not be adopted.

Rack and Pinion.—Sometimes we find either a rack or its pinion fitted with pins instead of ordinary teeth. In any case, however, the above rule must be attended to—viz., *the pins always to be placed on the follower.* Hence two cases arise—(1) the pinion may drive the rack, or (2) the rack may drive the pinion.

(1) *Suppose the Pinion to Drive the Rack.*—In this case, the pins must be placed upon the rack. Now the pitch line of a rack has been stated to be part of a pitch circle of infinite radius, and since the faces of the teeth on the driver are supposed to be described by a generating circle having the same diameter as the pitch circle of the follower, it follows that this generating circle must also be of infinite radius. Hence, the curves for the



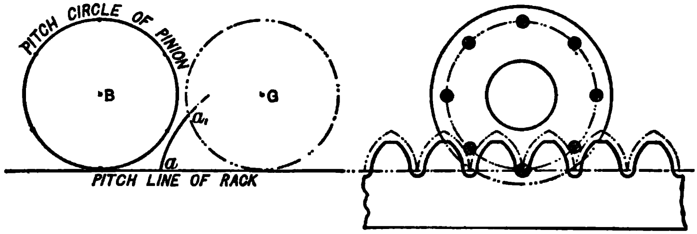
PINION IN GEAR WITH RACK FITTED WITH PINS.

faces of the teeth on the pinion will be involutes of its own pitch circle.

The principles of construction in this case will be understood from what has preceded and by a reference to the accompanying figures.

(2) *Suppose the Rack to Drive the Pinion.*—In this case, the pins must be placed upon the pinion. Hence, the faces of the teeth on the rack must be described by a generating circle, G, *equal in diameter to that of the pitch circle of the pinion.* The curves for the faces of these teeth will thus be *cycloids.* The method

of describing these as well as their appearance when complete will be easily understood from the figure.



PIN WHEEL IN GEAR WITH A RACK.

Disadvantage of Pin Wheels.—Pin wheels are now seldom used, except in clock and watch mechanism, owing to a practical disadvantage which they possess—viz., that the wheels required to gear with them have to be specially designed, and these latter can only be geared with one particular size of pin wheel, and with no other kind of toothed wheel.

Lecture XIII.—Questions.

1. Explain the terms "pitch" (circular and diametral), "pitch circle," and "pitch point" as applied to toothed gearing. State the relation between the circular and diametral pitches.

2. Explain, by aid of sketches, the meanings of the terms "face," "flank," "addendum," and "clearance" as applied to toothed gearing. State the usual proportions for the addendum and clearance (side and bottom) in terms of the pitch. What is the effect of clearance on the action of the teeth?

3. What is meant by the *pitch* of a tooth in a spur wheel? What are the usual forms of teeth and how are they described? Sketch two consecutive teeth of a spur wheel, and give the relative proportions of the different parts of a tooth in terms of the pitch.

4. Design by any method you know the tooth of a spur wheel—pitch = 2 inches; diameter = 7 inches; and show by dimensions the correct proportions. (C. and G. of L. Mech. Eng. Hons. Exam., 1891.)

5. What is meant by the term arc of action? State the usual length of arc of action in terms of the pitch.

6. Upon what principle are teeth of wheels of the epicycloidal and hypocycloidal form constructed? Show under what conditions they will work properly. What is to be done in order that any wheels of a set may work accurately together? (S. and A. App. Mechs. Hons. Exam., 1893.)

7. In forming the teeth of wheels, the geometrical condition is that the common perpendicular to the surfaces of two teeth in contact shall always pass through the point of contact of the pitch circles of the wheels. Write out a proof of this general proposition. (S. and A. App. Mechs. Hons. Exam., 1889.)

8. Give the theory for constructing teeth of wheels with radial flanks which shall work accurately together, the distance between the centres of the pitch circles of two such wheels being 24 inches, and the required velocity-ratio of the wheels 3 to 1, find the diameter of the rolling circles for describing the teeth of each wheel. (S. and A. App. Mechs. Hons. Exam., 1885.) *Ans.* 18 ins. and 6 ins.

9. A toothed spur wheel is 4 inches pitch. Sketch a tooth and mark on it suitable dimensions. Draw accurately a suitable curve for such a tooth, taking the pitch line straight as in a rack, and using a describing circle of 5 inches radius. (S. and A. Mach. Const. Hons. Exam., 1882.)

10. What geometrical condition must be satisfied by the acting surfaces of the teeth of a pair of wheels in order that the velocity-ratio communicated may be constant? Show that this condition is fulfilled by epicycloidal and hypocycloidal curves.

11. Show, by sketches, what cycloidal curves should be used or approximated to in the faces and flanks of the teeth in the following cases:—(*a*) Pair of wheels in external contact; (*b*) Pinion and rack. It is only necessary to mention the proper curves, without attempting to draw them. (S. and A. Mach. Const. Hons. Exam., 1883.)

12. Define, and show roughly by sketches, the following curves:—The cycloid, epicycloid, hypocycloid, and involute. Mention one property of those curves which make them so useful for engineering purposes. Discuss the various particular forms assumed by those curves under particular circumstances, and state some of their applications.

13. Describe the form of Gee's patent wheel teeth, and mention what is their advantage. (S. and A. Mach. Const. Hons. Exam., 1883.)

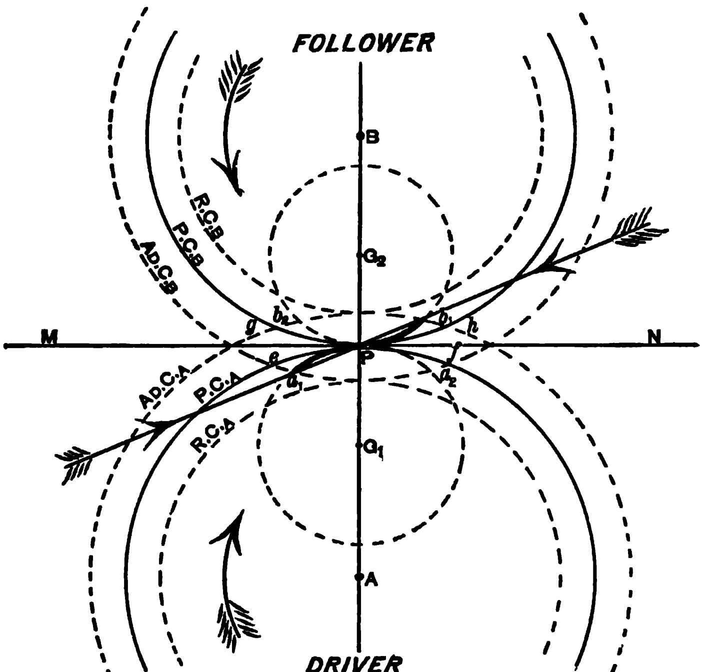
14. In wheels with pins for teeth the pins are always placed upon the follower; will you explain this? What are the chief disadvantages of pin wheels?

15. A toothed wheel drives a pin wheel; investigate the proper form for the curves of the teeth. The diameter of each pin being known, how do you proceed to set out the teeth, preserving their theoretical outline? Sketch the necessary diagram. (S. and A. App. Mechs. Hons. Exam., 1888.)

LECTURE XIV.

CONTENTS.—Path of Contact with Cycloidal Teeth—Obliquity of Reaction—Length of Cycloidal Teeth for given Arcs of Approach and Recess—Calculation of the Length of Cycloidal Teeth—Examples I. and II.—Diameter of Generating Circle—Least Number of Cycloidal Teeth to be placed upon a Wheel—Cycloidal Teeth for Wheels with Internal Contact—Path of Contact with Internal Gearing—Formulæ for Length of Teeth of Internal Gearing—Questions.

Path of Contact with Cycloidal Teeth.—Let the accompanying figure represent portions of two pitch circles with their addendum



circles and generating circles, G_1, G_2. From the figure, it will be seen that contact between two teeth begins at a_1 and terminates

at b_1, the points a_1 and b_1 being determined by the intersection of the addendum and generating circles. During motion the point of contact of the pair of teeth in question travels along the curve, $a_1 p b_1$, which is made up of the arcs, $a_1 p$, $p b_1$, of the two generating circles, G_1, G_2.

DEFINITION.—**The path $a_1 p b_1$, along which the point of contact of a pair of teeth moves, is called the Path of Contact.**

The whole path of contact is divided at the pitch point, p, into two parts called, respectively, the **Path of Approach**, $a_1 p$, and the **Path of Recess**, $p b_1$.*

If the direction of motion of the wheels be reversed, then the *path of contact* will be $a_2 p b_2$.

The path of contact in the case of cycloidal teeth is always circular, but in some forms of teeth, for example involute teeth, the path of contact may be a straight line. The student should examine all the preceding particular cases and ascertain the nature of the path of contact. He will then see that in teeth with involute faces part of this path is a straight line.

Obliquity of Reaction.—We have seen that (neglecting friction) the direction or line of action of the mutual pressure, or reaction, between a pair of properly constructed teeth in contact always passes through the pitch point, p. *The angle which this direction makes with the common tangent to the two pitch circles at their point of contact is called the* **Obliquity of Reaction.** Thus, in the previous figure, the direction of the mutual pressure or reaction at the beginning of contact of a pair of teeth is along $a_1 p$, and at the end of contact along $p b_1$. The obliquities of reaction at these two particular points are denoted by the angles $a_1 p$ M, $b_1 p$ N respectively. When the point of contact of the teeth reaches the pitch point, p, the direction of the reaction is along M N, the common tangent at p, and at this point the obliquity is zero. Thus, the obliquity of reaction in the case of cycloidal teeth, varies from a maximum at the beginning and end of contact of a pair of teeth, to zero at the pitch point. With such teeth,

* The student must carefully distinguish between the terms *path of contact* and *arc of contact*. The latter term refers to the *arc on either pitch circle* turned through by that pitch circle during contact of a pair of teeth, while the former refers to the *actual path* traversed by the point of contact during the same period. Nevertheless, it should be noticed, that *with cycloidal teeth*:—

Length of *arc of approach*, $e p$ or $g p$ = length of *path of approach*, $a_1 p$.
,, *arc of recess*, $p f$ or $p h$ = ,, *path of recess*, $p b_1$.

so that:—

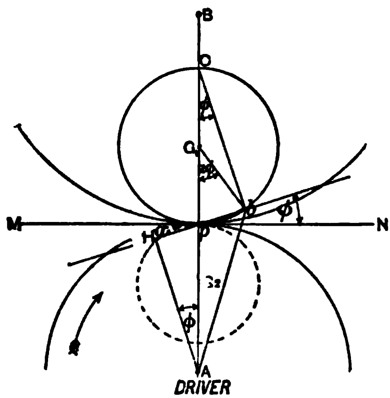
Length of *arc of contact* = length of *path of contact*.

the maximum obliquity should never be allowed to exceed 30°. In those teeth of which the path of contact is a straight line, the obliquity remains constant during contact, and should never exceed about 15°.

Length of Cycloidal Teeth for given Arcs of Approach and Recess.—When the arcs of approach and recess are given for a pair of wheels, we can then determine the lengths to be given to the teeth on the two wheels respectively. Referring to the last figure, let arcs $p\,e$, $p\,f$ represent the given lengths of the arcs of approach and recess respectively. On the given generating circles, G_1, G_2, cut off the arcs $p\,a_1$, $p\,b_1$, equal in length respectively to $p\,e$, $p\,f$. Through the points a_1, b_1 draw the circles $A_D\,C_B$, $A_D\,C_A$, about the centres B and A respectively. These are the addendum circles for the two wheels. After making allowances for bottom clearance, the root circles, $R\,C_A$, $R\,C_B$, can be drawn. From this construction, the sizes of the teeth on the two wheels can be determined.

Calculation of the Length of Cycloidal Teeth.—We shall now show how the previous problems may be solved by calculation.

In the accompanying figure, let wheel A be the driver, B the follower, and G_1, G_2 the generating circles. Then $a\,p\,b$ is the path of contact.



LENGTH OF CYCLOIDAL TEETH.

Let R_1 = Radius of pitch circle, A.
„ r_1 = Radius of circle, G_1, used in describing *faces* of teeth on A.
„ δ_1 = Addendum of teeth on A.
„ θ, φ = Maximum obliquities of action during approach and recess respectively.
„ α, β = Lengths of arcs of approach and recess respectively.

Then, since *Path of approach or recess = Arc of approach or recess.*

$$\therefore \qquad \alpha = \text{arc } a\,p\,; \text{ and } \beta = \text{arc } p\,b.$$

Join A b, $G_1\,b$, C b, and $p\,b$. Draw A H perpendicular to $b\,p$ produced. Then clearly, $\angle p\,A\,H = \angle p\,C\,b = \angle b\,p\,N = \varphi$.

Since b A H is a right angled triangle, we get :—

$$A\,b^2 = A\,H^2 + H\,b^2.$$

But, $A\,b = R_1 + \delta_1.$

$A\,H = R_1 \cos \varphi.$

And, $H\,b = H\,p + p\,b = R_1 \sin \varphi + 2\,r_1 \sin \varphi.$

$\therefore \quad (R_1 + \delta_1)^2 = R_1^2 \cos^2 \varphi + (R_1 + 2\,r_1)^2 \sin^2 \varphi.$

$\therefore \quad R_1^2 + 2\,R_1\,\delta_1 + \delta_1^2 = R_1^2 (\cos^2 \varphi + \sin^2 \varphi) + 4\,(R_1 + r_1) r_1 \sin^2 \varphi.$

$$\therefore \quad 2\,R_1\,\delta_1 + \delta_1^2 = 4\,(R_1 + r_1)\,r_1 \sin^2 \varphi. \quad \text{(I)}$$

Also, $\text{Arc}\; p\,b = r_1 \times 2\,\varphi.$

$$\left.\begin{array}{ll} i.e., & \beta = 2\,r_1\,\varphi. \\ \text{Or,} & \varphi = \dfrac{\beta}{2\,r_1}. \end{array}\right\} \quad \text{(II)}$$

Similarly, if R_2, r_2, δ_2 apply to the follower, B.

$$\text{Then,} \quad 2\,R_2\,\delta_2 + \delta_2^2 = 4\,(R_2 + r_2)\,r_2 \sin^2 \theta \quad (\text{I}_a)$$

$$\left.\begin{array}{ll} \text{And,} & \alpha = 2\,r_2\,\theta \\ \text{Or,} & \theta = \dfrac{\alpha}{2\,r_2} \end{array}\right\} \quad (\text{II}_a)$$

From these equations the addenda of the teeth on the two wheels can be calculated, if we know the sizes of the generating circles and the arcs of approach and recess.

When the wheels are large we may neglect δ^2 in equations (I) and (I_a), since this quantity will be small compared with R, and we get the approximate formulæ :—

$$\left.\begin{array}{ll} & \delta_1 = 2\left(1 + \dfrac{r_1}{R_1}\right) r_1 \sin^2 \varphi \\ \text{And,} & \delta_2 = 2\left(1 + \dfrac{r_2}{R_2}\right) r_2 \sin^2 \theta \end{array}\right\} \quad \text{(III)}$$

Again, the sizes of the generating circles are generally stated in terms of the size of the pitch circles inside which they roll.

Hence, let:— $r_1 = m_2\,R_2$, $r_2 = m_1\,R_1$.

Where m_1 and m_2 are fractions seldom greater than one-half.

Then, equations (I) and (III) become :—

$$\left.\begin{array}{l} 2\,R_1\,\delta_1 + \delta_1^2 = 4\,m_2\,(R_1 + m_2\,R_2)\,R_2 \sin^2 \varphi \\ 2\,R_2\,\delta_2 + \delta_2^2 = 4\,m_1\,(R_2 + m_1\,R_1)\,R_1 \sin^2 \theta \end{array}\right\} \quad \text{(IV)}$$

$$\left.\begin{array}{ll} \text{Or, approximately,} & \delta_1 = 2\,m_2 \left(1 + \dfrac{m_2\,R_2}{R_1}\right) R_2 \sin^2 \varphi \\ & \delta_2 = 2\,m_1 \left(1 + \dfrac{m_1\,R_1}{R_2}\right) R_1 \sin^2 \theta \end{array}\right\} \quad \text{(V)}$$

Let $\quad p$ = Pitch of teeth on wheels.
" $\quad n_1, n_2$ = Number of teeth on respective wheels.

Then, $\quad R_1 = \frac{n_1 p}{2\pi}, \qquad R_2 = \frac{n_2 p}{2\pi}.$

And we get the following final equations :—

$$\left.\begin{aligned} \pi n_1 p\, \delta_1 + \pi^2 \delta_1^2 &= m_2 (n_1 + m_2 n_2) n_2 p^2 \sin^2 \phi \\ \pi n_2 p\, \delta_2 + \pi^2 \delta_2^2 &= m_1 (n_2 + m_1 n_1) n_1 p^2 \sin^2 \theta \end{aligned}\right\} \quad . \quad \text{(VI)}$$

Or, approximately,

$$\left.\begin{aligned} \pi n_1 \delta_1 &= m_2 (n_1 + m_2 n_2) n_2 p \sin^2 \phi \\ \pi n_2 \delta_2 &= m_1 (n_2 + m_1 n_1) n_1 p \sin^2 \theta \end{aligned}\right\} \quad . \text{(VII)}$$

And,

$$\left.\begin{aligned} \pi \alpha &= m_1 n_1 p\, \theta \\ \pi \beta &= m_2 n_2 p\, \phi \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \quad \text{(VIII)}$$

We would recommend the student to use equations (I) and (II) in working out problems, instead of attempting to remember all the above particular forms which they assume.

EXAMPLE I.—The flanks of the teeth of a pair of wheels are radial. The number of teeth on the wheels are 21 and 120. The addendum to each wheel is $\frac{3}{10}$ pitch. Find the lengths of the arcs of approach and recess, supposing the small wheel to be the driver.

ANSWER.—Here $n_1 = 21$; $n_2 = 120$; $\delta_1 = \delta_2 = \frac{3}{10} p$.

$\therefore \qquad R_1 = \frac{n_1 p}{2\pi} = \frac{21 p}{2\pi},$

And, $\qquad R_2 = \frac{n_2 p}{2\pi} = \frac{120 p}{2\pi}.$

Since the flanks are radial $r_1 = \frac{1}{2} R_2 = \frac{120 p}{4\pi}.$

And, $\qquad r_2 = \frac{1}{2} R_1 = \frac{21 p}{4\pi},$

From equation (I), we get :—

$$2 R_1 \delta_1 + \delta_1^2 = 4 (R_1 + r_1) r_1 \sin^2 \phi.$$

$$\therefore \quad 2 \times \frac{21 p}{2\pi} \times \frac{3 p}{10} + \frac{9 p^2}{100} = 4 \left(\frac{21 p}{2\pi} + \frac{120 p}{4\pi}\right) \times \frac{120 p}{4\pi} \sin^2 \phi.$$

$$\therefore \frac{2 \times 21 \times 3 \times \pi}{10} + \frac{9 \times 2 \pi^2}{100} = 162 \times 60 \sin^2 \phi.$$

$$\therefore \qquad \sin \phi = \cdot 0652.$$

Or, $\qquad \phi = 3\frac{3}{4}^\circ$, nearly $= \cdot 065$ radian.

Hence, **Arc of recess** $= \beta = 2\, r_1\, \phi$

,, $= 2 \times \dfrac{120\,p}{4\,\pi} \times \cdot065 = 1\cdot25\,p$, **nearly.**

Next, to find the arc of approach.

From equation (I_a), we get :—

$$2\, \mathrm{R}_2\, \delta_2 + \delta_2^2 = 4\,(\mathrm{R}_2 + r_2)\, r_2 \sin^2 \theta.$$

$$\therefore \quad 2 \times \frac{120\,p}{2\,\pi} \times \frac{3\,p}{10} + \frac{9\,p^2}{100} = 4\left(\frac{120\,p}{2\,\pi} + \frac{21\,p}{4\,\pi}\right) \times \frac{21\,p}{4\,\pi} \sin^2 \theta.$$

$$\therefore 2 \times 12 \times 3 \times 2\pi + \frac{9 \times 4\,\pi^2}{100} = 261 \times 21 \sin^2 \theta.$$

$$\therefore \qquad \sin \theta = \cdot2885.$$

$$\therefore \qquad \theta = 16\tfrac{3}{4}^\circ \text{ nearly} = \cdot292 \text{ radian.}$$

Hence, **Arc of approach** $= \alpha = 2\, r_2\, \theta$

,, $= 2 \times \dfrac{21\,p}{4\,\pi} \times \cdot292 = \cdot976\,p$, **nearly.**

Had we neglected δ^2 in the above solution, and taken the approximate formulæ (III), we would have got :—

$$\sin \phi = \cdot0638, \text{ instead of } \cdot0652.$$

Now the difference in those two angles is only about 5 minutes, the first sine corresponding to an angle of 3° 39′, the second corresponding to an angle of 3° 44′.

Again, the exact value of θ is 16° 46′, while the approximate value (neglecting δ^2) would be 16° 42′.

Had we, therefore, assumed the approximate formulæ the results would practically not have been different from what we have just obtained.

Example II.—In Example I., find the addenda, when the arcs of approach and recess are each equal to half the pitch.

Answer.—Here, $\alpha = \beta = \dfrac{1}{2}p.$

From equation (II_a), we get :—

$$\theta = \frac{\alpha}{2\,r_2} = \frac{\frac{1}{2}p}{2 \times \frac{21\,p}{4\,\pi}} = \frac{\pi}{21}.$$

$$\therefore \qquad \theta^\circ = \frac{\pi}{21} \times \frac{180}{\pi} = 8^\circ\, 34'.$$

$$\therefore \qquad \sin\theta = \cdot 149.$$

And from II,

$$\phi = \frac{\beta}{2\,r_1} = \frac{\frac{1}{2}p}{2 \times \dfrac{120\,p}{4\,\pi}} = \frac{\pi}{120}.$$

$$\therefore \qquad \phi^\circ = \frac{\pi}{120} \times \frac{180}{\pi} = 1^\circ 30'.$$

Or, $\qquad \sin\phi = \cdot 0262.$

Hence, taking the approximate formulæ (III), we get:—

$$\delta_1 = 2\left(1 + \frac{r_1}{R_1}\right) r_1 \sin^2\phi$$

$$\text{,,} = 2\left(1 + \frac{60}{21}\right)\frac{120\,p}{4\,\pi} \times \cdot 0262^2 = \cdot\mathbf{05}\,p.$$

Similarly,

$$\delta_2 = 2\left(1 + \frac{r_2}{R_2}\right) r_2 \sin^2\theta.$$

$$\text{,,} = 2\left(1 + \frac{21}{240}\right)\frac{21\,p}{4\,\pi} \times \cdot 149^2 = \cdot\mathbf{08}\,p.$$

In this example we might have further simplified our calculations by writing $\sin\theta = \theta$ and $\sin\phi = \phi$, since these angles are very small. Doing this and combining equations (I) and (II), we get the following approximate formulæ:—

$$\left.\begin{aligned} \delta_1 &= \tfrac{1}{2}\left(\frac{1}{R_1} + \frac{1}{r_1}\right)\beta^2 \\ \delta_2 &= \tfrac{1}{2}\left(\frac{1}{R_2} + \frac{1}{r_2}\right)\alpha^2 \end{aligned}\right\} \quad \ldots\ldots \quad \text{(IX)}$$

Substituting for α, β, R_1, R_2, r_1, r_2, we get:—

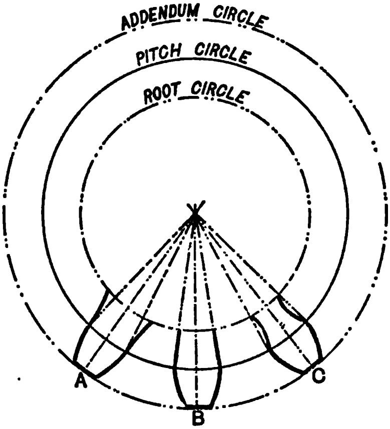
$$\delta_1 = \tfrac{1}{2}\left\{\frac{2\,\pi}{21\,p} + \frac{4\,\pi}{120\,p}\right\} \times \left(\frac{p}{2}\right)^2 = \cdot 05\,p,$$

$$\delta_2 = \tfrac{1}{2}\left\{\frac{2\,\pi}{120\,p} + \frac{4\,\pi}{21\,p}\right\} \times \left(\frac{p}{2}\right)^2 = \cdot 08\,p,$$

which are exactly the same results as before.

Diameter of Generating Circle.—We have already had instances of the effects produced on the form of cyloidal teeth, by the size of the generating circle employed. We have seen that if the size

of the generating circle be half that of the pitch circle inside which it rolls, the flanks of the teeth so described are radial. If the generating circle be larger than this, the flanks described by it will be undercut and comparatively weak at the roots. The accompanying figure illustrates the influence of the size of the generating circle on the form of the teeth. The flanks of the teeth, A, B, and C, are described by generating circles, having respectively diameters less than, equal to, and greater than the radius of the pitch circle. From these figures it is evident that the generating circle employed in describing the flanks of the teeth on a wheel should never have a diameter greater than half that of the pitch circle of the wheel.



INFLUENCE OF SIZE OF GENERATING CIRCLE ON THE FORM OF TEETH.

If a set of wheels, such as the change wheels for a screw-cutting lathe, have to gear together in different arrangements, it is clear that the same generating circle must be used for describing both faces and flanks of the teeth of every wheel in the set. This being the case, it follows from what has been said above that **the diameter of the generating circle employed in describing the faces and flanks of the teeth of a set of wheels must not be greater than half that of the pitch circle of the smallest wheel in the set,** otherwise the flanks of the teeth on the smallest wheel would be undercut and weak at the roots.* Since it is desirable to have as large a generating circle as possible, it is usual to construct the teeth on the smallest wheel with radial flanks. *The size of the generating circle must then have a diameter equal to the radius of the smallest wheel in the set.*

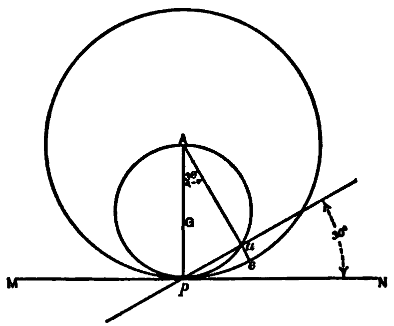
Least Number of Cycloidal Teeth to be placed upon a Wheel.—It has been pointed out, that with any pair of wheels gearing together, *there should never be fewer than two pairs of consecutive*

* Cases are not wanting where the flanks have been described by a generating circle larger than the radius of the pitch circle; but in such cases fillets are made at the roots in order to strengthen the teeth.

teeth in action at any time, and, further, *that the maximum obliquity of reaction should never exceed* 30°, in the case of cycloidal teeth. These conditions being premised, it is easy to determine the least number of teeth which must be placed upon the smallest wheel of a set.

Let A be the centre of the pitch circle of the smallest wheel in the set; G the generating circle, the diameter of which is half that of pitch circle, A.

Let contact between a pair of teeth begin at a, then $\angle a p \mathrm{N} = 30°$. When one pair of teeth are in contact at a, the preceding consecutive pair will be in contact at the pitch point, p. Make arc $p e$ = arc $p a$. Then in this case, $p e$ is the arc of approach, and is equal to the pitch of the teeth. Join A e. This line will



Least Number of Cycloidal Teeth on a Wheel.

pass through the point a; hence, it is easy to see that $\angle p \mathrm{A} e = \angle a p \mathrm{N} = 30°$. We then get:—

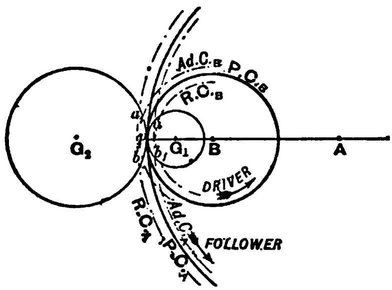
$$\text{Arc } p\,e = \text{arc of } 30° \text{ on pitch circle A,}$$
$$\text{,, } = \tfrac{1}{12} \text{ of circumference of pitch circle A.}$$
$$\text{But arc } p\,e = \text{pitch of teeth,}$$
$$\therefore \text{ Pitch of teeth} = \tfrac{1}{12} \text{ of circumference of pitch circle A,}$$

i.e, **the least number of teeth on the smallest wheel of a set must be 12.**

In a similar way, it can be shown that **the least number of pins to be placed upon a pin or lantern wheel is 6.**

Cycloidal Teeth for Wheels with Internal Contact.—Let A be the centre of an internal toothed wheel; B the centre of a pinion gearing with wheel A. The faces of the teeth on A and the flanks of the teeth on B are described by the same generating circle, G_1, while the flanks of the teeth on A and the faces of the teeth on B

are described by the generating circle G_2. Since the *faces* of the teeth on an internal toothed wheel lie *inside* the pitch surface, and the *flanks outside* the pitch surface, it is clear that the curves for the former are arcs of *hypocycloids* obtained by rolling G_1 inside the pitch circle A, while the curves for the flanks are *epicycloids* obtained by rolling G_2 outside the pitch circle A. Hence, the curves for the faces of the teeth on A and the flanks of the teeth on B are hypocycloids, while the curves for the flanks of the teeth on A and the faces of the teeth on B are epicycloids. The size of



CYCLOIDAL TEETH FOR WHEELS WITH INTERNAL CONTACT.

generating circle, G_1, must not exceed half that of pitch circle of the pinion, B, otherwise the roots of the teeth on the pinion will be undercut and weak. The size of generating circle, G_2, may be anything we like, since the curves described by it on both pitch circles are epicycloids.

Path of Contact with Internal Gearing.—Let P C, Ad C, and R C, with the suffixes A and B, denote the pitch, addendum, and root circles of wheels A and B respectively. Let the pinion be the driver. Contact between a pair of teeth begins at *a*, the point of intersection of circles G_1 and Ad C_A, and terminates at *b*, the point of intersection of circles G_2 and Ad C_B; *a p b* is, therefore, the path of contact. If the annular wheel were the driver, then the curve, $a_1\,p\,b_1$, would represent the path of contact, the direction of motion being the same as before. The student should experience little difficulty in applying all the preceding principles to internal toothed gearing if he has followed intelligently what has already been said regarding wheels with external contact.

Formulæ for Length of Teeth of Internal Gearing.—The student should now prove the following formulæ for internal gearing, the method of arriving at the results being similar to that previously given.

(1) *When the Pinion is the Driver.*

For Pinion:— $2\,R_1\,\delta_1 + \delta_1^2 = 4\,(R_1 + r_1)\,r_1 \sin^2 \varphi$

And, $2\,r_1\,\varphi = \beta.$

For Annular Wheel, $2\,R_2\,\delta_2 - \delta_2^2 = 4\,(R_2 - r_2)\,r_2 \sin^2 \theta$

And, $2\,r_2\,\theta = \alpha.$

(2) *When the Pinion is the Follower.*

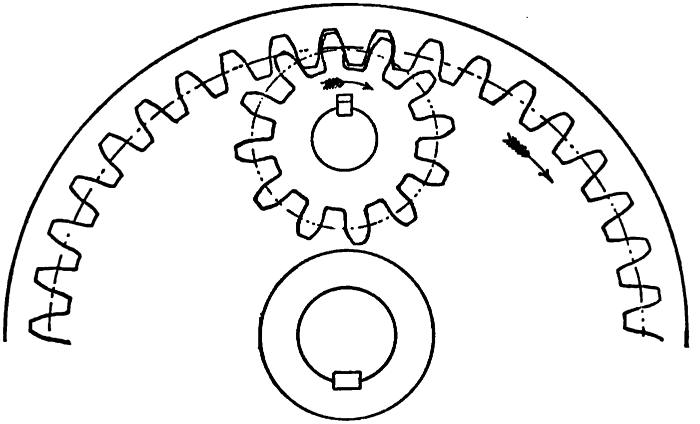
For Pinion:— $2\,R_2\,\delta_2 + \delta_2^2 = 4\,(R_2 + r_2)\,r_2 \sin^2 \theta$

And, $2\,r_2\,\theta = \alpha.$

For Annular Wheel, $2\,R_1\,\delta_1 - \delta_1^2 = 4\,(R_1 - r_1)\,r_1 \sin^2 \varphi$

And, $2\,r_1\,\varphi = \beta.$

The various symbols have the same meanings as before.



ANNULAR WHEEL AND PINION WITH CYCLOIDAL TEETH.

Lecture XIV.—Questions.

1. Distinguish between the terms *arc of approach* or *recess* and *path of approach* or *recess.* Given the pitch circles of a pair of wheels, sizes of generating circles, and arcs of approach and recess, show by a construction how to set out the path of contact.

2. Upon what principle are teeth of wheels of the epicycloidal and hypocycloidal form constructed? Show under what conditions they will work properly. What is to be done in order that any wheels of a set may work accurately together? (S. and A. Hons. Exam., 1893.)

3. In the consideration of the form suitable for the teeth of spur wheels, state:—(*a*) What geometrical condition should be satisfied as to the position of the common normal at the point of contact of two teeth, and explain why that condition should be satisfied; (*b*) within what limits it is desirable to keep the obliquity of the line of action of the pressure between two teeth, and why within those limits; (*c*) the least number of pairs of teeth which it is desirable should be engaged at the same time. Explain also (*d*) why it is undesirable that the action between two teeth should extend far from the pitch point. By a graphical construction, determine the arc of action, and the greatest obliquity of the line of action of a pair of cycloidal teeth, according to the following data, and state how many pairs of teeth will be in action at the same time:—Pitch of teeth = 2 inches; number of teeth in wheels = 30 and 50; diameter of rolling or describing circles = $8\frac{3}{4}$ inches; addenda of teeth = $\frac{5}{8}$ inch. (S. and A. Mach. Const. Hons. Exam., 1892.) *Ans.* 4 inches; 13·5°; 3.

4. Show by construction how you would determine the correct form for the teeth of a spur wheel 4 feet in diameter. The diameter of the smallest wheel in the train being 8 inches, what sized rolling circle would you use? (C. and G. Mech. Eng. Ord. Exam., 1892.) *Ans.* 4 inches.

5. A pair of wheels have 25 and 130 cycloidal teeth respectively. Find the addendum of each wheel, that the arcs of approach and recess may each be equal to the pitch, the flanks being radial. *Ans.* ·17 p; ·28 p.

6. The diameter of the pitch circle of an annular wheel by means of which a water-wheel communicates motion to a mill is to differ as little as possible from 24 feet. The pitch of the teeth is to be 4 inches. Find actual diameter and number of teeth. If the velocity of the periphery be $5\frac{1}{2}$ feet per second, and the pinion in gear with the wheel is required to make 30 revolutions per minute, find the necessary diameter of the pinion and the number of teeth. Again, if both faces and flanks of all the teeth be described by a constant generating circle 12 inches in diameter, find the arcs of approach and recess, the addendum of both wheels being $1\frac{1}{4}$ inches. *Ans.* 23·98 ft.; 226 teeth; 3·5 ft.; 33 teeth; $\alpha = 3·42$ ins.; $\beta = 3·96$ ins.

LECTURE XV.

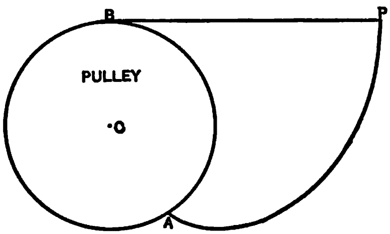
CONTENTS.—Involute Teeth—Size of Base Circle to be employed—Length of Involute Teeth for given Arcs of Approach and Recess—Calculation of the Length of Involute Teeth—Least Number of Involute Teeth to be placed upon a Wheel—Rack and Pinion with Involute Teeth—Wheels with Involute Teeth and Internal Contact—Calculations for Involute Teeth with Internal Contact—Examples I. and II.—Bevel Wheels—Teeth of Bevel Wheels—Mortice Wheels—Gear Cutting Machine—Questions.

Involute Teeth.—In a previous Lecture we have shown that an involute curve is a particular case of an epicycloid, and with those particular conditions we have had instances of teeth with *involute faces*. But no case has yet been considered wherein both faces and flanks are involute in form. Involute teeth—*i.e.*, those forms of teeth whose faces and flanks are described by involutes of circles—possess certain peculiar properties, and on that account may be studied as a class independent of all other forms.

In this Lecture, we shall first show that the involute form of tooth satisfies all the primary conditions for correct working, and we shall thereafter explain its unique properties.

In order to properly understand what follows, we shall first explain how an involute curve can be drawn.

Let C represent the centre of a thin pulley with a fine string wound round its circumference. Fix a sheet of drawing paper to one of the faces of the pulley, and tie a pencil, P, to the free end of the string. By keeping the string taut and unwinding it from the pulley, the pencil at P will trace on the drawing paper the involute, A P.



HOW TO TRACE AN INVOLUTE CURVE.

For our present purpose it is much better to conceive the curve described in the following manner:—

Let the pulley and its attached paper be capable of turning round C as an axis. Take hold of the pencil, P, and pull it along the straight line, B P. By this means the pulley and

paper will be made to rotate about C, while the pencil will trace out the involute, A P, on the moving paper just as before. Similarly, if the string be wound on to the pulley, by rotating the pulley and the paper, the pencil will retrace the curve P A, if P be made to move in the direction, P B.

The circle of which the curve is the involute is called the *base circle.*

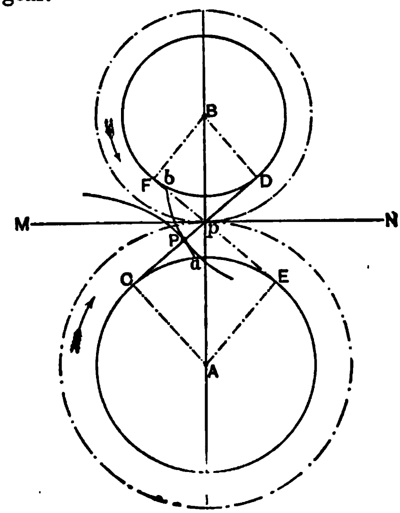
In the next figure, let the dotted circles represent the pitch circles of a pair of wheels in gear.

With A and B as centres, describe the circles C a E, D b F, of such sizes that

$$A C : B D = A p : B p.$$

These are the base circles for the pair of wheels.

Now imagine these base circles to represent pulleys over which a *crossed* string, C D F E, is stretched. Then, clearly, the motion transmitted by means of these pulleys and the crossed string will be identically the same as that obtained by the rolling of the pitch circles.



TRACING THE CURVES FOR INVOLUTE TEETH.

Suppose a sheet of paper to be fixed to pulley, A, and capable of rotating with it. Then a pencil, P, anywhere on the string, C D, will, during rotation in the direction shown, describe the involute, a P, on the rotating paper. The curve, a P, is an involute to the base circle, C a E. Similarly, by supposing a sheet of paper to have been fixed to pulley, B, the pencil would have, simultaneously described the involute, b P, to the base circle, D b F.

The two involutes, a P, b P, being simultaneously described by the tracing point, P, will always be in contact at that point, and have a common normal, C D. The locus of P is C D, and therefore the common normal always passes through the pitch point, p. Hence the principal condition for the curves of all properly-constructed teeth is satisfied by involutes traced in the above manner.

By fixing a pencil to the string, E F, and proceeding as before, the curves for the opposite working surfaces of the teeth can be drawn.

To complete the curves representing the working surfaces of the teeth, it is only necessary to describe the addendum and root circles; the parts of the involutes intercepted between these circles will represent the acting surfaces of the complete teeth.

The following properties of involute teeth should be noted :—

(1) *Both face and flank of an involute tooth form one continuous curve.*

On this account it will be much easier to set out the curves for involute teeth than for cycloidal teeth, since, in the latter, the curves for face and flank are always of opposite convexity.

(2) *With involute teeth the centres of the wheels can be pushed further apart or brought closer together without affecting their velocity-ratio or smoothness of action.*

This is a most valuable and unique property of such teeth. The reasons for this property will be apparent from an inspection of the previous figure. Pushing the wheels further apart, or bringing them closer together, alters the sizes of the pitch circles without altering their ratio, but does not alter the sizes of the base circles, and, therefore, does not affect the curvature of the involutes. It does, however, affect the direction of the normal thrust between the teeth. The direction of this thrust is always along the common tangent to the base circles. More will be said about this immediately.

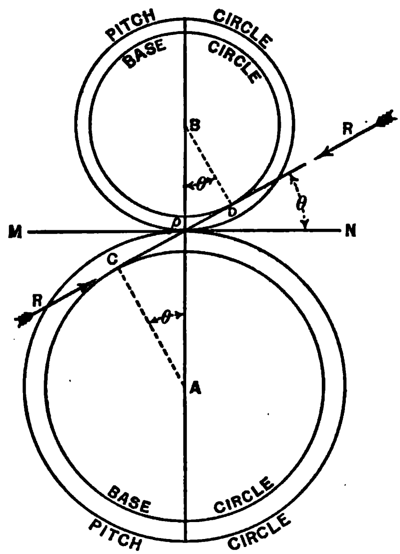
(3) *All wheels with involute teeth of equal pitch and obliquity of action work accurately with each other.*

The reasons for this will also be apparent from what has just been said.

From the above properties it will be seen, that involute teeth are singularly well suited for most purposes, and in our opinion it would be well if engineers would universally adopt this form, and thus save endless expense and trouble in patterns, &c. As matters now stand, each maker of toothed wheels has his own method of constructing the teeth; the result being, that the wheels of one maker will not work correctly, nor approximately correct, with those of equal pitch by other makers. The same state of affairs with regard to screw threads existed prior to the establishment of a standard thread by the late Sir Joseph Whitworth. At that time, if the nut of a bolt were lost, it was ten chances to one if another could be found to fit it, with the result that many good bolts had to be thrown into the scrap heap for want of nuts. The nuts, too, were of all sizes, even for the same size of bolt, and as a consequence every fitter had to be supplied

with a multitude of spanners whenever he had a series of bolts to deal with.

One objection which has been urged by some engineers against involute teeth is the normal thrust between the teeth, which is always constant in direction, and, being oblique to the common tangent at the pitch point, tends to push the wheels out of gear. We are of opinion, however, that too much importance has been attached to this; for, if the obliquity of reaction be kept less than 15°, no very serious results will follow. With cycloidal teeth, the obliquity of reaction varies from zero, when the pair of teeth are in contact at the pitch point, to a maximum, when the teeth are just beginning or just ending contact.



SIZE OF BASE CIRCLE FOR INVOLUTE TEETH.

Size of Base Circle to be employed.—We have shown that the line of action of a pair of involute teeth in contact is always along a common tangent to the base circles. The direction of the mutual thrust, or, in other words, the obliquity of reaction, is constant.

Let $\quad R$ = Thrust between a pair of teeth in contact.
,, $\quad \theta$ = Obliquity of reaction = $\angle D\,p\,N$.
,, $\quad R_1\,R_2$ = Radii of pitch circles, A and B.
,, $\quad r_1\,r_2$ = ,, base ,, ,,

Then, $\quad$ *Component of* R *along* M N = R cos θ.

Component of R *along* A B = R sin θ.

The former of these represents the effective pressure causing motion, while the latter tends to throw the wheels out of gear, and has, therefore, to be resisted by the bearings at A and B. To reduce this separating force to a minimum, θ must be made as small as possible. In practice, θ never exceeds $14\frac{1}{2}$° or 15°.

Taking θ at 15°, we can easily calculate the size of base circle for any given size of pitch circle.

Referring to the above figure, we see that :—

$$\angle p\,A\,C = \angle p\,B\,D = \theta.$$

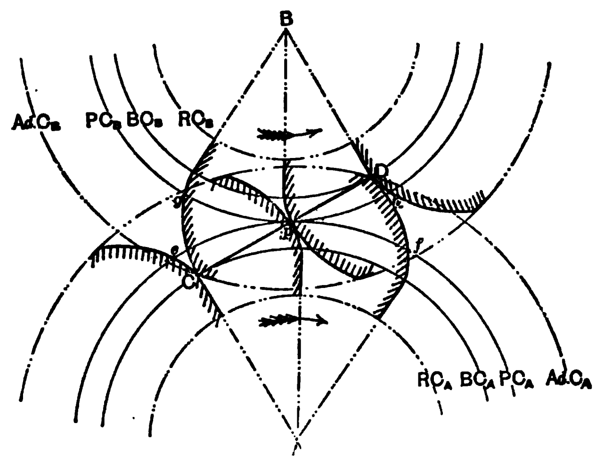
Hence, $\quad r_1 = R_1 \cos\theta.$
$\qquad\qquad r_2 = R_2 \cos\theta.$

But, $\quad \cos\theta = \cos 15^\circ = \cdot 966 \text{ or } \frac{63}{65}, \text{ nearly.}$

$$\therefore \qquad r_1 = \cdot 966\ R_1 = \frac{63}{65} R_1 \,. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{(I)}$$

Or, **Diameter of base circle** $= \frac{63}{65}$ **(diameter of pitch circle).**

Length of Involute Teeth for given Arcs of Approach and Recess.—With involute teeth the path of contact is a straight line, which is a tangent to the base circles. In the accompanying figure (which is much exaggerated for the sake of clearness) three pairs of teeth are shown in contact. One pair is beginning contact



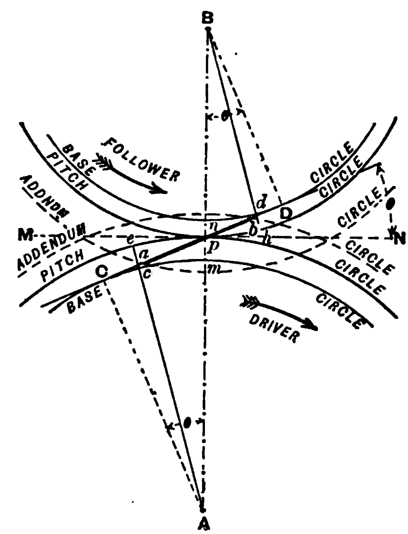
Length of Involute Teeth for given Arcs of Approach and Recess.

at C, a second pair is in contact at the pitch point, p, while the third pair is terminating contact at D. Contact cannot commence before C, nor be carried beyond D. If, therefore, the maximum length of contact be utilised, then the addendum circles for A and

B must be drawn through the points D and C respectively. After allowing for bottom clearance, the root circles, R C_A, R C_B, can be drawn.

From the figure it will be seen, that the root circles fall inside the base circles and are concentric with them. Now the curves forming the working surfaces of the teeth, being involutes of the base circles, do not pass beyond those base circles, and consequently those parts of the roots of the teeth lying between the base and root circles must be formed by some line straight or curved. Since, however, those parts are not portions of the working surfaces they are usually made straight and radial.

If arc $e\,p$ = arc $g\,p$ = arc of approach, and arc $p\,f$ = arc $p\,h$ = arc of recess, then, manifestly, e, f, g, and h are points of inter section of the curves of the teeth with the pitch circles.



To Find the Addenda of Involute Teeth.

We are now able to find the addenda of the teeth for given arcs of approach and recess; or conversely, to find the arcs of approach and recess when the addenda of the teeth are given.

Draw the pitch circles for the two wheels. Through p draw the line of mutual pressure, C D, making an angle, θ, with M N. As already explained, θ should not exceed 15°. With A and B as centres draw the base circles tangential to C D. Along pitch circle A set off arc $p\,e$ equal to given arc of approach, and on pitch circle B set off arc $p\,h$ equal to given arc of recess. Join A e, B h. These radii cut the base circles at c and d respectively. Then arc $c\,m$ is the arc turned through by base circle, A, during approach, while arc $n\,d$ is the arc turned through by base circle, B, during recess.

Along C D set off $p\,a$ = arc $c\,m$, and $p\,b$ = arc $d\,n$. Then the straight line $a\,p\,b$ is the path of contact.

Through b and a draw the addendum circles as shown. After allowing for clearance the root circles can be drawn in.

The solution of the converse proposition should not present much difficulty to the student.

Calculation of the Length of Involute Teeth.—Suppose wheel A to be the driver.

Let R_1, R_2 = Radii of pitch circles of wheel A and B.
" n_1, n_2 = Number of teeth on " "
" p = Pitch of teeth.
" δ_1, δ_2 = Addenda of teeth on A and B.
" α, β = Arcs of approach and recess respectively.
" θ = Obliquity of mutual pressure.

Referring to the previous figure, join A to b,* then, A b C is a right-angled triangle.

$$\therefore \quad A\,b^2 = A\,C^2 + C\,b^2.$$

But, $$A\,b = R_1 + \delta_1,$$
$$A\,C = R_1 \cos\theta,$$

And, $$C\,b = C\,p + p\,b = R_1 \sin\theta + p\,b.$$

Now, $$p\,b = \text{path of recess}$$

$$\therefore \quad \text{"} = \text{arc } n\,d \text{ of base circle B.}$$

But, $$\frac{\text{Arc } n\,d}{\text{Arc } p\,h} = \frac{\text{Radius of base circle B}}{\text{Radius of pitch circle B}} = \cos\theta.$$

$$\therefore \quad \text{Arc } n\,d = \beta \cos\theta.$$

$$\therefore \quad C\,b = R_1 \sin\theta + \beta\cos\theta.$$

$$\therefore \quad (R_1 + \delta_1)^2 = R_1^2 \cos^2\theta + (R_1 \sin\theta + \beta\cos\theta)^2.$$

$$\therefore \quad 2\,R_1\,\delta_1 + \delta_1^2 = 2\,R_1\,\beta \sin\theta\cos\theta + \beta^2\cos^2\theta$$
$$\text{"} \quad = R_1\,\beta\sin 2\theta + \beta^2\cos^2\theta.$$

$$\left.\begin{aligned} \text{Or,} \quad 2\,R_1\,\delta_1 + \delta_1^2 &= (R_1 \sin 2\theta + \beta\cos^2\theta)\,\beta \\ \text{Similarly,} \quad 2\,R_2\,\delta_2 + \delta_2^2 &= (R_2 \sin 2\theta + \alpha\cos^2\theta)\,\alpha \end{aligned}\right\} \quad . \quad . \quad \text{(II)}$$

Generally δ is small compared with R, therefore we may neglect δ^2, and we get the approximate formulæ :—

$$\left.\begin{aligned} 2\,R_1\,\delta_1 &= (R_1 \sin 2\theta + \beta\cos^2\theta)\,\beta \\ 2\,R_2\,\delta_2 &= (R_2 \sin 2\theta + \alpha\cos^2\theta)\,\alpha \end{aligned}\right\} \quad . \quad \text{(III)}$$

* The line A b has been accidently omitted, and the student should now draw it on the figure before proceeding further.

Again, since $R_1 = \frac{n_1 p}{2\pi}$, $R_2 = \frac{n_2 p}{2\pi}$, and usually $\theta = 15°$.

$$\therefore \qquad \sin 2\theta = \sin 30° = \frac{1}{2}$$

And,
$$\cos^2\theta = \cos^2 15° = \frac{2+\sqrt{3}}{4}.$$

Making these substitutions in (III), we get :—

$$2 \times \frac{n_1 p}{2\pi} \times \delta_1 = \left(\frac{n_1 p}{2\pi} \times \frac{1}{2} + \beta \times \frac{2+\sqrt{3}}{4}\right)\beta.$$

$$\therefore \qquad n_1 p\, \delta_1 = \left(\frac{n_1 p}{4} + \beta \times \frac{3{\cdot}732 \times 3{\cdot}1416}{4}\right)\beta$$

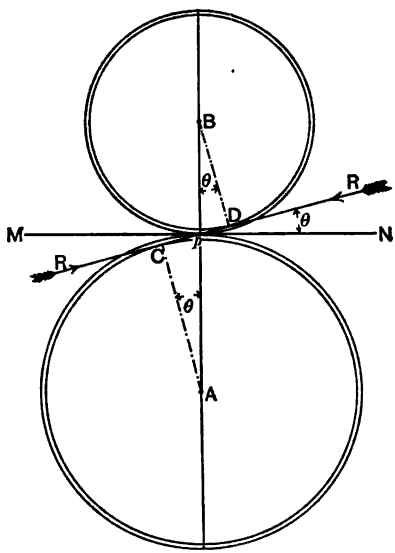
$$,, \quad = \left(\frac{n_1 p}{4} + 2{\cdot}931\,\beta\right)\beta.$$

$$\therefore \qquad \left.\begin{aligned} \delta_1 &= \left(\frac{1}{4} + \frac{3\beta}{n_1 p}\right)\beta \\ \text{Similarly,} \quad \delta_2 &= \left(\frac{1}{4} + \frac{3\alpha}{n_2 p}\right)\alpha \end{aligned}\right\} \quad \ldots\ldots\ldots \quad \text{(IV)}$$

Least Number of Involute Teeth to be placed upon a Wheel.—Let A and B be the centres of a pair of wheels in gear. Set out the obliquity line, C D, making an angle, θ, with the common tangent, M N, to the pitch circles. Draw the base circles tangential to C D. Then C D is the maximum length of path of contact.

Let B be the smaller wheel, then p D is less than p C.

Now, in order that there may not be less than two pairs of teeth in contact at any instant, it is clear that if one pair of teeth be just ending contact at D, another pair must be just



To Find the Least Number of Involute Teeth for a Wheel.

beginning contact at some point near C, while an intermediate pair is in contact at p.

Let r = Radius of base circle of smaller wheel B.
,, n = Least number of teeth on ,, ,,

Then the minimum number of teeth on B will occur when the whole path p D is utilised and when θ has its maximum value.

Hence, $n \times p\,\mathrm{D}$ = Circumference of base circle B $= 2\pi r$.

But, $p\,\mathrm{D} = \mathrm{B\,D}\tan\theta = r\tan\theta$.

$\therefore \quad n\,r\tan\theta = 2\pi r$.

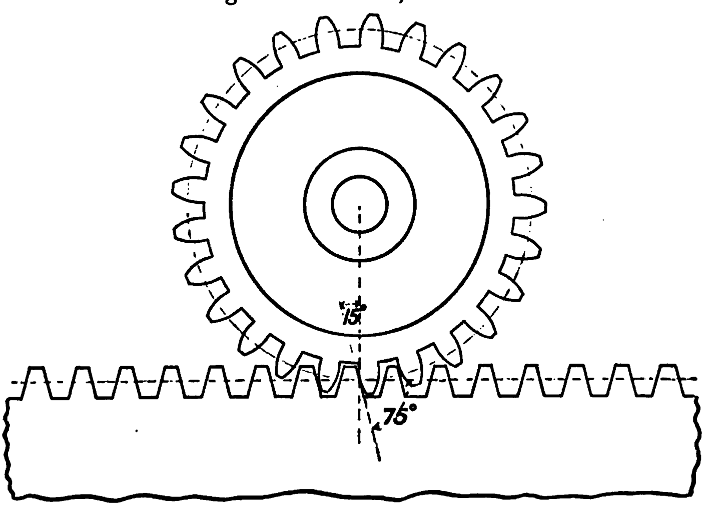
$$\therefore \quad n = \frac{2\pi}{\tan\theta}.$$

Since θ must not exceed 15°, we get :—

$$n = \frac{2 \times 3{\cdot}1416}{{\cdot}2679} = 23{\cdot}45.$$

i.e., The least number of involute teeth to be placed upon a wheel is 24.

Rack and Pinion with Involute Teeth.—When a pinion with involute teeth has to gear with a rack, then the teeth of the latter



RACK AND PINION WITH INVOLUTE TEETH.

must also be involute. Now, the pitch circle of the rack is infinite in size, hence its base circle is also infinite in size, and, therefore,

the involute corresponding to it will be a straight line. Hence, the face and flank of a tooth on the rack will be a straight line.

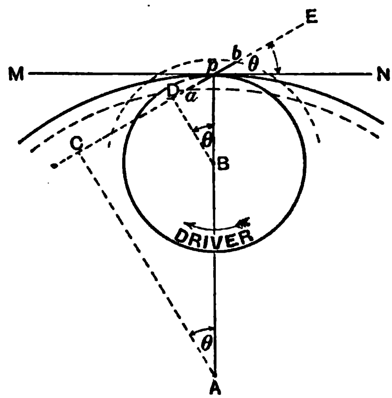
Since the working surface of the teeth during contact must be perpendicular to the line of obliquity; and, since this line makes an angle of about 15° with the common tangent to the pitch circles, it follows that the working surfaces of the teeth on the rack (being straight) make constant angles of 75° with this common tangent. And clearly, in the case of a rack and pinion, the common tangent coincides with the pitch line of the rack.

The previous figure represents a rack and pinion with involute teeth.

Wheels with Involute Teeth and Internal Contact.—Draw the pitch circles of the annular wheel, A, and its pinion, B. Let M N be the common tangent to the pitch circles at the pitch point, p. Through p draw the obliquity line, C p E, making an angle, θ, with M N.

From A and B draw the perpendiculars A C, B D upon C E. These are radii of the base circles for wheels A and B respectively.

Next draw in the addendum circles for the two wheels as indicated. Let these circles intersect the obliquity line in the points a and b respectively. Then, for the direction of motion shown in the figure, $a\,b$ will be the path of contact, $a\,p$ the path of approach, and $p\,b$ the path of recess.



WHEELS WITH INVOLUTE TEETH AND INTERNAL CONTACT.

The preceding principles for wheels with external contact apply equally to the case of wheels with internal contact, so that the student should not experience much difficulty in applying them. He should, however, note that the base circle for the annular wheel must be less than its addendum circle, which, in this case, is inside the pitch circle.

Calculations for Involute Teeth with Internal Contact.—The student should now prove the following formulæ for internal gearing, the method of arriving at the results being similar to that previously given.

Let the symbols with the suffixes 1 and 2 refer to the driver and follower respectively. Then :—

(1) *When the Pinion is the Driver.*

$$\left.\begin{aligned} 2\,R_1\,\delta_1 + \delta_1^2 &= (R_1 \sin 2\,\theta + \beta \cos^2 \theta)\,\beta \\ 2\,R_2\,\delta_2 - \delta_2^2 &= (R_2 \sin 2\,\theta - \alpha \cos^2 \theta)\,\alpha \end{aligned}\right\} \quad . \quad . \quad . \quad (V)$$

(2) *When the Annular Wheel is the Driver.*

$$\left.\begin{aligned} 2\,R_1\,\delta_1 - \delta_1^2 &= (R_1 \sin 2\,\theta - \beta \cos^2 \theta)\,\beta \\ 2\,R_2\,\delta_2 + \delta_2^2 &= (R_2 \sin 2\,\theta + \alpha \cos^2 \theta)\,\alpha \end{aligned}\right\} \quad . \quad . \quad . \quad (V_a)$$

The approximate formulæ corresponding to these are :—

(1)
$$\left.\begin{aligned} \delta_1 &= \left(\frac{1}{4} + \frac{3\,\beta}{n_1\,p}\right)\beta \\ \delta_2 &= \left(\frac{1}{4} - \frac{3\,\alpha}{n_2\,p}\right)\alpha \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (VI)$$

(2)
$$\left.\begin{aligned} \delta_1 &= \left(\frac{1}{4} - \frac{3\,\beta}{n_1\,p}\right)\beta \\ \delta_2 &= \left(\frac{1}{4} + \frac{3\,\alpha}{n_2\,p}\right)\alpha \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (VI_a)$$

The student should also notice that the formulæ for internal gearing are at once deduced from those for external gearing by considering the radius of the annular wheel as negative.

EXAMPLE I.—A pair of wheels with involute teeth have 30 and 120 teeth respectively. The addendum to each wheel is $\frac{3}{10}$ pitch. Find the lengths of the arcs of approach and recess, supposing the obliquity to be 15°, and the small wheel the driver.

ANSWER.—Here $n_1 = 30$; $n_2 = 120$; $\delta_1 = \delta_2 = \frac{3}{10}\,p$.

From equation (IV), we get :—

$$\delta_2 = \left(\frac{1}{4} + \frac{3\,\alpha}{n_2\,p}\right)\alpha.$$

$$\therefore \qquad \frac{3}{10}\,p = \left(\frac{1}{4} + \frac{3\,\alpha}{120\,p}\right)\alpha.$$

$$\therefore \quad \alpha^2 + 10\,p\,\alpha - 12\,p^2 = 0.$$

$$\therefore \qquad \alpha = \frac{-\,10 \pm \sqrt{10^2 + 4 \times 12}}{2}\,p = 1{\cdot}08\,p.*$$

i.e., **Arc of approach = 1·08 × pitch.**

* The positive sign for the radical must be taken, since neither α nor β can be negative.

Again, $$\delta_1 = \left(\frac{1}{4} + \frac{3\,\beta}{n_1\,p}\right)\beta$$

$$\therefore \quad \frac{3}{10}p = \left(\frac{1}{4} + \frac{3\,\beta}{30\,p}\right)\beta.$$

$$\therefore \quad 2\,\beta^2 + 5\,p\,\beta - 6\,p^2 = 0.$$

$$\therefore \quad \beta = \frac{-5 \pm \sqrt{5^2 + 4 \times 2 \times 6}}{2 \times 2}\,p = \cdot 885\,p.^*$$

i.e., **Arc of Recess = ·885 × pitch.**

Example II.—With the same sizes of wheels as in last example, find the addenda of the wheels, the arcs of approach and recess being each $\frac{7}{8}$ of the pitch.

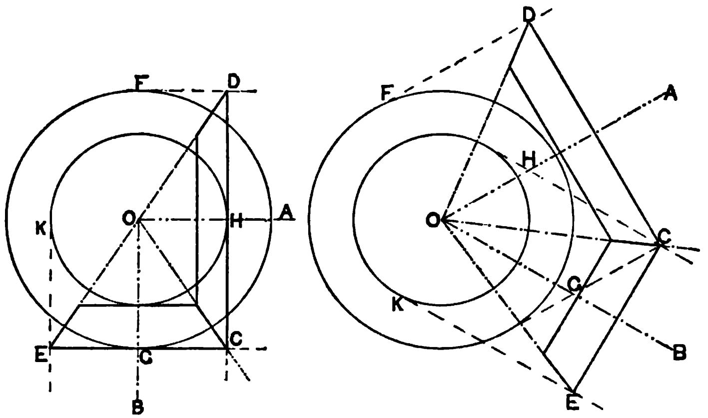
Answer.—Here $\alpha = \beta = \frac{7}{8}\,p$.

From equation (IV), we get :—

$$\delta_1 = \left(\frac{1}{4} + \frac{3 \times \frac{7}{8}\,p}{30\,p}\right) \times \frac{7}{8}\,p = \cdot 295 \times \textbf{pitch}.$$

Again, $$\delta_2 = \left(\frac{1}{4} + \frac{3 \times \frac{7}{8}\,p}{120\,p}\right) \times \frac{7}{8}\,p = \cdot 237 \times \textbf{pitch}.$$

Bevel Wheels.—The teeth on bevel wheels are constructed upon precisely the same principles as those on spur wheels. In the case of bevel wheels, however, the pitch surfaces are conical,



Method of Setting out the Pitch Cones for Bevel Wheels.

* The positive sign for the radical must be taken, since neither α nor β can be negative.

and on this account it becomes necessary to give a brief description of the application of the preceding principles to such cases. In the first place we shall explain how to set out the pitch cones for a pair of bevel wheels whose angular velocity-ratio is given.

Two principal cases are shown by the foregoing figure—(1) when the axes of the shafts intersect at right angles, and (2) when they intersect at an acute angle. The letters on the two diagrams are so arranged that the following description is applicable to both.

Let O be the intersection of the axes, O A, O B, of the shafts. Suppose the angular velocity-ratio to be 2 : 3; *i.e.*, let:—

Angular velocity of shaft, O A : *Angular velocity of shaft*, O B = 2 : 3.

When the size of one of the wheels is given, that of the other wheel can be found in the usual way when the angular velocity-ratio is known. Thus:—

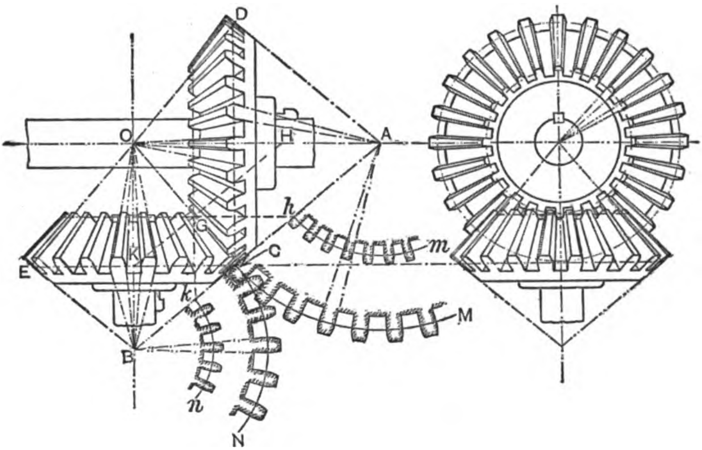
Diameter of wheel on shaft, O A : *Diameter of wheel on shaft*, O B = 3 : 2.

With centre, O, draw two circles of diameters equal or proportional to those of the wheels. From the larger circle draw the tangents, F D, G C, parallel to the axis, O A; and from the smaller circle draw the tangents, H C, K E, parallel to the axis, O B. The tangents, G C, H C, intersect at C. The line, O C, is then the line of contact of the two pitch cones. By drawing C D and C E perpendicular to the axes, O A, O B, respectively, and meeting the tangents, F D, K E, in the points, D and E, and by joining O D, O E we get the complete pitch cones, C O D, C O E. In practice, frusta only of the pitch cones are used. These are shown by full lines on the figures.

Another method of setting out the pitch cones is as follows:— Along the axes, O A, O B, measure off distances, O H, O G, respectively proportional to the angular velocities of the shafts, O A and O B; *i.e.*, in this particular case, let O H : O G = 2 : 3. Complete the parallelogram, O H C G; then the diagonal, O C, is the line of contact of the pitch cones as before. The pitch cones can then be completed by making $\angle$ A O D = $\angle$ A O C, and $\angle$ B O E = $\angle$ B O C. The remainder of the construction is obvious.

Teeth of Bevel Wheels.—We shall now give a brief explanation of the usual method adopted in setting out the teeth for a pair of bevel wheels in gear. Let O A, O B be the axes of the shafts. Having drawn the pitch cones, C O D, C O E, draw A C B through C perpendicular to the line of contact, O C, and join A D, B E. Imagine C A D, C B E to be conical surfaces whose vertices are A and B respectively. Now, if we further imagine these conical

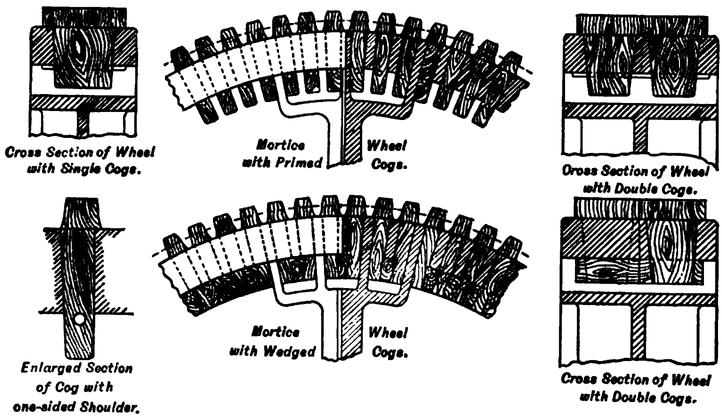
surfaces *developed*—that is, flattened out into circular segments, A C M, B C N—then these segments would roll, without slipping, upon their circular edges, C M, C N, during the rotation of the wheels. Hence, we may look upon A C M, B C N as portions of the pitch circles, with reference to which the *outer* ends of the teeth are described. On these *virtual* pitch circles the curves for the outer ends of the teeth are to be described in the usual way. The teeth may be either cycloidal or involute. From the figures it will be seen that the teeth taper towards the point, O.



METHOD OF SETTING OUT CURVES FOR TEETH ON BEVEL WHEELS.

In a similar way to the above we could draw the virtual pitch circles for the inner ends of the conical frusta, and then construct the curves for the inner ends of the teeth. These inner virtual pitch circles are usually drawn concentric with the outer ones as follows:—Let O G be the breadth of the face of the wheels. Through G draw H G K perpendicular to O G, and meeting O A in H and O B in K. Then, just as before, H G and K G are the radii of the virtual pitch circles for the smaller ends of the frusta. It is more convenient to draw these circles about A and B as centres than about H and K as centres. Hence project G on to A C and B C, by drawing G h, G k parallel to O A and O B respectively. Then, A h = H G, and B k = K G. With A and B as centres, draw the virtual pitch circles h m, k n. The curves for the inner ends of the teeth are set out on these virtual pitch circles. The dimensions of the teeth on h m, k n are reduced in the proportion A h : A C, or B k : B C.

Mortice Wheels.—When ordinary toothed wheels are run at a high speed their mutual actions become very rough and noisy, and severe vibrations are usually set up. To obtain a smoother and more regular action the method is sometimes adopted of placing wooden teeth on one of each pair of wheels in gear. These wooden teeth, or "*cogs*" as they are called, are *morticed* into the iron rims of the wheels, and hence such wheels are termed **mortice wheels.** The action of mortice wheels is very smooth and noiseless. Some of the common methods of securing the cogs to the rims are shown by the accompanying figures. In the upper



MORTICE WHEELS.

figures the cogs are secured by pegs or pins passing through the tenons of the cogs in a direction parallel to their length. In the lower figures they are shown secured by dove-tailed wooden keys driven into correspondingly-shaped grooves at the projecting ends of the tenons. Double cogs are sometimes used when the wheels are very broad; these are shown by the cross sectional views on the right-hand side. The cogs are usually made with side shoulders, as shown by the longitudinal sections of the rims of the wheels, but occasionally there is only one side shoulder, as shown by the lower left-hand figure.

The cogs may be cycloidal or involute, according to the shape of the iron teeth in gear with them, and are usually shaped by hand. To prevent undue wear of the cogs by the iron teeth, it is usual to "pitch" and "trim" the acting surfaces of the latter by carefully chipping, and afterwards filing, them to a high degree of smoothness. With machine-moulded teeth, which are much

smoother, more uniform and perfect in shape than those cast from a pattern in the ordinary way, it is sufficient to merely file the teeth to smoothness.

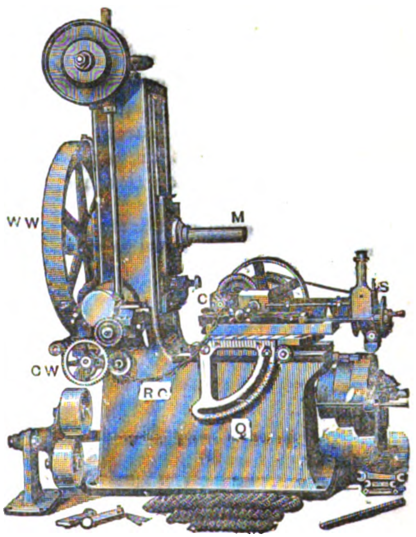
The cogs are made of hard, tough wood, such as oak, beech, hornbeam, holly, or apple tree. Since the material of the cogs is softer and weaker than that of cast iron, it follows, as a matter of economical distribution of material, that the thickness of the cogs should be greater than that of the iron teeth. The following are the usual proportions for the teeth of mortice gearing:—

Thickness of wood cogs at pitch circle		= 0·60 × pitch.		
„	*iron teeth*	„	= 0·40	„
Height of teeth above		„	= 0·25	„
Depth „	*below*	„	= 0·30	„

From these proportions it will be seen that there is no side clearance between the teeth when new.

Bevel wheels can also be fitted with wooden cogs, which are secured to the rim by methods similar to those for spur wheels.

Gear Cutting Machine.—It is now becoming more and more common to *cut* the teeth of wheels from a plain blank, and not to *cast* them. The involute form of tooth is especially suitable for this, as each side of the tooth is one continuous curve. With machine-cut teeth there is no side clearance, and, therefore, no back lash. The figure shows a machine for cutting wheel teeth, as made by Messrs. John Lang & Sons of Johnstone (who have kindly supplied the illustration). The wheel to be cut is placed on the mandril, M, which carries a worm wheel, W W, on its back end. (This worm wheel is covered by a guard, which prevents its teeth being seen in the figure.) The bearing for the mandril can be



Gear Cutting Machine.

moved up and down between the vertical guides by a screw and hand-wheel, and the height of M is thus adjusted to suit wheels of different diameters. The cutter is ground to the exact form of the spaces between the teeth, and is fixed to a spindle carried by the slide, S, as shown at C. As the cutter rotates it is advanced by a screw, and cuts a space right across the rim of the blank. When it has gone through, the slide shifts a tappet and reverses the motion of the screw. The cutter is then moved back and is ready to cut another space. The wheel being cut is turned through the space of the pitch of its teeth by the worm wheel, W W, and change wheels, C W. The screw, gearing with the worm wheel, carries a small friction clutch which is always in motion, but while the cutter is in action, the screw is prevented from rotating by a hook and cam at the end of the train of change wheels. Every time the slide nears the end of its back stroke it pulls away this hook by the releasing chain, R C, and so releases the cam. The friction clutch then turns the screw and worm wheel until again stopped. As the cam can only make exactly one revolution before being again stopped by the hook, the angle through which the mandril, and wheel being cut, turn, can be made anything required by putting in a suitable train of change wheels. The machine is driven by a belt in the usual way and is self-acting. When used for cutting bevel wheels the table carrying the slide and cutter is inclined by the quadrant, Q, to suit the angle of the face of the wheel. As the teeth of bevel wheels are not of the same size at their inner and outer ends, they cannot be made at one cut, but each side has to be cut separately.*

* The student should see *The Practical Engineer* of 26th July, 1895, p. 60, for a paper on "Cutting Bevel Gears in a Universal Milling Machine."

Lecture XV.—Questions.

1. A pair of wheels have 25 and 130 involute teeth respectively. The addenda of the teeth in both cases is $\frac{1}{3}$ the pitch. Find the lengths of the arcs of approach and recess, assuming the obliquity to be 15°, and the larger wheel the driver. *Ans.* $\alpha = \cdot 925\,p$; $\beta = 1\cdot 19\,p$.

2. Prove the formulæ for the addenda of involute teeth in terms of the arcs of approach and recess, &c. Hence show that, if the arcs of approach and recess are each equal to the pitch, the addenda should be calculated from the formula:—Addendum $= \left(\frac{1}{4} + \frac{3}{n}\right) \times$ pitch, where n represents the number of teeth on wheel.

3. What are bevel wheels? Two axes intersect at an angle of 60°, and it is required to connect them by bevel gearing, so that their angular velocities shall be as 3 : 2. Construct the pitch cones of the bevel wheels.

4. Under what conditions will two cones roll together? Motion is to be communicated between two shafts inclined at an angle of 90°, and one is to make three rotations while the other makes four. Set out the pitch cones in a diagram, marking dimensions. (S. and A. Exam., 1689.)

5. Two shafts intersecting at right angles are connected by bevel wheels with 22 and 44 teeth respectively, of 1 inch pitch. Draw, to a scale of $\frac{1}{4}$, the pitch surfaces of the wheels, and find the development of the conical surfaces on which the shape of the ends of the teeth are set out. Having given the shape of the end of a tooth, explain how the shape of the surface of the tooth is determined. (S. and A. Mach. Const. Adv. Exam., 1891.)

6. Draw the pitch cones for two bevel wheels in gear, having 60 and 45 teeth respectively with $2\frac{1}{2}$ inches pitch, measured at larger end of conical frustum, the shafts to make an angle of 60° with each other. Explain fully the method adopted by engineers in setting out the teeth for a pair of bevel wheels, by applying it to the example given.

7. Sketch to scale a pair of bevel wheels in gear with each other, the gearing ratio to be 3 to 1, the mean pitch to be $1\frac{1}{4}$ inches, and the number of teeth in the smaller wheel to be 16. Make sufficient sketches to show the construction of the wheels and the shape of the teeth fully. Given the shape of the tooth of a rack in a set of interchangeable wheels, show how to develop by graphic construction the proper shape of tooth for a wheel of any given number of teeth. (C. and G. of L. Mech. Eng. Hons. Exam., 1884.)

8. Distinguish between a bevel wheel, a mitre wheel, and a mortice wheel. Draw a section of two mitre wheels in gear. Sketch a mortice tooth for (1) a spur wheel, (2) a bevel wheel, and describe with sketches two methods of fixing it in position.

9. Give two views of a tooth of a mortice spur wheel, showing how it is fitted into the rim of the wheel and held in position. Under what circumstances would you use mortice wheels? (S. and A. Mach. Const. Adv. Exam., 1888.)

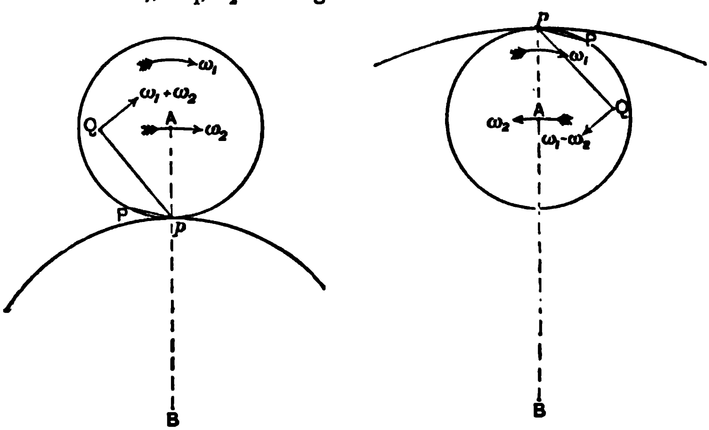
LECTURE XVI.

CONTENTS.—Friction of Toothed Gearing—The Principle of Combined Rotations—Example I.—Strength of Wheel Teeth—Case I.—Strength of Teeth when Contact between the Various Pairs of Teeth in Gear is Perfect—Case II.—Strength of Teeth when Contact is Imperfect—Breadth of Wheel Teeth—Example II.—Flanged or Shrouded Teeth—Hooke's Stepped Gearing—Helical Gearing—Double Helical Wheels—Questions.

Friction of Toothed Gearing.—The action between a pair of teeth in contact is partly rolling and partly sliding, but the frictional resistance of the former is small compared with that of the latter, and may, therefore, be conveniently neglected. The sliding between a pair of teeth takes place in a direction perpendicular to the common normal at their point of contact, and in order to find its amount we require to know the relative motion of the teeth in this direction. We proceed, in the first place, to determine this relative motion of the teeth, and in doing so shall make use of:—

The Principle of Combined Rotations.—Let A and B be the centres of two spur wheels in gear, p being the pitch point.

Let R_1, R_2 = Radii of wheels A and B.
,, ω_1, ω_2 = Angular velocities of A and B.



TO ILLUSTRATE THE PRINCIPLE OF COMBINED ROTATIONS.

The wheels will rotate in the opposite or in the same direction, according as the contact is external or internal. It is convenient to indicate this distinction by the sign attached to ω. Thus, in the case of external contact, let the angular velocities be $+ \omega_1$ and $- \omega_2$; for internal contact, let these quantities be $+ \omega_1$ and $+ \omega_2$.

The *relative* angular velocities of wheels A and B will not be altered, if to each we impart an equal angular velocity. Thus, suppose each of the wheels in external contact receives an angular velocity $+ \omega_2$ about the axis, B. Then the angular velocity of B will be $\omega_2 - \omega_2 = 0$; *i.e.*, it will be at rest. The wheel A will be rotating about B with angular velocity, ω_2, and about its own axis with angular velocity, ω_1. Hence, the *resultant* angular velocity of wheel A is $(\omega_1 + \omega_2)$ about some axis which we are about to determine. During the rotation of the wheel A about B, that line on A, which is, for the instant, in contact with B, is at rest. At that instant, the wheel A is rotating about this line as an *instantaneous axis*. Clearly, this instantaneous axis passes through the pitch point, p, and is parallel to the axis of A or B. Hence, at a given instant any point, P, on wheel A is rotating about the axis through p, with an angular velocity $(\omega_1 + \omega_2)$.

By similar reasoning, if each of the wheels in internal contact receive an angular $- \omega_2$, then wheel B will be brought to rest, and any point on wheel A will be rotating about the instantaneous axis through p, with an angular velocity $(\omega_1 - \omega_2)$.

If, then, P be a point of contact between a pair of teeth, Pp is the direction of the common normal to the two teeth at that point, and the velocity of P perpendicular to Pp is the velocity of sliding between the teeth.

Let V = Velocity of pitch circles.

,, v = Velocity of P perpendicular to Pp.

,, r = Distance of P from pitch point, p.

Then, from what has been demonstrated above, we get :—

Or, $v = (\omega_1 + \omega_2)\, r$, for external gearing,

,, $= (\omega_1 - \omega_2)\, r$, ,, internal ,,

But, $V = \omega_1 R_1, = \omega_2 R_2$.

$$\therefore \qquad v = \left\{ \frac{1}{R_1} \pm \frac{1}{R_2} \right\} r V.$$

Let arc p P be denoted by x, and suppose the wheels to receive a small circular displacement, $d\,x$, as measured along the pitch

circles. During this displacement, let ds represent the distance through which the teeth slide on one another.

Then, $$ds = \frac{dv}{dt} = \left\{\frac{1}{R_1} \pm \frac{1}{R_2}\right\} r \frac{dV}{dt},$$

$$,, = \left\{\frac{1}{R_1} \pm \frac{1}{R_2}\right\} r\,dx.$$

We may take r as being approximately constant, since, for a small movement ds, at right angles to pP, it does not perceptibly change.

Let P_n = Normal pressure between the teeth.
,, μ = Coefficient of friction.
,, α, β = Length of arcs of approach and recess respectively.

Then, *Work lost in friction during displacement*, dx, $= \mu P_n ds.$

$$,, \quad ,, \quad = \mu\left\{\frac{1}{R_1} \pm \frac{1}{R_2}\right\} P_n r\,dx.$$

∴ Total work lost in friction during whole arc of contact $= W = \mu\left\{\frac{1}{R_1} \pm \frac{1}{R_2}\right\}\left\{\int_0^\alpha P_n r\,dx + \int_0^\beta P_n r\,dx\right\}$ (I)

The law according to which P_n varies is not definitely known, being dependent upon the number of teeth in contact, the state of the teeth, and other causes. Its magnitude may not vary much, and probably has a mean value somewhere between $\frac{1}{2}$ P and P, where P denotes the driving or tangential force at the pitch circles of the wheels.

Taking $P_n = \frac{2}{3}$ P, which is quite a legitimate assumption, and putting chord pP = arc pP, or $r = x$, we get the following approximate equations:—

$$W = \tfrac{2}{3}\mu P\left\{\frac{1}{R_1} \pm \frac{1}{R_2}\right\}\left\{\int_0^\alpha x\,dx + \int_0^\beta x\,dx\right\}$$

$$,, = \tfrac{1}{3}\mu P\left\{\frac{1}{R_1} \pm \frac{1}{R_2}\right\}(\alpha^2 + \beta^2). \quad . \quad . \quad . \quad \text{(II)}$$

Let N_1, N_2 = Number of teeth on wheels A and B respectively.
,, p = Pitch of teeth.

Then, $$W = \tfrac{2}{3}\pi\mu P\left\{\frac{1}{N_1} \pm \frac{1}{N_2}\right\}\frac{\alpha^2 + \beta^2}{p}. \quad . \quad . \quad . \quad . \quad (\text{II}_a)$$

If, further, we suppose the arcs of approach and recess each equal to the pitch, we get:—

$$W = \tfrac{4}{3}\pi\mu P\left\{\frac{1}{N_1} \pm \frac{1}{N_2}\right\} p. \quad . \quad . \quad . \quad (\mathrm{II}_b)$$

From these equations, we learn that the greater the number of teeth (*i.e.*, the smaller the pitch), and the shorter the arcs of approach and recess, the smaller is the loss due to friction.

The above results are equally true for bevel gearing.

The loss due to the friction of toothed gearing is usually very small, being about 3 per cent. It has been stated by some authorities that the friction during approach is greater than that during recess. This explains why, in some kinds of wheelwork (such as in watches and clocks), the teeth are shaped so that there is no arc of approach.

EXAMPLE I.—In a pair of spur wheels with external contact, the number of teeth on the wheels is 30 and 70 respectively. Assuming the arcs of approach and recess each equal to the pitch of the teeth, and taking the coefficient of friction at ·1, find the efficiency of the gearing.

ANSWER.—The *work lost* by friction during the action between one pair of teeth is given by equation (II_b), viz.:—

$$W = \frac{4}{3}\pi\mu P\left\{\frac{1}{N_1} + \frac{1}{N_2}\right\} p.$$

The *total work expended* during the same action (arc of approach + arc of recess) is:—

$$W_T = P_n\,(\text{arc of approach} + \text{arc of recess})$$

$$,, \quad = \frac{2}{3}P \times 2p = \frac{4}{3}Pp.$$

$$\therefore \quad \textit{Efficiency} = \frac{\textit{Useful work done}}{\textit{Total work expended}}$$

$$,, \quad = \frac{W_T - W}{W_T} = 1 - \frac{W}{W_T}$$

$$,, \quad = 1 - \pi\mu\left\{\frac{1}{N_1} + \frac{1}{N_2}\right\}$$

$$,, \quad = 1 - \frac{22}{7} \times \frac{1}{10}\left\{\frac{1}{70} + \frac{1}{30}\right\}.$$

Or, **Efficiency = 1 − ·015 = ·985, or, 98·5 per cent.**

This example serves to show how small is the loss due to the friction of the teeth of wheels.

Strength of Wheel Teeth.—The power which can be transmitted by toothed gearing depends upon the circumferential speed of the wheels and the strength of the teeth. Thus :—

Let H.P. = Horse-power transmitted.
,, P = Tangential pressure in lbs. at pitch circle.
,, V = Velocity in feet per minute at pitch circle.

Then, $$\text{H.P.} = \frac{\text{P V}}{33{,}000} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{(III)}$$

We now proceed to determine P in terms of the dimensions of the teeth, &c.

The strength of a tooth depends upon the manner in which the pressure on that tooth is distributed, and this latter depends upon the accuracy with which the wheels are made and adjusted in gear. Two cases occur, according as the contact between a pair of teeth is perfect or imperfect. We shall consider these cases separately.

Case I.—**Strength of Teeth when Contact between the Various Pairs of Teeth in Gear is Perfect.**—When the wheels are accurately adjusted in gear, and the teeth well formed, any pair of teeth should be in contact along a line across the breadth of the teeth. In such a case the mutual pressure between the teeth will probably be uniformly distributed along that line. Let this line be taken at the end or point of the tooth which we are about to consider, so that the bending moment due to the distributed pressure may be a maximum. We may also neglect the curved form of the tooth, and assume it to be a rectangular block fixed to the rim of the wheel. It may then be looked upon as a short beam fixed at one end (the root) and loaded uniformly along a transverse line at the other or free end.

Let P_n = Total pressure acting on the tooth.
,, b, l, t = Breadth, length, and thickness of the tooth.
,, f = Safe stress for the material.

If the material of the tooth be of uniform strength throughout, then the tendency of P_n will be to break the tooth along the root E F G H.

The bending moment at section E F G H is, B.M. = $P_n\,l$.

The resisting moment offered by the material at section E F G H is :—

$$\text{R.M.} = \frac{1}{6} b\, t^2 f.$$

$$\therefore \qquad P_n\, l = \frac{1}{6} b\, t^2 f.$$

Or,
$$P_n = \frac{1}{6}\left(\frac{b\, t^2}{l}\right) f.$$

Now, the magnitude of P_n is not exactly known; but, if there are never fewer than two pairs of teeth in contact at once, it seems quite a fair assumption to take $P_n = \frac{2}{3}$ P. Hence:—

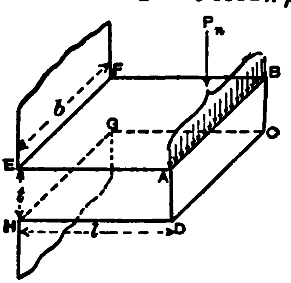
$$P = \frac{1}{4}\left(\frac{b\, t^2}{l}\right) f \quad . \quad . \quad . \quad . \quad . \quad . \quad (\text{IV})$$

Usually the dimensions of a tooth are stated in terms of the pitch of the teeth, and the ordinary proportions for *new* teeth were stated at the beginning of Lecture XIII. Making an allowance for wear, we may take:—

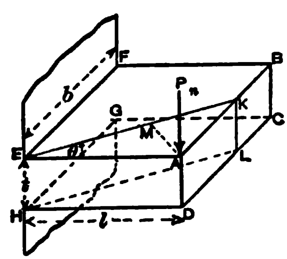
$l = \cdot 7\, p$ for iron teeth. $\qquad t = \cdot 36\, p$ for iron teeth.
$l = \cdot 6\, p$ for wooden teeth. $\qquad t = \cdot 45\, p$ for wooden teeth.

The breadth, b, varies considerably, the average being, $b = 2 \cdot 5\, p$. In the meantime, denote the breadth by $n\, p$. Making these substitutions in equation (IV), we get:—

$$\left.\begin{array}{l} P = 0\cdot 0463\, n\, p^2 f, \textit{ for iron teeth} \\ P = 0\cdot 0844\, n\, p^2 f, \textit{ for wooden teeth} \end{array}\right\} \quad . \quad . \quad . \quad . \quad (\text{V})$$



ILLUSTRATING DISTRIBUTED PRESSURE ON TOOTH.



ILLUSTRATING CONCENTRATED PRESSURE ON TOOTH.

From these equations we see that ***the driving force*, P, *varies as the square of the pitch of the teeth.***

By substituting these values for P in equation (III), the maximum H.P. transmitted can be obtained.

CASE II.—**Strength of Teeth when Contact is Imperfect.**—When the wheels are badly adjusted in gear, or the teeth badly shaped, contact between the latter will most likely be very imperfect. Instead of the teeth bearing along a line, as in the case just considered, they may bear at a few points, or perhaps at one point only. This state of affairs may be caused by one or more of the following defects:—(1) in spur gearing, the shafts may not be strictly parallel; or, in bevel gearing, the shafts may not be coplanar—*i.e.*, in the same plane. Although these defects may not exist when the gearing is newly erected, the subsequent wear of the shaft bearings may ultimately bring about this state of affairs. (2) The severe stresses to which the parts of the gearing (especially at the shaft supports) are sometimes subjected cause imperfect contact between the teeth. (3) The teeth may have been badly shaped to begin with. With wheels which have been moulded from a pattern in the ordinary way, the teeth are slightly tapered across their breadth, caused by the pattern which is purposely made thus to allow its being withdrawn from the mould. Hence, care is needed in the erection of such wheels, to see that they are so placed that the thick parts of the teeth on the one come in contact with the thin parts of the teeth on the other. Attention to this rule is not always given. This defect does not exist with machine-moulded or machine-cut teeth.

The worst, and most likely, case occurs when the one tooth presses upon a corner of the other. We shall, therefore, consider this case.

Let P_n act at the corner A. Then its tendency is to break off a triangular portion, E A K, along a section, E K L H, passing through E H.

Let θ = angle A E K. Draw A M perpendicular to E K.

Then bending moment about section, E K L H, is:—

$$\text{B.M.} = P_n \times AM = P_n l \sin\theta.$$

Resisting moment of material at section, E K L H, is:—

$$\text{R.M.} = \frac{1}{6}.\,EK\,.\,t^2 f = \frac{1}{6}.\,l\sec\theta\,.\,t^2 f.$$

$$\therefore \quad \frac{1}{6}\,l\sec\theta\,.\,t^2 f = P_n l\sin\theta$$

$$\therefore \quad f = \frac{3\,P_n}{t^2}.\sin 2\theta.$$

This gives the stress in the material along the section, EKLH, in terms of P_n, and will be a maximum when $\theta = 45^\circ$, or $\sin 2\theta = 1$. Hence, P_n tends to break off the portion, E A K, along a plane, E K L H, inclined at an angle of 45° with E A or E F.

$$\therefore \qquad P_n = \frac{1}{3} \cdot t^2 f.$$

Or, putting $P_n = \frac{2}{3}P$, we get :—$P = \frac{1}{2} \cdot t^2 f.$ . . . . (VI)

This equation shows that in this case P is independent of the breadth and length of the tooth.

Substituting for t its value in terms of the pitch, p, we get :—

$$\left.\begin{array}{l} P = 0{\cdot}065\, p^2 f, \textit{ for iron teeth} \\ P = 0{\cdot}100\, p^2 f, \quad \text{,,} \textit{ wooden ,,} \end{array}\right\} \quad . \; . \; . \quad \text{(VII)}$$

Equations (VII) again show that *the driving force*, P, *varies as the square of the pitch of the teeth.*

From what has been said above regarding the uncertainty of the distribution of the pressure on the teeth, it will be evident that the results expressed by equations (VII) should be taken in the design of wheel teeth.

Hence, combining equations (III) and (VII) we get the following :—**When the teeth of different wheels are proportioned according to the same rules, the power which they are capable of transmitting is proportional to the pitch circle velocity and to the square of the pitch of the teeth.**

$$\text{Or,} \qquad \text{H.P.} \propto V p^2.$$

$$\therefore \qquad p \propto \sqrt{\frac{\text{H.P.}}{V}}.$$

The value of f in equations (VII) varies according to circumstances. In machinery subjected to shocks, vibrations, or sudden reversals (as in pumping and rolling-mill gears) a larger factor of safety, and, therefore, a smaller value of f must be employed, than in those other cases (such as hand-worked or slow moving machinery), which are not so severely stressed. The following average values for f are given by Prof. Unwin :—

Iron teeth subjected to little shock, f = 9,600 lbs. per sq. in.
,, ,, moderate ,, f = 6,100 ,, ,,
,, ,, excessive ,, f = 4,300 ,, ,,

For wooden teeth we may take $f = 2{,}740$ lbs. per square inch, since these should never be subjected to severe shocks.

Breadth of Wheel Teeth.—The greater the breadth of the teeth the greater is their durability. When, however, the teeth are made too broad, there is a difficulty in fixing the wheels accurately in gear, since the slightest amount out of truth may cause the mutual pressure between the teeth to act over a very limited area. The breadth varies from $2p$ to $4p$ in ordinary gearing, the average being $2{\cdot}5\,p$.

The above formulæ for the strength of teeth, though deduced for the case of spur wheels, are equally true for bevel gearing. In the latter case, however, the velocity, V, and the pitch, p, are to be measured at a pitch circle half way between the larger and smaller ends of the conical pitch frustum.

Example II.—A spur wheel of 2 inches pitch and 4 inches width of face transmits 30 H.P. when its pitch line velocity is 10 feet per second. What power could be transmitted by a spur wheel of 4 inches pitch and 8 inches width of face with a pitch line velocity of 3 feet per second? (S. and A. Mach. Const. Hons. Exam., 1881.)

Answer.—Let the various quantities in the two cases be distinguished by the suffixes 1 and 2 respectively.

From equation (III) $\quad \text{H.P.} = \dfrac{\text{P V}}{33{,}000}$.

Now, assuming contact between the teeth to be perfect, as explained in the text, we get:—

From equation (IV) $\quad \text{P} = \frac{1}{4}\left(\dfrac{b\,t^2}{l}\right)f.$

Or, " (V) $\quad \text{P} = {\cdot}0463\,b\,p\,f.$

$$\therefore \quad \text{H.P.} = \frac{{\cdot}0463}{33{,}000}\,f\,b\,p\,\text{V}.$$

Hence, assuming f to be the same in both cases, we get:—

$$\frac{\text{H.P.}_2}{\text{H.P.}_1} = \frac{b_2\,p_2\,\text{V}_2}{b_1\,p_1\,\text{V}_1}.$$

$$\therefore \quad \frac{\text{H.P.}_2}{30} = \frac{8 \times 4 \times (3 \times 60)}{4 \times 2 \times (10 \times 60)} = \frac{6}{5}.$$

$$\therefore \quad \text{H.P.}_2 = \frac{6}{5} \times 30 = 36.$$

Flanged or Shrouded Wheels.—Sometimes the rims of toothed wheels are made broader than the teeth, and extend to the pitch circle or even to the points of the teeth. The wheels in such cases are said to be "*flanged*" or "*shrouded*." Shrouding has the effect of increasing the strength of the teeth, and it is for this purpose they are so made. With ordinary proportions of teeth, shrouding to the points may have the effect of nearly doubling the strength of the teeth. It is clear, however, that only one of a pair of wheels can be strengthened in this way. When the two wheels in gear are about equal in size, both may be shrouded to near their pitch circles. In other cases, it is usual to fully shroud the smaller wheel only, since the teeth of this wheel are subjected to greater wear than those on the larger one. Sometimes both wheels are shrouded to the points of their teeth on one side only, the shrouded side of the one being opposite the unshrouded side of the other. This

Longitudinal Section of Wheel Shrouded to Points of Teeth. *Shrouded to the Points.* *Shrouded on one Side.* *Shrouded to Pitch Line.* *Longitudinal Section of Wheel Shrouded to the Pitch Line.*

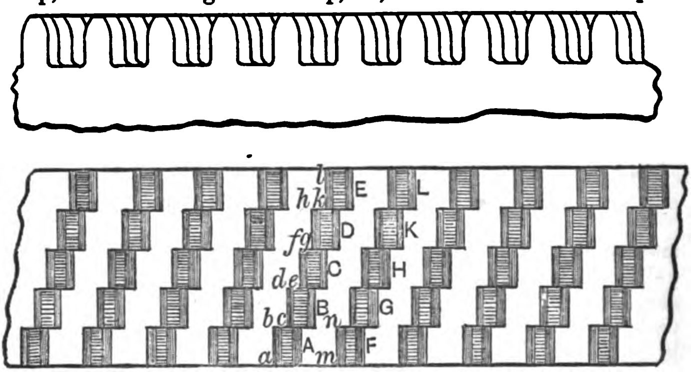
SHROUDED TEETH.

method is also adopted in those cases where the wheels are required to be thrown out of gear, by sliding one of the wheels along its shaft. These three methods of shrouding will be easily understood from the accompanying figures. Owing to the difficulty in moulding shrouded wheels, they are never adopted except in heavy machinery subjected to severe shocks.

Hooke's Stepped Gearing.—The smoothness of action with toothed gearing depends upon the number of pairs of teeth which are in contact simultaneously, the greater the number the sweeter and smoother the motion. This is of the greatest importance with some kinds of machinery, and noisy action should be avoided, as far as possible, in every case. We have seen in a previous Lecture that the number of pairs of teeth which are in action at once may be increased, either by reducing the pitch of the teeth, or by increasing the length of the path of contact. But reducing the pitch of the teeth reduces their strength, as we have just shown; and increasing the length of the path of contact causes an increase in the length of the teeth, which has also the effect of reducing their strength. Hence, neither of these methods can be advantageously adopted. To overcome these difficulties, Dr. Hooke invented his *stepped gearing*, which results in the smoothness of action due to fine pitched teeth without the reduction in strength.

To understand this form of gearing, imagine an ordinary spur

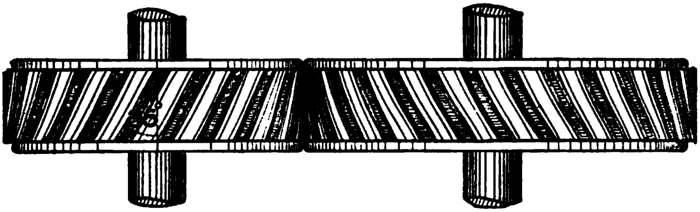
wheel built up of n (five on the figure) narrow spur wheels rigidly fixed together side by side, the pitch and other dimensions of the teeth being proportioned by the ordinary rules. Instead of the teeth being placed end to end in a straight line parallel to the shaft, let them be arranged in steps, as shown at A, B, C, D, E, so that each successive tooth is a short distance behind the previous one. The action with a pair of such wheels will clearly be similar to that of ordinary wheels wherein the pitch is only $b\,c$. The number of steps, n, and the length of the steps, $b\,c$, may be anything to suit circumstances, but the former is generally so arranged that the face of the last tooth, E, may just be to the left of the face of the first tooth, F, on the next series of teeth, by the length of a step, $b\,c$. The length of a step, $b\,c$, is then one nth of the pitch



Hooke's Stepped Gearing.

of the teeth, and should be small compared with the thickness of the teeth, otherwise the teeth will be weakened for want of sufficient connection with those on either side. If the teeth are designed in the usual way, so that two consecutive teeth on any of the rings composing a wheel are always in contact with two consecutive teeth on the corresponding ring of the other wheel, it is evident that for the two wheels there will never be fewer than $2\,n$ pairs of teeth in contact. Thus, under ordinary circumstances, with five steps on each wheel, there would always be at least ten pairs of teeth in contact. The motion would, therefore, be much smoother and sweeter than with ordinary gearing. Stepped gearing is sometimes used for the rack and pinion arrangements for moving the tables of planing machines, which require to be very uniformly and steadily moved. This form of gearing might be conveniently adopted in many other cases where regularity and smoothness of motion are of primary importance.

Helical Gearing.—If we suppose *b c* to become infinitely small, and, therefore, the number of steps, *n*, infinitely great, the broken line *a b c d e* . . . . would become a continuous curve, which is clearly a helix, or screw line, traced on the pitch surface of the wheel. The series of stepped teeth, A, B, C, D, E, would then form a single helical tooth. Wheels having their teeth formed in this manner are called **Helical Wheels.** When accurately made



SINGLE HELICAL WHEELS IN GEAR, SHOWING OPPOSITE OBLIQUITIES.

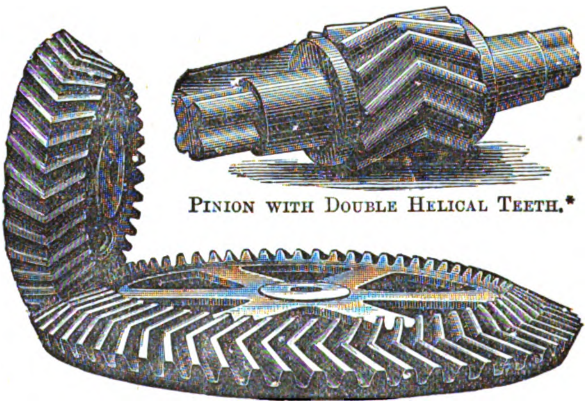
and erected, there is almost perfect line contact between the pairs of teeth in gear, thus augmenting the strength of the gearing, while the smoothness of action is about the greatest attainable with ordinary materials.

The obliquity of the teeth is the angle which their directions make with a plane containing the axis of the shaft, and in ordinary circumstances is about 35°. Just as screws are right-handed or left-handed, according to the direction of the helical thread on the bolt, so we speak of right-handed or left-handed obliquities with respect to helical teeth. A pair of helical wheels to gear together must have right- and left-handed obliquities respectively, as shown by the above figures.

Double Helical Wheels.—Owing to the oblique direction of the teeth their mutual pressure tends to separate the wheels axially, and thereby produces considerable lateral pressure on the bearings. To neutralise this prejudicial action, the teeth are now formed in two equal parts with opposite obliquities, and united at a common section, as shown by the accompanying figures. Such wheels are termed **Double Helical Wheels.** They are remarkable for their strength and smoothness of action. The pinion shown is shrouded to the pitch circle, which further increases its strength. No reliable information has as yet been forthcoming regarding the relative strength of helical and ordinary gearing, but it is believed that, under similar circumstances, the former are at least 30 per cent. stronger than the latter, and they are certainly more durable and easier in their action. It is, however, more difficult to obtain perfect bearing between the teeth of helical wheels than with

ordinary ones. The smallest displacement of the middle planes of the wheels from coincidence may cause the action between the teeth to be confined to one half of each tooth, thus virtually diminishing their strength. The method of overcoming this difficulty is to allow one of the shafts a small amount of lateral motion in its bearings, so that the wheel which it carries may always accommodate itself to the other.

Helical teeth are designed in the same way and according to the same rules as ordinary teeth, by setting off their dimensions,



PINION WITH DOUBLE HELICAL TEETH.*

BEVEL WHEELS WITH DOUBLE HELICAL TEETH.*

&c., along a pitch circle of the wheel. The teeth, however, must lie along the face of the wheel in a helical direction.

Helical wheels are extensively used in the construction of heavy gearing, such as that required for large cogging and rolling mills and submarine cable machinery, and they are said to give great satisfaction. The casting of such wheels presented considerable difficulties at first, but these have been overcome, and now helical bevel wheels can be as easily cast as spur wheels. Their cost is little more than that of ordinary toothed wheels.

* We are indebted to Messrs. P. R. Jackson & Co., Ltd., Manchester, and Messrs. Bodley Bros. & Co., Exeter, for these figures.

Lecture XVI.—Questions.

1. Deduce an expression for the work lost by friction between the teeth of a pair of wheels in gear. In a pair of spur wheels with external contact, the number of teeth on the wheels are 25 and 75 respectively, and the coefficient of friction is 0·1. Assuming the arcs of approach and recess each equal to the pitch, find the efficiency of the gearing. *Ans.* 98·3 °/ₒ.

2. In spur-wheel gearing, explain how to estimate the pitch of the teeth to transmit a given horse-power with a given speed of periphery. Show that under some circumstances the pitch should be proportional to $\sqrt{P}$, and under other circumstances to $\frac{P}{b}$, where P is the pressure between two teeth, and b is the breadth of the face of the teeth. (S. and A. Mach. Const. Hons. Exam., 1890.)

3. A toothed wheel, 18 inches diameter, makes 150 revolutions per minute, and transmits 30 horse-power, what is the maximum pressure on one tooth, assuming it to take the whole load? If the width of the teeth is 2 inches, what pitch should you adopt? (C. and G. of L. Mech. Eng. Hons. Exam., 1892.) *Ans.* $P = 1{,}400$ and $p = 1·4$ inches.

4. A cast-steel spur wheel transmits 80 horse-power. The pressure comes upon one tooth, and may be supposed to act uniformly along its point—that is to say, the tooth may be regarded as a cantilever loaded at the extremity. Diameter of spur wheel, 3 feet; number of revolutions per minute, 150; length of tooth, 1¼ inches; width of tooth, 4 inches; safe tensile and compressive strength of cast steel (allowing for vibrations) per square inch, 8,000 lbs. Find thickness of root of tooth. (C. and G. of L. Mech. Eng. Hons. Exam., 1890.) *Ans.* ·6 inch.

5. Sketch the rim of a spur wheel with shrouded teeth. Explain the object of shrouding the teeth.

6. Give sketches showing Hooke's stepped gearing, and explain its construction and action. What advantages are derived from making toothed gearing of this form?

7. Give sketches showing single and double helical wheels. Explain the principles upon which they are constructed and act. Why are double helical wheels used in preference to single helical wheels? Discuss the advantages and disadvantages of helical and ordinary toothed wheels.

LECTURE XVII.

CONTENTS.—Belt, Rope, and Chain Gearing — Materials for Belting — Curing, Cutting, and Splicing Leather for Belts—Different Methods of Jointing Leather Belts—Average Strength of Leather Belt Joints—Manufacture of Long and Broad Leather Belting—Which Side of the Leather should Face the Pulley—Double and Treble Belting—Compound Belting—Link Chain Belting—Victoria Belting—Waterproof Canvas Belts—India-rubber Belts—Guttapercha and Composite Guttapercha Belts — Strength of, Working Tension in, and Horse-power Transmitted by, Belts—General Requirements for Belting—Rope Gearing — Sizes of Ropes and Pulleys — Strength of Cotton and Hemp Ropes—Rope Pulleys—Multigroove Rope Drives—Speed of, and Horse-power Transmitted by, Ropes — Power Absorbed by Rope Driving—Telodynamic Transmission—Pulleys—Wire-Rope Haulage and Transport—Questions.

Belt, Rope, and Chain Gearing.—The transmission of power between distant shafts is usually effected by means of pulleys and belts, ropes, or chains.* Although belts are most commonly

* The transmission of power between distant shafts is often effected by means of dynamos, wires, and motors, but this case is evidently outside of the range of the present work. The following is a list of books and papers treating of belt and rope driving:—

E. & F. Spon's *Dictionary of Engineering* (E. & F. Spon, London).
Paper on "Transmission of Power by Wire, Ropes, and Turbines," by H. M. Morrison. *Proceedings of Inst. of Mech. Engineers*, 1874, p. 56.
Paper on "Rope Gearing for Transmission of Large Powers in Mills and Factories," by J. Durie. *Proceedings of Inst. of Mech. Engineers*, 1876, p. 372.
Sir W. Siemens' prize "Essay on Machine Belting," by A. H. Barendt, read before the Liverpool Polytechnic Society, January 29, 1883.
Paper on "Belt Driving," by J. Tullis, communicated to the Convention of British and Irish Millers in Glasgow, June 17, 1885. See Messrs. Tullis & Son's *Guide to Belt Driving*, 1891.
"Rope Driving," by Chas. W. Hunt, of N. Y. City. See vol. of *Trans. Am. Soc. of Mech. Engs.* for 1890.
A Treatise on the Use of Belting for the Transmission of Power, by John H. Cooper. Fourth edition, 1891. Published by Edward Meeks, Walnut Street, Philadelphia, U.S.A., and E. & F. Spon, London.
"Experiments on the Transmission of Power by Belting," made by Messrs. Wm. Sellars & Co. See vol. vii. *Am. Soc. Mech. Engineers.*
Two Papers, by Wilfred Lewis and Prof. Lanza, read before the American Society of Engineers, Chicago. See vol vii. of the *Transactions*, 1886.
"Comparative Tests of Leather and Canvas Rubber Belts," by S. Webber. See *Trans. of the Am. Soc. Mech. Engs.*, vol. viii., 1887, and the *Electrical Review* of September 5, 1890.

employed for this purpose, yet in certain cases, where great power has to be transmitted between two distant shafts at high speeds, rope gearing is preferred on account of its flexibility, lightness, quiet smooth working, easy repair, small first cost, and the facility with which the desired tension can be regulated by means of a tightening pulley when one continuous rope is used.*

Chain gearing is employed in cases where the motion is slow and the power greater than could be safely transmitted by narrow belts or ropes. It is also used where slipping is inadmissible, and where, as in the case of rolling mills, &c., belts or ropes would soon become perished or burned by the heat from the materials being acted upon by the rolls.

Materials for Belting.—The most common and, generally speaking, the best material for belting is leather; although many substitutes, such as cotton, india-rubber, Dick's canvas and balata, woven gut, camels' and Llama hair, have been devised, and found very serviceable under special conditions.

Curing, Cutting, and Splicing Leather for Belts.—The best leather for belting is made from skins taken from the backs of full-grown Highland oxen. The hides are thoroughly cured by being immersed for a long time in an "orange tan" liquid,† which possesses the property of condensing and contracting the raw hide instead of causing it to swell and become heavier, as is the case when they are tanned with "oak bark." This process produces a

Paper on "Belting for Machinery," by H. A. Mavor, M.Inst.E.E., read before the Institution of Engineers and Shipbuilders, Glasgow, on February 21, 1893. See *Proceedings of the Institution*.

Elements of Machine Design, Part I., "Belt and Rope Gearing," by Prof. W. C. Unwin, F.R.S. (Longman, Green & Co., London).

Pocket Diary of the *Mechanical World* (Emmot & Co., Manchester).

The *Practical Engineer* Pocket-Book and Diary (Technical Publishing Co., Manchester).

Paper by Prof. W. C. Unwin, F.R.S., being the Howard Lectures delivered before the Society of Arts, 1893. Printed in the *Society of Arts Journal*, and reprinted in the *Practical Engineer*, December, 1893, and January, 1894.

A series of excellent articles on "Rope Driving," by Prof. J. J. Flather in *The Electrical World* (W. J. Johnston Co., Limited, New York City), from October 21, 1893, to April, 1895.

* When two or more independent ropes are employed, tightening pulleys are found to be impracticable.

† From the Patent Specification No. 8,165 of 1884 we learn that this liquid consists of "'Spanish extract,' having borax or saltpetre dissolved therein; the said extract being an astringent solution obtained by grinding and boiling the rind of the orange and lemon. The quantities of borax and saltpetre may be considerably varied. After the hides, when thus saturated, are treated by the ordinary operation of currying, it converts them into leather fit for commercial use."

thin, firm, practically stretchless and very light, strong leather, which is particularly well suited for the transmission of power. The thickness of the leather thus obtained is generally about $\frac{3}{16}$ inch, its extreme length is about 4 feet 6 inches, and width about 4 feet. The tanned hides are usually cut up into parallel strips of the required breadth, and these strips are then joined end to end by overlap tapered splices in order to make up the desired length of belting.

In forming these splices the ends of two strips are *first* carefully tapered to a thin edge by planing machines.* *Secondly*, the tapered faces are covered with glue. *Thirdly*, they are cemented together under great pressure. *Fourthly*, as an additional precaution the splices are laced with untanned ox-hide, riveted with copper rivets and washers, or sewn with wax thread and copper wire.

If a greater thickness should be required than that afforded by a single strip of the hide (as in the case of double or treble belting), then two or three such strips are thoroughly glued together under great pressure. The final ends are left square until the roll of belting is taken from the store for the purpose of making up an endless belt of the required length for any particular set of pulleys. A sufficient length is then cut from the roll and stretched before making the final joint by one or other of the following methods.

Different Methods of Jointing Leather Belts.—Fig. (1) shows a joint which is generally made in a belt factory where the necessary scarfing planes and compressing gear are available. Cemented joints made in this way are nearly as strong as the other joints. They last longer and drive better than when cut up by sewing or riveting (see following table).

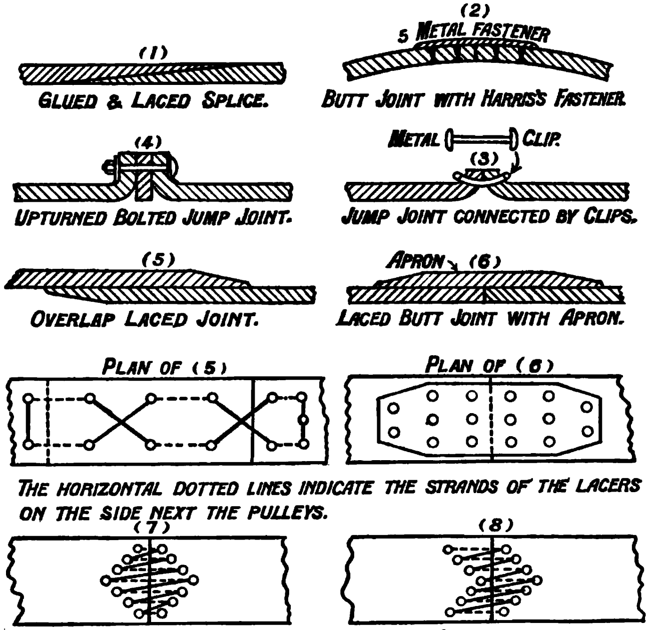
Fig. (2) represents a good connector for new and stout leather belts. This Harris's fastener is laid down upon an iron block with the teeth upwards. First one end of the belt is driven home upon one half of the teeth, and then the other end upon the other half, care being taken not to disturb the curve of the fastener as this gives the necessary holding bite to the teeth.

Fig. (3). The tough yellow metal fasteners shown serve very well for belts running at high speeds over small pulleys. The slits for these clips are made by a special form of cutting pliers and should be cut fully $\frac{1}{4}$ inch from the end of the belt, being so spaced that the crossheads of the fasteners come close to each other across

* Care should be taken to plane the one strip from the hair side and the other strip from the flesh side, so that the belt as a whole may present a uniform surface on each face.

the strap. In order to prevent the body of the metal clip from being bent up and down when passing over the pulleys the fastener should be as short as practicable.

Fig. (4). The strength of an upturned joint of this form is increased by inserting a leather strip or washer between the turned up ends as shown. For this kind of joint Jackson's patent bolt and washer fasteners are used by Messrs. John Tullis & Son, of



TWO METHODS OF LACING BUTT JOINTS.

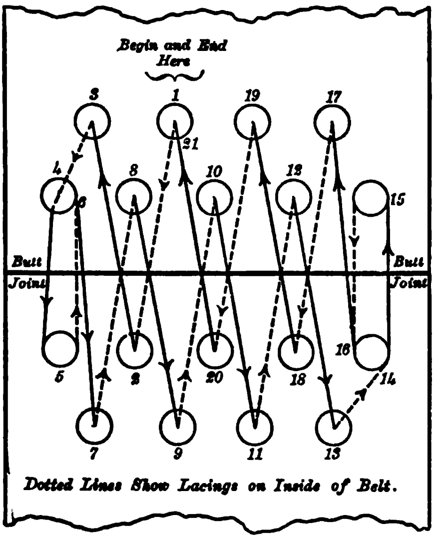
DIFFERENT METHODS OF JOINTING LEATHER BELTS.

Glasgow. They are also recommended by them for cotton and Llama hair belting.

Fig. (5) shows the ordinary overlap laced joint in both section and plan. Care should be taken to taper down and curve the ends to suit the smallest pulley over which the belt has to pass, otherwise the joint will be stiff, and, consequently, every time it travels on and off the pulley a sort of hinge action takes place, accompanied by a shock, which not only shortens the life of the belt, but also communicates a jar to the shaft and machine being driven.

Fig. (6). For heavy broad double belts, where a scarfed cemented joint cannot be used, this form of joint with leather apron makes a very good connection.

Figs. (7) and (8) show the best methods of lacing simple butt joints. The holes for the lacer are punched so as to form a diamond or pointed figure, whereby there are never more than two holes in line across the belt; and, consequently, it is possible to retain almost the entire strength of the belt without reducing its flexibility. Another plan of lacing a butt joint, which is very suitable for dynamo driving and ensures steady smooth running, is illustrated by the following figure:—

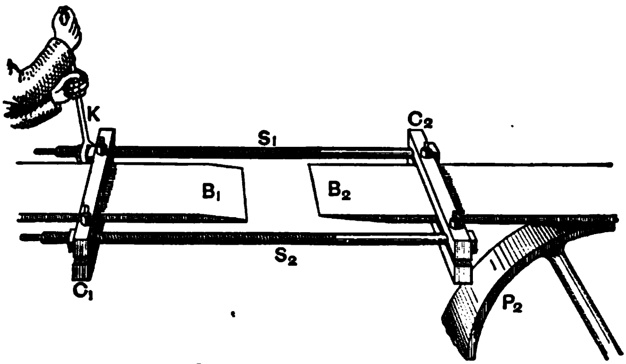


PLAN OF A LACED BUTT JOINT FOR A 6-INCH BELT.

The joint is made by a Helvation leather lacer of ½-inch width in one length, by beginning at hole marked 1, and continuing through the several holes in the order, 2, 3, 4, &c., as shown by the figure, ending with hole 1, which is also marked 21.

In order to facilitate the making of any one or other of the above-mentioned joints in belts more than 6 inches wide, it is usual to employ a "drawing-up frame" of the form shown by the accompanying figure. The two ends, B_1, B_2, of the belt, after being passed over the driving and driven pulleys, are brought towards each other and gripped by the clamps, C_1, C_2. The

spanner, K, is then applied alternately to the nuts of the long screws, S_1, S_2, until the tapers of the splice are brought fairly over each other, or until the ends come together fairly and squarely in the case of a butt joint. The lacing can then be done



STRETCHING APPARATUS.

without any other effort than that of merely passing the thong through the holes and drawing it home tightly.

The chief objects to be attained by a good joint are—*first*, to maintain a close approximation to the belt strength; and *second*, to keep the thickness and flexibility of the joint as nearly equal to that of the belt as possible in order to prevent jumping. From the following table it will be seen how far the first of these objects is attained in practice:—

AVERAGE STRENGTH OF LEATHER BELT JOINTS.

And percentage strength as compared with the average strength of leather belts, taken as 4,132 lbs. per square inch. From tests made by A. H. Barendt at the Liverpool School of Science.

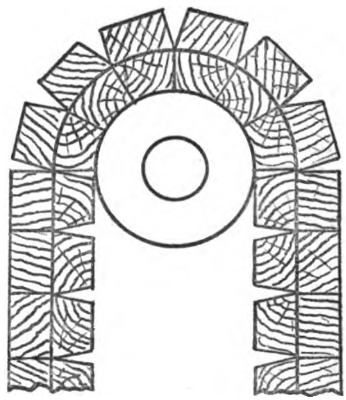
Kind of Joint.	Average Strength in lbs. per square inch.	Per Cent. Strength.
Cemented (only), ...	2,254	52·12
Ordinary white thong, without rivets, .	2,404	59·4
Waxed thread, without rivets, . .	3,240	78·4
Wire sewn or clinched, without rivets, .	3,272	79·18
White thong, with two rivets, . .	2,262	54·74
Waxed thread, with two rivets, . .	2,802	67·83
Copper wire sewn, with two rivets, .	3,050	75·81
White thong, with one rivet, . . .	3,226	78·06

Manufacture of Long and Broad Leather Belting.—By the ordinary plan of cutting up hides into straight strips, their length is limited to about 5 feet, thereby necessitating a corresponding number of splices. Messrs. Sampson & Co., of Stroud, have recently devised a method of making long continuous strips of leather. A disc of leather, 4 to 4½ feet in diameter, is cut from the hide. After being tanned and cured in the usual way, it is cut in a spiral direction from the outer edge to within about 9 inches of the centre. This spiral strip is then stretched and well rubbed until it becomes quite straight. It is found that the strength of the leather is not apparently diminished by this process. Of course, the length of the strip depends on its breadth. If the breadth be only 1·5 to 2 inches, then a strip of about 100 feet can thus be cut from a disc 4½ feet in diameter. By sewing a number of such strips together side by side, while stretched, a belt of any required width can be obtained. Belts 75 inches broad, ½ inch thick, and over 150 feet in length, have thus been built up for transmitting power in mills, directly from the flywheels of large engines to the main shafting. Long belts should never be made heavy, because the greater their weight, the greater becomes their tendency to oscillate up and down, and swing from side to side.

Which Side of the Leather should Face the Pulley.—The grain or hair side of the leather is naturally much smoother than the flesh side, and engineers differ in opinion regarding which of these sides should run in contact with the pulley. In this country, the rough or flesh side is almost invariably placed in contact with the pulley; but in America it is the smooth side. It is claimed for the latter method, that the life and efficiency of a belt is thereby increased. Whichever side of the belt is in contact with one pulley, the same side should be in contact with all the pulleys over which it passes, so as not to bend it alternately in different directions. When the flesh side is next the rim of the pulley, it should receive one coating of currier's dubbing, and three coatings of boiled linseed oil every year, in the case of a belt having to endure continuous hard work. This has the effect of rendering it about as smooth as the hair side, and its efficiency as a whole is said to be thereby increased. When the hair side is in contact with the pulley, it is usual to give that side an occasional coating of castor oil in order to render it more flexible, and, consequently, more durable than it would be in its natural state. Rosin or cobbler's wax should *never* be employed, as they gather dirt, and form lumps on the pulley and belt.

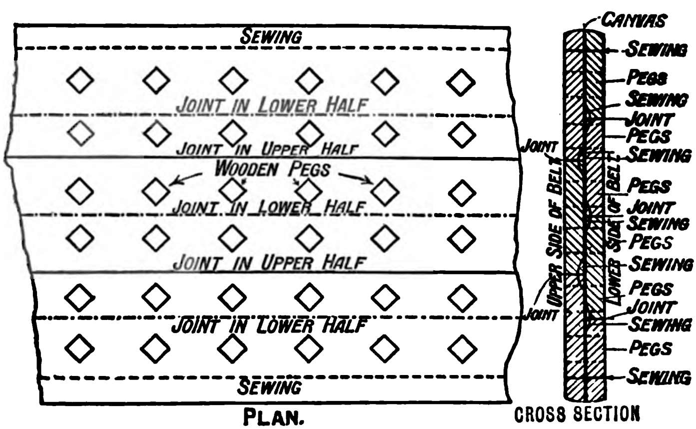
Double and Treble Belting.—Sometimes the breadth of a single belt necessary to transmit a given power would be inconveniently great, and hence double or treble belts are used. These are made

by cementing, and then riveting or sewing, two or three thicknesses of belting together. Owing to the greater rigidity of such belts, they do not work so satisfactorily as single ones, and are not well adapted for running at high speeds, or over pulleys having a diameter of less than 3 or 4 feet, or in cases where the pulleys are close together. This is clearly seen from the accompanying illustration, which shows the change that is continually going on in thick belts, especially when passing over small pulleys. The compression on the inner face of the belt is shown by the necessary reduction in the size of the inside blocks, and the stretching of the outer face by the parting of the blocks of the outside row, when passing over the pulley.



ACTION ON THICK BELTS PASSING OVER SMALL PULLEYS.

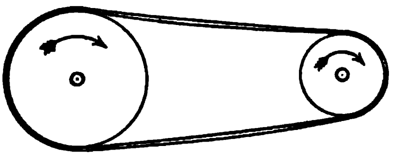
Messrs. Sampson & Co. build their double belts by sewing together and pegging with hard wood pins, a number of specially cut narrow leather strips, with a layer of canvas between



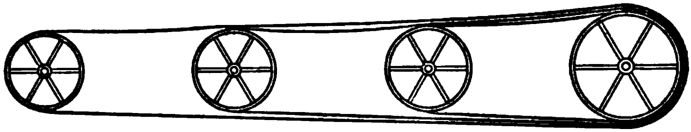
SAMPSON & CO.'S DOUBLE LEATHER BELTING.

them, as illustrated by the accompanying figure. There is always one more strip on the lower or working side than on the upper half. They claim the advantage of being able to produce in this way a belt of very nearly uniform weight throughout its length, having a hard, smooth surface formed on the flesh side, due to the hammering required to drive home and clench the numerous hard wooden pegs with which the two single belts are fixed together. Further, since the sewing is not exposed on either surface of this belt, the leather must be nearly worn through before the sewing is damaged.

Compound Belting.—In order to avoid the internal straining action of thick double or treble belting, it has been found advisable to simply put two or three thin single belts on the top of each other, without any cementing or riveting together, as shown by the accompanying figure. This gives perfect freedom of action to each belt to accommodate itself to the curves over which it passes without stressing its neighbours. The friction between the surfaces of these several belts is not found to have any observable objections, and it is said that 70 per cent. more power can be transmitted by compounding two single belts in this manner. By the addition of a third belt, still more power may be conveyed. Messrs. Tullis & Co. find that by placing plain double belting on the top of their linked chain belts (when arched to fit the curve of the pulley), twice as much power may be transmitted than by either of these alone.



COMPOUND BELT DRIVING.

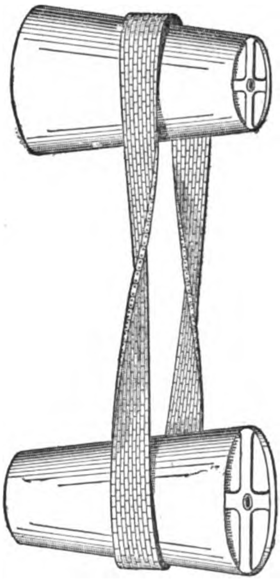


COMPOUND BELT DRIVING FOR THREE MACHINES.

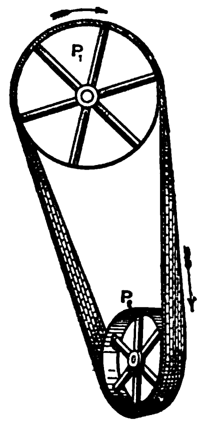
A simple modification of this compound belt drive is illustrated by the above figure. The power is transmitted from one main driving pulley to two or more driven pulleys by separate belts, all moving in the same plane, and in a direct line with each other. This arrangement saves considerable room in a workshop or factory. It has been proved to be very handy for driving several dynamos direct from one flywheel, instead of from independent drums.

It is often urged as an argument against single belting, that it does not work well under the action of shifting guide forks. This may be avoided by running the belt at a sufficient speed, say between 2,000 and 3,000 feet per minute, and, in all cases, by placing the guiding fork at least 1 foot from the point where the belt touches the ongoing side of the pulley, so as to give freedom to the belt to assume its new position.

Link Chain Belting.—Since flat belts fail to take a perfect grip, due to their retaining a cushion of air between them and their pulleys, Messrs. Tullis & Son, of Glasgow, have devised a complete



THICK-SIDED CHAIN BELTING.



TULLIS'S THICK-SIDED LEATHER CHAIN BELT, WORKING QUARTER TWIST.

system of link chain belting. This form of belting permits of the escape of the air through the spaces between the links, and thus enables the leather to bear uniformly over the face of the pulley.

It is composed of a series of short leather links bound together by steel pins and washers. These belts possess considerable flexibility, and can be made with a flexible central row of links to automatically suit the arch of any pulley. If desired, they may be specially curved to suit any camber. They further possess the advantage of being easily shortened and rejointed by simply bringing the ends together with a drawing-up frame, interlocking

the links until a row of rivet holes are brought fair in a line, and then inserting the rivet. When one side of this belting is made thicker than the other it is well adapted for working on tapered cones and for quarter twist driving.

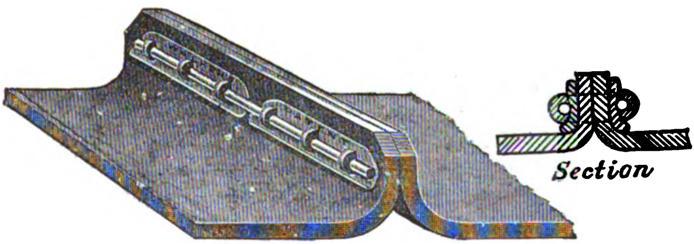
Victoria Belting.—A successful kind of belting has of late been introduced under the name "*Victoria Belting.*" This is simply ordinary belting from which the spongy parts of the leather on the flesh side have been pared away by aid of machinery, and the whole reduced to a uniform thickness. A belt of this description is more flexible than an ordinary one, and, although thinner, is about equal in strength. Double belts, made by cementing and riveting two thicknesses of Victoria belting, are little thicker than ordinary single belts, and are quite as flexible and very much stronger. The hair side of each of the parts cemented together being kept outside, these will always run in contact with the rims of the pulleys and guide pulleys over which they pass.

Waterproof Canvas Belts.—Next to leather, waterproof canvas belting of one kind or another is the most popular. It is especially well suited for paper-mills, dye-works, and other factories where moisture or steam is prevalent, or for working in the open air under varying climatic conditions. The best canvas belting is made from selected Egyptian cotton. After being folded to the required breadth and number of plies, it is stretched and then stitched throughout its length along the open seam. After being dried it is thoroughly saturated with linseed oil, painted with red silicate to make it grip, mangled to squeeze out the superfluous oil, and finally stretched for about a fortnight to render it as inextensible as possible under ordinary working conditions. It can be made in one piece of any required length, breadth, or thickness, and it has a greater breaking stress per square inch than oak tanned leather in the ratio of about 6,700 lbs. for canvas to 4,100 lbs. for the leather. It forms excellent main driving belts, but once it begins to wear and give way at any place it cannot be repaired or patched so readily as leather, neither does it so well endure cross driving or being acted upon by shifting forks. The latter defect is to a large extent overcome by protecting it with stitched leather edges throughout its entire length.

India-Rubber Belts.—These belts present a very smooth, well finished, and excellent gripping surface. They are made by taking stout canvas of the required breadth direct from the roll, passing it over a steam heated cylinder in order to expel the damp inherent in the canvas, which would otherwise rot the first layer of rubber. The web is next passed between two heated rollers. From the upper one it receives a thin plastic coating of rubber. It is then folded double; or, if the canvas is of the same breadth as the belt,

two or more plies similarly treated are pressed together. It is then entirely wrapped in rubber, trimmed along the edges, and vulcanised in steam heated moulds under considerable pressure. They do not wear nearly so well as leather belts, for the life of a hard worked one does not exceed eighteen months, and once the rubber begins to peal off the working side and the canvas heart to wear they suddenly break straight across. They do not give so much trouble as might be expected from stretching, and they can be taken up and jointed by the methods shown in Figs. 1, 4, and 6 at the beginning of this Lecture. They cannot be recommended for shifting belts, as the edges which come into contact with the shifting fork soon become ragged and cut.

Guttapercha and Composite Guttapercha Belts.—Solid guttapercha belts have been tried, but being expensive, liable to perish, and to stretch they have been abandoned. A very successful composite belt has, however, been introduced of late by Messrs. R. & J. Dick of the Greenhead Works, Glasgow, which possesses all the advantages of water-proofed canvas, together with good gripping and



WALKER'S JUMP-JOINT FASTENER.

lasting qualities. It is manufactured from the best "long-staple" Egyptian cotton canvas, which is cut to the required breadth, dried, and covered on each side with balata, dried again, folded into the number of plies and breadth of the belt, covered on the outside with guttapercha, and finally, is stretched in long lengths in order to render it as inextensible as possible under working conditions.

Any required length, breadth, and thickness of belt can be made in this way, and the final single joint may be so neatly scarfed, glued, and pressed that it is not thicker than the ordinary belt and nearly as strong and supple. Or Walker's jump-joint fasteners may be employed, as illustrated by the accompanying figures.

Strength of, Working Tension in, and Horse Power transmitted by, Belts.—The ultimate strength of ordinary bark tanned single leather belting varies from 3,000 to 5,000 lbs. per square inch of

cross-section.* It is, however, sometimes considered more convenient to state the tenacity of a belt in lbs. per inch of width. The thickness of single belting varies from $\frac{3}{16}$ to $\frac{5}{16}$ inch, and from $\frac{3}{8}$ to $\frac{5}{8}$ inch for double belting. Taking the mean thicknesses, we may say that the breaking stresses are from :—

750 to 1,250 lbs. per inch of width for single belts,
and 1,500 to 2,500 „ „ double „

It is found, however, that the strength of the joints is sometimes only about one-third of the strength of the solid leather. Hence, the *final* strength of an endless belt should not be reckoned at more than *one-third* of the above values. Further, the *safe working tension* should never exceed *one-fifth* of this final strength (of the joint), in order to provide for deterioration and sudden changes of load. From this point of view, we thus arrive at approximate values for the safe working tensions by taking one-fifteenth (*i.e.*, one-third for joint and one-fifth for factor of safety) of the breaking stress *per inch of width*, viz. :—

For single belting, 50 to 80 lbs.
„ double „ 100 „ 160 „

In *practice*, however, the rule is not to put on a greater tension than 50 lbs.† for single, and 80 lbs. for double belts per inch of width, since it is found that under these mild conditions, leather belts run for many years with a minimum of lubrication on the bearings, and with far less chance of heating the journals or stressing the shafts.

The theoretical safe working tension per inch of belt width to transmit a certain power, at a certain speed, without slipping, may be arrived at from another point of view, viz. :—By ascertaining the coefficient of friction between a belt and its pulley, and substituting the same in the formulæ given in Lectures VII. and VIII., from which it will be seen, that the power which may be transmitted by a belt is determined by the difference between the tensions of the driving and following sides, and the speed of the belt, and that the logarithm of the ratio of these ten-

* Messrs. Tullis & Sons, of Glasgow, claim the following from tests carried out at Lloyd's Proving House :—

For *their* orange tanned leather, a breaking stress of 8,244 lbs. per sq. in., with 1·3 per cent. elongation.

For the best oak tanned leather, a breaking stress of 5,746 lbs. per sq. in., with 3·8 per cent. elongation.

† Mr. John H. Cooper, in his *Treatise on the Use of Belting*, pp. 383, 384, tabulates the results of fifty-three rules for the H.P. and stress transmitted by belts, and also arrives at the above mean result of 50 lbs. per inch of width.

sions is proportional to the coefficient of friction, and to the arc of the pulley embraced by the belt.*

EXAMPLE I.—Referring to the following table, we see that one H.P. can be transmitted per inch of width of belt, when travelling at 700 feet per minute. Let the arc embraced by the belt be 180° and the coefficient of friction ·25. Find the tensions on the driving and slack sides of the belt, and their difference.

Let T_d = Tension on driving side in lbs.
" T_s = Tension on slack side in lbs.
" V = Velocity of belt in feet per minute = 700.
" μ = Coefficient of friction = ·25.
" θ = Ratio of length of arc of contact to radius of pulley.

Then, $$\text{H.P.} = \frac{(T_d - T_s)V}{33{,}000}.$$

$$\therefore \quad (T_d - T_s) = \frac{\text{H.P.} \times 33{,}000}{V}.$$

Or, $$T_d - T_s = \frac{1 \times 33{,}000}{700} = 47{\cdot}2 \text{ lbs.} \quad . \quad . \quad (1)$$

From Lecture VII., equation (XI.), we get:—

$$\log_e \frac{T_d}{T_s} = \mu\,\theta.$$

But, $$\theta = \frac{\text{arc}}{\text{radius}} = \frac{\frac{1}{2}(2\pi r)}{r} = \pi.$$

$$\therefore \log_e \frac{T_d}{T_s} = \log \frac{T_d}{T_s} \times {\cdot}4343 = {\cdot}25 \times 3{\cdot}1416 \times {\cdot}4343 = {\cdot}3411.$$

$$\therefore \quad \frac{T_d}{T_s} = 2{\cdot}2. \quad \text{Or, } T_d = 2{\cdot}2\,T_s \quad . \quad . \quad (2)$$

Inserting this value in equation (1), we get:—

$$2{\cdot}2\,T_s - T_s = 47{\cdot}2. \quad \therefore T_s = 39{\cdot}3 \text{ lbs.}$$

And, $$T_d = 47{\cdot}2 + T_s = 86{\cdot}5 \text{ lbs.}$$

These are the minimum working stresses per inch width of belt which must be applied to prevent slipping under the above-mentioned conditions.

* For a description of Morin's experiments on the tension of belts made at Metz in 1834, and his use of this formula, see Cooper on "Use of Belting," Chap. VII.

Horse-power that Different Leather Belts will Transmit per Inch in Width at Various Speeds.

(*By A. G. Brown, M.E., for Musgrave & Co.*)

Velocity of Belt per Minute.	Kind of Belts.								
	Best Oak-tanned Belts.			Best Link or Chain Belts.					
	Single belts.	Light double belts.	Heavy double belts.	3/8"	1/2"	5/8"	3/4"	7/8"	1"
Feet.	Horse-power they will Transmit.								
100	·15	·21	·27	·13	·15	·17	·20	·24	·27
200	·30	·42	·55	·25	·29	·35	·40	·47	·55
300	·45	·64	·82	·38	·44	·52	·60	·71	·82
400	·61	·85	1·09	·51	·58	·69	·80	·95	1·09
500	·76	1·06	1·36	·64	·73	·86	1·00	1·18	1·36
600	·91	1·27	1·64	·76	·87	1·04	1·20	1·42	1·64
700	1·06	1·49	1·91	·89	1·02	1·21	1·40	1·65	1·91
800	1·21	1·70	2·18	·92	1·16	1·38	1·60	1·89	2·18
900	1·36	1·91	2·45	1·05	1·31	1·55	1·80	2·13	2·45
1000	1·51	2·12	2·73	1·27	1·45	1·73	2·00	2·36	2·73
1100	1·67	2·33	3·00	1·40	1·60	1·90	2·20	2·60	3·00
1200	1·82	2·55	3·27	1·53	1·75	2·07	2·40	2·84	3·27
1300	1·97	2·76	3·55	1·65	1·89	2·25	2·60	3·07	3·55
1400	2·12	2·97	3·82	1·78	2·04	2·42	2·80	3·31	3·82
1500	2·27	3·18	4·09	1·91	2·18	2·59	3·00	3·55	4·09
1600	2·42	3·39	4·36	2·04	2·33	2·76	3·20	3·78	4·36
1700	2·58	3·61	4·64	2·16	2·47	2·94	3·40	4·02	4·64
1800	2·73	3·82	4·91	2·29	2·62	3·11	3·60	4·25	4·91
1900	2·88	4·03	5·18	2·42	2·76	3·28	3·80	4·49	5·18
2000	3·03	4·24	5·45	2·55	2·91	3·45	4·00	4·73	5·45
2100	3·18	4·45	5·73	2·67	3·05	3·63	4·20	4·96	5·73
2200	3·33	4·67	6·00	2·80	3·20	3·80	4·40	5·20	6·00
2300	3·49	4·88	6·27	2·93	3·35	3·97	4·60	5·44	6·27
2400	3·64	5·09	6·55	3·05	3·49	4·15	4·80	5·67	6·55
2500	3·79	5·30	6·82	3·18	3·64	4·32	5·00	5·91	6·82
2600	3·94	5·52	7·09	3·24	3·78	4·49	5·20	6·15	7·09
2700	4·09	5·73	7·36	3·28	3·85	4·66	5·40	6·38	7·36
2800	4·24	5·94	7·64	3·31	3·86	4·73	5·60	6·62	7·64
2900	4·39	6·15	7·91	3·32	3·87	4·78	5·80	6·85	7·91
3000	4·50	6·36	8·18	3·31	3·86	4·75	5·97	7·09	8·18
3100	4·60	6·58	8·45	3·30	3·85	4·73	5·96	7·33	8·45
3200	4·69	6·79	8·70	3·28	3·82	4·71	5·94	7·37	8·73
3300	4·77	7·00	8·86	3·24	3·77	4·70	5·92	7·35	8·88
3400	4·84	7·21	8·96	3·19	3·71	4·64	5·87	7·32	8·86
3500	4·90	7·31	9·06	3·13	3·61	4·50	5·78	7·26	8·80
3600	4·95	7·40	9·16	3·05	3·50	4·37	5·67	7·16	8·73
3700	4·99	7·48	9·24	2·96	3·39	4·26	5·55	7·01	8·58
3800	5·03	7·54	9·29	2·84	3·28	4·15	5·41	6·87	8·41
3900	5·03	7·60	9·34	2·72	3·13	4·02	5·20	6·70	8·27
4000	5·08	7·64	9·37	2·58	2·95	3·84	5·01	6·48	8·04

General Requirements for Belting.—The ultimate strength of a belt is, however, a secondary consideration, since it is so very much greater than the normal working stress. The properties of greatest importance are—(1) straightness, (2) stretchlessness, (3) pliability, (4) homogeneity, (5) uniformity in thickness, (6) good flexible joints, (7) ease of repair, (8) endurance or longevity. These can only be determined by practical experience. Further, the very best belting cannot be expected to work well upon pulleys which are unbalanced or out of truth and line, or which are too close together.* The pulleys should be as far apart as possible and their diameters as large as possible. For example, at Messrs. Clark & Co.'s thread works at Paisley there are pulleys as far apart as 90 feet. For high speeds the pulleys should have little or no crowning; for, when a speed of about 3,000 feet per minute is reached, the sides of the belt will rise up from the face of a heavily crowned pulley due to centrifugal force, and thus greatly diminish the area of contact, inducing slipping, wearing of the belt, and unsatisfactory driving.

Rope Gearing.—The transmission of power is frequently accomplished by means of ropes instead of belts. In addition to their other advantages mentioned at the beginning of this Lecture, ropes seldom give way without due warning by slackening or fraying in one or more of the strands, thus reducing the risk of accident and stoppage of the works to a minimum. The working stress in the ropes being but a small fraction of their breaking strength, any signs of weakness in an individual rope would allow it to be removed and the engine run with the remaining ropes until a convenient opportunity is offered for the replacement of the weak member. Further, their comparative slackness between the pulleys facilitates their cancelling any small irregularity in the motive power.

These ropes are made of manilla-hemp, cotton, leather, and raw ox hide. Hemp ropes are preferred to cotton ropes for main drives with large pulleys since they are cheaper, stronger, and last nearly as long if spun with a soft greased core. In Messrs. J. & P. Coats' great thread works, Paisley, cotton ropes are, however, universally employed. They are made "hawser laid," from the best "long staple" Egyptian, white, untarred cotton. By "hawser laid" is meant that the fibres of the material are first spun into yarns having a *right-handed twist*. These yarns are

* Mr. Henry A. Mavor in his reply to the discussion on his paper "Belting for Machinery," read before the Institution of Engineers and Shipbuilders in Scotland, states that (where possible) the pulleys should be kept apart not less than three times, and not more than four times the diameter of the larger pulley.

next twisted *left-handedly* into strands. Lastly, three such strands are twisted together *right-handedly* so as to complete the rope. It will be noticed that when the strand is twisted it untwists *each* of the threads, and that when the three strands are twisted together into rope this action untwists the strands, but at the same time re-twists the threads. It is this opposite twist that serves to keep the rope in its proper form.* Cotton ropes, being softer and more pliable than manilla ropes, can be used with smaller pulleys without undue injury to the fibres. This is also considerably aided by the natural wax in the structure of the long staple variety which acts as a lubricant, and permits of greater freedom of motion between the several fibres. The life of a good cotton rope is usually about thirteen years if properly adjusted and well treated.

Sizes of Ropes and Pulleys.—The size and number of ropes, as well as the least diameter of pulley, for any given power, are points of importance and should be considered for each case.

The ropes commonly used for the transmission of power in factories or mills vary from 3 to 5 inches in circumference. No matter what the diameter of the pulley may be, ropes of $1\frac{3}{4}$ inches diameter should not be exceeded for main drives, and $1\frac{1}{4}$ inches diameter for secondary drives. The diameter of the smallest pulley should not be less than thirty times the diameter of the rope, as the larger the pulley the less will be the internal friction, and consequent injury to the rope from bending and unbending (see the following table).

Strength of Cotton and Hemp Ropes.—The ultimate strength of white untarred cotton ropes may be taken at 9,000 lbs. per square inch of the nett sectional area, which is about 90 per cent. of the area of the circumscribing circle. The normal working stress should not exceed $\frac{1}{30}$ of the ultimate breaking stress, or say 300 lbs. per square inch, although ropes are frequently worked at even a less tension. Messrs. Musgrave & Sons, of Bolton, allow a working stress of about 300 lbs. per square inch of sectional area. Of this about 20 per cent. or 60 lbs. is absorbed in overcoming back tension, wedging of rope, &c., leaving 240 lbs. for centrifugal force and transmission of power.

The following table from *The Practical Engineer* gives useful data regarding these points.

The ultimate strength of *new manilla* ropes is about 11,000 lbs. per square inch of the nett sectional area, which in a three stranded hawser is only about 80 per cent. of the area of its circumscribing circle. The necessary lubrication, however, reduces the strength by

* See *The Practical Engineer* of June 22, 1894, p. 486, &c., for "Notes on the Manufacture of Ropes," by W. C. Popplewell, M.Sc.

20 to 30 per cent., but the lubrication of the fibres is of much greater importance than the actual breaking stress. The greater freedom of movement amongst the fibres permits a heavier working stress to be carried, and ensures a much longer life; for a properly lubricated manilla rope will outlast from two to four similar dry laid ropes working under the same conditions. There are many ways of lessening the internal friction; one of the best being that of coating the several yarns with a mixture of black lead and tallow prior to twisting the same into strands. Under favourable conditions, when thus treated, it is practically waterproof, and will last for about eight years.

Cotton Driving Ropes and Pulleys.

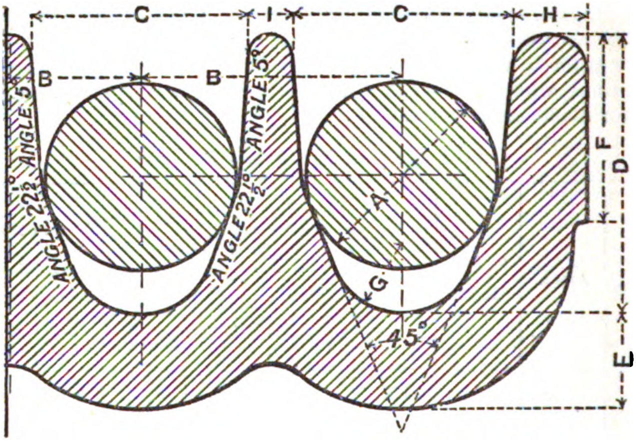
Diameters.	Areas.	Weight per Foot.	Ordinary Working Tension including Centrifugal Force.	Velocity of Ropes, 4,700 Ft. per Minute (their most effective Speed). Stress due to Centrifugal Force.	Effective Tension.	Power each Rope will Transmit.	Rope Pulleys. Centre to centre of ropes	Diam. of smallest pulley.
Ins.	Ins.	Lbs.	Lbs.	Lbs.	Lbs.	H.P.	Ins.	Ins.
1/2	·1963	·081	47	16	31	4·43	7/8	15
5/8	·3067	·125	72	24	48	6·84	1	18
3/4	·4417	·184	106	35	71	10·07	1 1/8	22
7/8	·6013	·25	144	48	96	13·67	1 5/16	26
1	·7854	·33	190	63	127	18·05	1 1/2	30
1 1/4	1·2272	·51	294	98	196	27·90	1 13/16	37
1 1/2	1·7671	·74	426	142	284	40·48	2 1/8	45
1 3/4	2·4053	1·00	576	192	384	54·70	2 1/2	52
2	3·1416	1·30	750	250	500	71·10	2 3/4	60

Splicing Ropes.—The splicing of ropes is a matter of great importance, and should never be left to unskilled hands. In order to secure a sufficiently strong joint its length requires to be from forty to fifty times the diameter of the rope, or say from 6 to 7 feet for main, and from 4 to 5 feet for secondary drives. The different strands should be neatly interlocked, so that the thickness of the rope is not increased. The strength of a good splice is only about 70 per cent. of that of the rope.

Rope Pulleys.—The pulleys used for rope gearing are made with V-shaped grooves around their rims, as shown by the accompanying figures.

The sides of the grooves usually make an angle of 45° with each other. The ropes must never be allowed to rest on the bottom of the grooves, but only on the sides, as shown. They are thus

wedged into the grooves, and the resistance to slipping is thereby greatly increased. This has the effect, however, of producing greater wear and tear of the ropes. In the case of guide pulleys, the rope should always rest on the bottom of the groove.



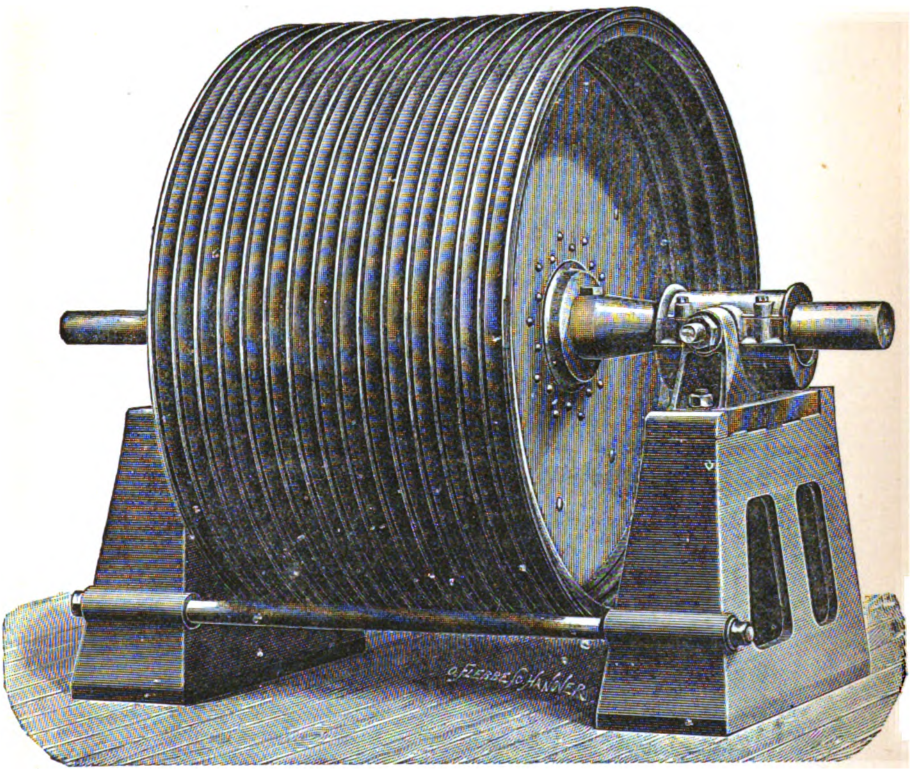
SECTION OF GROOVES FOR ROPE PULLEYS.

The following table gives the proportions of grooves of the form shown in the above figure, for ropes from ½ inch diameter to 2⅛ inches diameter:—

PROPORTIONS OF GROOVES FOR ROPE PULLEYS.

Diameter of Rope. A	B	C	D	E	F	G	H	I
Ins.	Ins.	Ins.	Ins.	In.	Ins.	In.	In.	In.
½, . . .	1	¾	$\frac{13}{16}$	$\frac{5}{16}$	½	$\frac{3}{16}$	$\frac{5}{16}$	¼
⅝, . . .	1⅛	⅞	1	⅜	⅝	$\frac{7}{32}$	⅜	¼
¾, . . .	$1\frac{5}{16}$	1	$1\frac{3}{16}$	$\frac{7}{16}$	¾	$\frac{7}{32}$	⅜	¼
⅞, . . .	$1\frac{7}{16}$	1⅛	1⅜	½	⅞	$\frac{5}{16}$	⅜	$\frac{5}{16}$
1, . . .	1⅝	1¼	$1\frac{9}{16}$	$\frac{9}{16}$	1	⅜	⅜	$\frac{5}{16}$
1¼, . . .	1⅞	$1\frac{9}{16}$	$1\frac{15}{16}$	⅝	1¼	$\frac{13}{32}$	½	$\frac{11}{32}$
1½, . . .	$2\frac{3}{16}$	$1\frac{13}{16}$	2¼	¾	1½	$\frac{9}{16}$	$\frac{9}{16}$	⅜
1⅝, . . .	$2\frac{5}{16}$	$1\frac{15}{16}$	2½	$\frac{13}{16}$	1⅝	⅝	⅝	⅜
1¾, . . .	2½	$2\frac{1}{16}$	2⅝	⅞	1¾	$\frac{11}{16}$	$\frac{11}{16}$	$\frac{13}{32}$
2, . . .	2¾	$2\frac{5}{16}$	3	1	2	¾	¾	$\frac{7}{16}$
2⅛, . . .	2⅞	$2\frac{7}{16}$	$3\frac{3}{16}$	1⅛	2¼	$\frac{13}{16}$	$\frac{13}{16}$	$\frac{15}{32}$

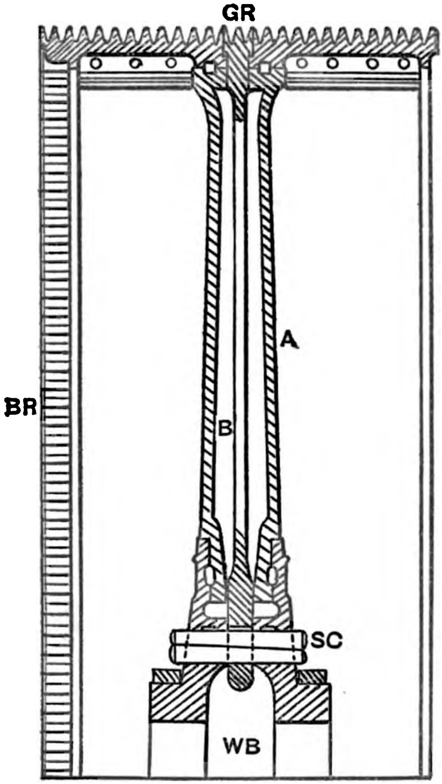
In the case of belts running on flat-rimmed pulleys, we have seen that sufficient adhesion to prevent slipping between the belt and pulleys is obtained by stretching the belt over the pulleys with an initial tension. But with ropes, it is found that smoother



SPECIAL FORM OF ROPE PULLEY BY WATSON, LAIDLAW & Co.

working and less wear and tear take place when the rope is put on the pulleys with as little initial stress as possible. Hence the reason for increasing the frictional resistance between the rope and its pulleys, by allowing the rope to wedge itself into the grooves in the pulley rims. The necessary pressure between the rope and

the sides of the grooves to ensure sufficient frictional adhesion at the commencement of motion, is simply that due to the weight of the hanging parts of the ropes between the pulleys. The pulleys should, therefore, be large, and be placed at a sufficient horizontal distance apart, so as to have the arcs of contact between the rope and the pulleys as great as possible.



Section of Large Rope Pulley.

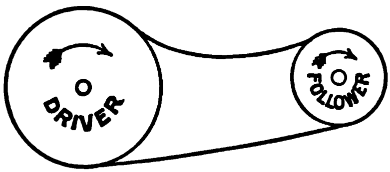
The accompanying figure shows the construction of a large rope flywheel, in which the wrought-iron bolt, B, connects the grooved rim, G R, with the wheel boss, W B, and thus receives the tensile stress due to centrifugal action on G R.

In horizontal drives, the tight side of the rope should always be in contact with the lower parts of the pulleys, and the slack side above, so as to obtain a maximum arc of contact between the rope and the pulleys, as shown by the following right-hand figure.

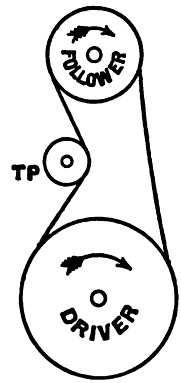
Index to Parts.

W B for Wheel boss.	A for Arm.
S C ,, Steel cotters.	G R ,, Grooved rim.
B ,, Wrought-iron bolt.	B R ,, Barring rack.

If the drive be vertical, then the rope may require a tightening pulley, in order to insure sufficient frictional resistance between it and the under side of the lower pulley, as shown by T P in the right-hand figure. A tightening pulley naturally shortens the life of the rope.

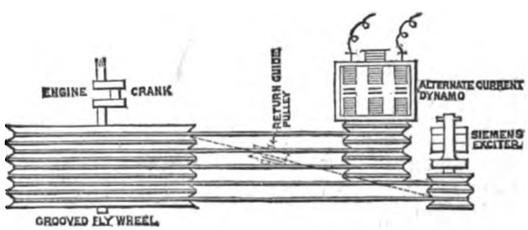


HORIZONTAL ROPE DRIVING.



VERTICAL ROPE DRIVING.

Multigroove Rope Drives.—Where great power has to be transmitted it is neither convenient nor advisable to use very thick ropes. The usual practice is to have a number of ropes running in parallel grooves on one large pulley or wheel. The grooves in each pulley must be of the same size and depth, and all the ropes of the same thickness. They should also be stretched as equally as possible between the pulleys. These conditions are necessary to prevent some of the ropes being more severely strained than others. Since ropes stretch, it is advisable to put them all on at



MULTIGROOVE ROPE DRIVE WITH ONE ROPE AND GUIDE PULLEY.

the same time when they are intended to work on the same pair of pulleys. In certain cases, such as when driving an alternator and its exciter, a single rope is used with a guide pulley and facilities for sliding the machines towards or away from the driving pulley, as shown by the figure, or the guide pulley may also be used independently for tightening the rope.

HORSE-POWER THAT GOOD COTTON DRIVING ROPES WILL TRANSMIT AT VARIOUS SPEEDS.*

(*By A. G. Brown, M.E., for Musgrave & Co.*)

Velocity in Feet per Minute.	Diameter of Ropes in Inches.								
	½"	⅝"	¾"	⅞"	1"	1¼"	1½"	1¾"	2"
	Horse-Power they will Transmit.								
600	·84	1·30	1·91	2·60	3·43	5·30	7·69	10·40	13·52
700	·98	1·52	2·23	3·03	4·00	6·18	8·96	12·12	15·75
800	1·12	1·73	2·54	3·45	4·56	7·05	10·22	13·82	17·96
900	1·26	1·94	2·86	3·88	5·12	7·92	11·48	15·52	20·17
1,000	1·39	2·15	3·16	4·30	5·67	8·76	12·72	17·18	22·34
1,100	1·53	2·35	3·47	4·71	6·22	9·61	13·94	18·83	24·48
1,200	1·66	2·56	3·77	5·12	6·76	10·44	15·15	20·47	26·61
1,300	1·79	2·76	4·07	5·53	7·29	11·27	16·35	22·10	28·73
1,400	1·92	2·96	4·36	5·93	7·83	12·10	17·55	23·72	30·83
1,500	2·05	3·16	4·65	6·32	8·34	12·89	18·70	25·27	32·86
1,600	2·18	3·36	4·94	6·74	8·86	13·70	19·88	26·86	34·92
1,700	2·30	3·55	5·22	7·10	9·37	14·48	21·01	28·39	36·90
1,800	2·42	3·74	5·50	7·47	9·86	15·25	22·12	29·89	38·85
1,900	2·54	3·92	5·76	7·83	10·34	15·97	23·18	31·32	40·71
2,000	2·66	4·10	6·03	8·20	10·82	16·72	24·26	32·79	42·62
2,100	2·77	4·27	6·29	8·54	11·28	17·43	25·29	34·17	44·42
2,200	2·88	4·45	6·55	8·90	11·75	18·16	26·35	35·60	46·29
2,300	2·99	4·62	6·80	9·24	12·19	18·84	27·34	36·94	48·03
2,400	3·10	4·78	7·04	9·56	12·62	19·51	28·31	38·26	49·73
2,500	3·20	4·94	7·28	9·89	13·05	20·17	29·26	39·55	51·41
2,600	3·30	5·09	7·50	10·18	13·44	20·77	30·14	40·73	52·96
2,700	3·39	5·24	7·71	10·48	13·83	21·37	31·00	41·90	54·47
2,800	3·48	5·38	7·92	10·75	14·20	21·94	31·84	43·02	55·93
2,900	3·57	5·51	8·12	11·03	14·56	22·50	32·64	44·11	57·35
3,000	3·66	5·65	8·31	11·30	14·91	23·04	33·44	45·18	58·74
3,100	3·74	5·78	8·50	11·56	15·25	23·57	34·20	46·22	60·08
3,200	3·83	5·90	8·69	11·81	15·59	24·09	34·95	47·23	61·40
3,300	3·90	6·01	8·85	12·02	15·87	24·53	35·59	48·10	62·53
3,400	3·96	6·12	9·01	12·23	16·15	24·96	36·21	48·94	63·62
3,500	4·03	6·22	9·15	12·44	16·42	25·37	36·81	49·75	64·67
3,600	4·09	6·31	9·29	12·63	16·67	25·76	37·38	50·51	65·66
3,700	4·15	6·41	9·43	12·81	16·91	26·13	37·92	51·24	66·62
3,800	4·20	6·48	9·53	12·95	17·10	26·43	38·35	51·82	67·36
3,900	4·25	6·56	9·65	13·12	17·32	26·76	38·83	52·48	68·22
4,000	4·29	6·62	9·75	13·24	17·48	27·01	39·20	52·97	68·86
4,100	4·33	6·68	9·83	13·36	17·63	27·25	39·53	53·42	69·44
4,200	4·36	6·73	9·91	13·46	17·77	27·46	39·84	53·84	69·99
4,300	4·39	6·78	9·98	13·55	17·89	27·65	40·11	54·21	70·47
4,400	4·41	6·80	10·01	13·60	17·95	27·75	40·26	54·40	70·72
4,500	4·42	6·82	10·04	13·64	18·00	27·82	40·36	54·55	70·91
4,600	4·43	6·83	10·06	13·66	18·03	27·87	40·44	54·64	71·04
4,700	4·43	6·84	10·07	13·67	18·05	27·90	40·48	54·70	71·10
4,800	4·43	6·84	10·07	13·67	18·05	27·90	40·48	54·70	71·10
4,900	4·43	6·83	10·06	13·66	18·03	27·87	40·44	54·64	71·04
5,000	4·41	6·80	10·01	13·60	17·95	27·74	40·25	54·40	70·70

* In practice, Messrs. Clark & Co. of Paisley find these powers to be rather high for the larger ropes. For example, they only allow 35 to 40 H.P. for a 1¾-inch cotton rope at 3,600 feet per minute.

Speed of, and Horse-Power Transmitted by, Ropes.—The speed of ropes is generally very high, being from 3,000 to 5,000 feet per minute. The usual or average speed may be stated to be about 4,500 feet per minute, although some engineers have used as high as 5,600 with advantage. The preceding table shows the power which good cotton ropes will transmit at various speeds.

Power Absorbed by Rope Driving.—It is stated by some engineers that rope gearing absorbs 10 per cent. less power than toothed gearing. I am assured, however, that this is an error, for well designed and well applied tooth gearing consumes little more than 4 per cent., belts from 5 to 5½ per cent., and ropes about 7 per cent.

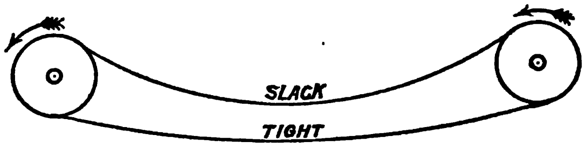
Telodynamic Transmission.*—The successful transmission of power by round endless wire ropes commences where a belt, or cotton and hemp ropes, would be too long to be used profitably (*i.e.*, say about 50 feet between the driver and follower for belting and 100 feet for ordinary ropes), and ends economically at distances of from 10,000 to 13,000 feet. For, the efficiency rapidly decreases as the distance increases, being about 95 per cent. for 100 yards, 90 per cent. for 500 yards, and only 60 per cent. for 5,000 yards under the most favourable conditions.

This system has been much more extensively employed on the Continent than in this country, although the author has seen numerous instances of its adoption in Scotland, and in Orkney, for driving ordinary thrashing mills where the water power was down in a hollow and removed from the steadings about 200 to 400 yards. Messrs. Rochling and Trenton, N.J., state, that in point of economy, this system costs only about $\frac{1}{15}$ of an equivalent amount of belting and $\frac{1}{25}$ of shafting. This is not to be wondered at, since steel wire ropes are cheap and strong, and can be run at very high speeds so that great power may be transmitted by them with comparatively light gearing. The range in the size of the cables used is, however, small, for the employment of a large wire rope means self destruction and loss of power due to its bending and unbending over the pulleys; and further loss of power due to moving it at the required velocity over great distances. For example, a rope of ⅜ inch diameter will transmit 20 H.P. or less, and a 1-inch rope 300 H.P., whereas a 1¼-inch one would not

* See Chapter VIII. of the fourth edition of a Treatise *On the Use of Belting*, by John H. Cooper (Edward Meeks, Walnut Street, Philadelphia, or E. & F. Spon, London, 1891); *Elements of Machine Design*, by Prof. W. C. Unwin (Longmans, Green & Co., London); also the *Howard Lectures*, by Prof. Unwin, *Society of Arts Journal*, 1893, where an interesting account is given of the rise and progress of Telodynamic Transmission as well as details of the latest practice.

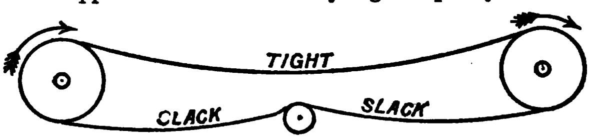
successfully transmit any more power, with the same speed and size of pulleys and the same extreme distance, owing to its greater stiffness and weight.

The first of the following figures shows a single span with the slack part of the rope uppermost :—



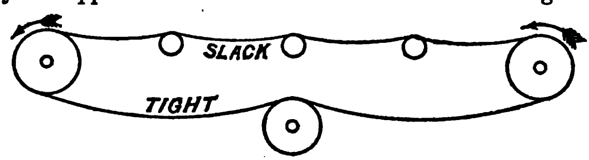
SINGLE SPAN.

The second also represents a single span but with the slack part below and supported in the middle by a guide pulley.



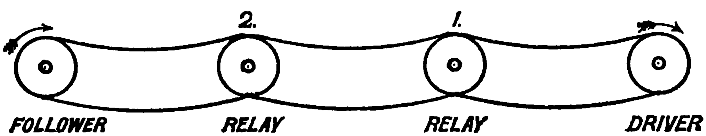
SPAN WITH SINGLE SUPPORTING PULLEY.

When the length of a span is great, and the height of the pulleys not sufficient to prevent the rope trailing on the ground, it may be supported in the manner shown in the third figure.



SINGLE SPAN WITH MULTIPLE SUPPORTING PULLEYS.

When the power has to be conveyed over a very great distance, it is advisable to split up the length into intermediate stations or "relays," each relay being worked by a separate rope, as shown by the following figure. The pulleys at the relays are double grooved,

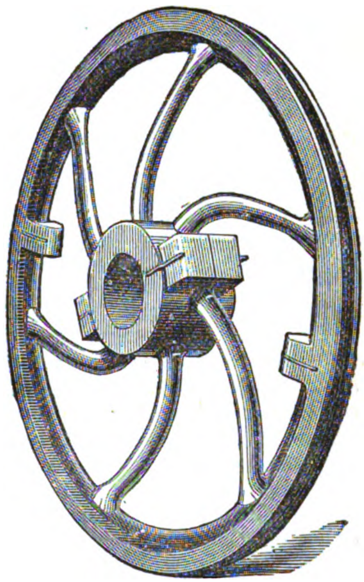


INDEPENDENT SPANS.

so that the two ropes run together on the same pulley. The length of a relay may be from 400 to 5C0 feet, with guide pulleys

every 50 or 60 feet. A good example of this is to be found at Schaffhausen on the Rhine, where about 650 horse-power are transmitted through a distance of 2,700 feet to twenty-three customers, where a small rope of 1-inch diameter, moving at 62 feet per second, transmits 280 horse-power.

Pulleys.—Since bending is more injurious to wire ropes than those made of hemp or cotton, it is necessary to use very large pulleys with the former. This not only diminishes the damage done to the cable, but also the power expended in bending and unbending the rope. Further, the arcs of contact should be as large as possible, in order to secure sufficient frictional resistance between the rope and the pulleys. Wire ropes, unlike hemp or cotton ones, must not be pressed laterally against the sides of the V-shaped grooves, but allowed to rest on the bottom. The grooves for wire ropes are much wider than those for hemp or cotton ropes, and it is necessary to line the bottom of the grooves with some material softer than iron, such as wood, guttapercha, old rope, tarred oakum, or leather, so as to protect the rope, and increase its resistance to slipping. The last material is most extensively used for this purpose. The leather is cut into sections of the dove-tailed shape shown shaded dark in the figures on the next page, and set in on end around the rim. Scrap leather, cut from old shoes or pieces of belting, does very well, but, being very thin, it takes at least a thousand of them for a 7-foot wheel. When many are wanted, it is worth while to make a die to cut them out accurately and quickly. This is the most durable filling that can be made, but it is reported that even leather does not last more than six months' continuous work.* The guide or supporting pulleys do not require to be lined in this way.

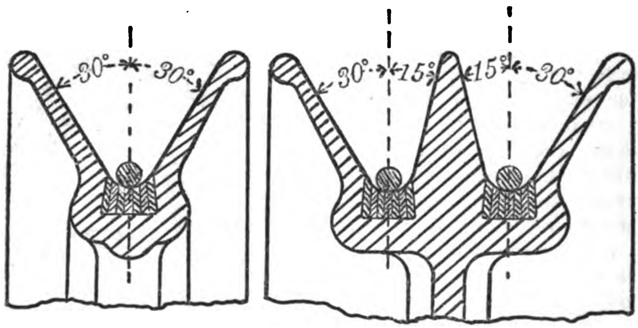


LARGE PULLEY FOR WIRE ROPES.

* It occurs to the author that compressed brown paper might make a good lining for the bottoms of the grooves of these wire-rope pulleys.

22

The ropes used for telodynamic transmission are generally made of steel wires. There are usually six strands in each rope. Each strand is composed of six wires twisted around a hempen core, and these six strands are then laid around a central hempen core. As with hemp or cotton ropes, the strands are twisted in the opposite direction to the wires composing them. Finally,



SINGLE GROOVE. DOUBLE GROOVE FOR RELAYS.

the rope is protected from oxidation by a coating of boiled linseed oil. Considerable trouble is caused by the stretching of these cables, but this may, to a large extent, be prevented by passing them (before use) between grooved compression rollers, which kills this tendency to stretch, although, at the same time, it of necessity slightly diminishes their diameter.

The splicing of these ropes must be done by a practised hand, in order that the splices may not be distinguishable in size, strength, and appearance from the factory made cable. The splices should be of such a length—say 20 to 30 feet for 1-inch ropes—that the friction between the interlaid wires may easily withstand the tension.

These wire ropes, when at work, are subjected to three different stresses—(1) the longitudinal tension due to the power transmitted and their own weight, (2) the bending stress when passing over the pulleys, and (3) the centrifugal stress. As a rule, the longitudinal tension on the tight side is made twice that on the slack side, and the diameter of the pulleys is so chosen that the bending stress is about equal to the maximum longitudinal stress. The centrifugal stress is usually neglected, unless the velocity of the ropes is exceptionally great. The working tension is seldom greater than 15,000 lbs. per square inch of section, although the steel wires composing the same withstand from 70 to 100 tons per

square inch.* The life of these ropes does not appear to be much more than one year when working continuously; consequently, it is found advisable to keep a spare rope spliced and ready for action in case of accident.

Wire Rope Haulage and Transport.—Although this large subject naturally follows what has been said on telodynamic transmission, there is neither space nor time for a full treatment of the question in this book. Consequently, we must refer students who may desire to pursue this subject to those books and papers wherein the different systems are described and discussed.† The various Rope Tramways to be met with in San Francisco, Chicago, New York, Melbourne, London, and Edinburgh are excellent examples of fast speed rope haulage. These instances are, however, excelled as far as speed and distance are concerned by the District Subway of Glasgow. Here, there are two parallel circular iron tunnels, each 6½ miles long, with thirteen stations. Through these run seven trains of two cars each, at a speed of between 18 and 20 miles per hour. These trains are worked by an endless steel wire rope kept in continuous motion by stationary engines coupled to grooved drums and the necessary accessories. When about to leave a station, all that the train driver has to do is to bring the cable-grip into action with the moving rope, and when arriving at one he has simply to disengage it and apply the brake, as may be seen from the accompanying figures.‡

* See Paper on "Wire Ropes," by A. S. Biggart. *Proceedings of the Institute of Civil Engineers*, vol. ci., p. 231, 1889-90.

† See Paper on "The Monte Penna Wire Ropeway," by W. P. Churchward. *Proc. Inst. Civil Engineers*, vol. lxiii., p. 273.

Paper on "Three Systems of Wire Rope Transport," by W. T. H. Carrington. *Proc. Inst. Civil Engineers*, vol. lxv., p. 299.

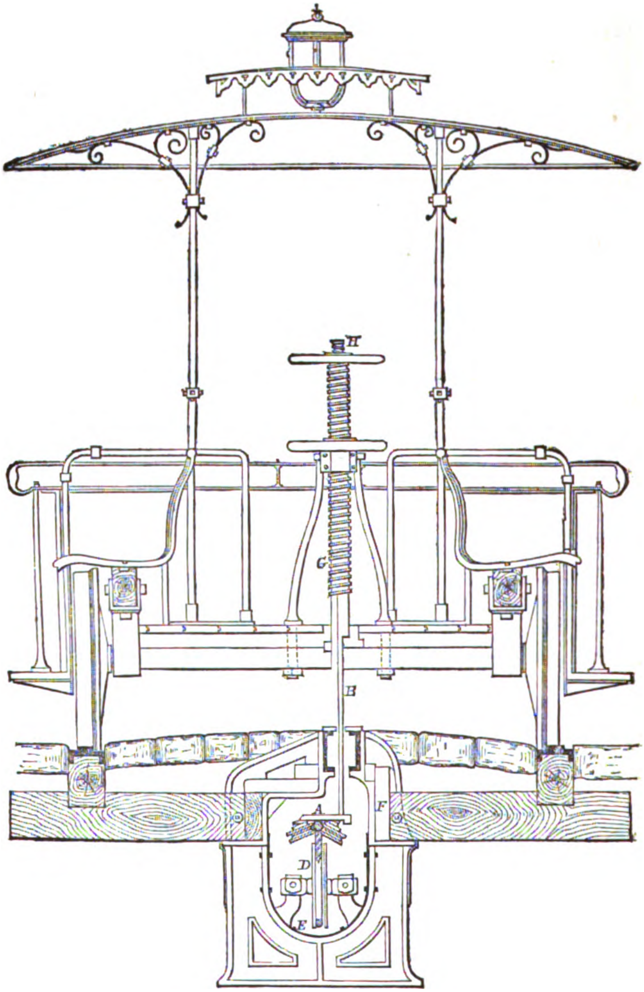
Paper on "Wire Rope Street Railways of San Francisco and Chicago," by W. Morris. *Proc. Inst. Civil Engineers*, vol. lxxii., p. 210.

Paper on "The Temple Street Cable Railway, Los Angelos, California," by F. W. Wood and H. Hawgood. *Proc. Inst. Civil Engineers*, vol. cvii., p. 323.

Street Railways: Their Construction, Operation, and Maintenance, by C. B. Fairchild (The Street Railway Publishing Co., World Buildings, New York).

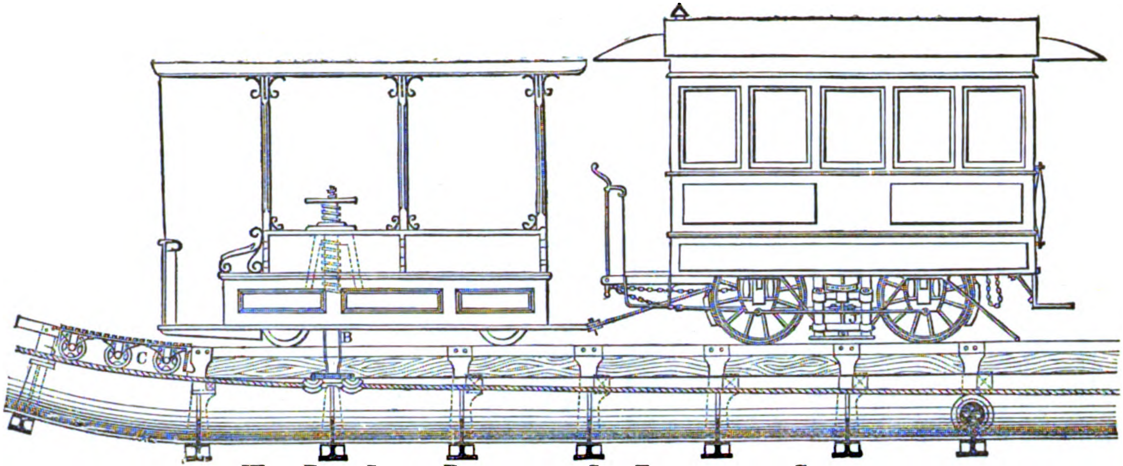
‡ These two figures, and the one of a "Multigroove Rope Drive" for one rope and guide pulley, were kindly provided by the Institution of Civil Engineers with the approval of the Council.

The gripper attachment is fixed to the end of the shank, B, which is a flat bar of iron $5\frac{1}{2}$ inches wide by $\frac{5}{8}$ inch thick, working



CROSS SECTION OF CAR AND GRIPPER.
WIRE-ROPE STREET RAILROADS IN SAN FRANCISCO AND CHICAGO.

through a longitudinal slot. This gripper, A, consists of two pairs of small sheaves about $3\frac{1}{2}$ inches in diameter, and placed at



WIRE-ROPE STREET RAILROADS IN SAN FRANCISCO AND CHICAGO.

an angle, each pair being about 10 inches apart and secured to sliding frames, in which there are also placed jaws, for the purpose of taking a firm hold of the wire rope. The small sheaves act as guides, in the first instance, for the rope to pass between the jaws. When starting, these sheaves are gradually tightened on the rope by the screw, G, and this allows the front, or hauling, car to gradually acquire the speed of the rope. The gripping jaws are then brought into action by means of the small screw, H, and take a firm hold of the rope.

The second car, besides the usual brakes to the wheels, has between each pair of wheels a brake, J, which presses vertically on the rails. It is 28 inches long, and is strong enough to lift the car off the track when empty.

Lecture XVII.—Questions.

1. Under what respective circumstances would you use belt, rope, and chain gearing, and state the advantages and disadvantages of each?

2. Explain how the best leather belts are made and spliced.

3. What is meant by "compound" belting? For what reasons is it preferred to "double" or "treble" belting?

4. Give a short description of "chain," "Victoria," and "composite guttapercha" belting. State their respective advantages and disadvantages.

5. The tension on the slack side of a belt is half that on the tight side. The limiting tension is 40 lbs. for each inch in width of the belt. Find the breadth of the belt to transmit 40 horse-power from a pulley 3 feet in diameter making 100 revolutions per minute (Adv. Sc. & A. Exam., 1894). *Ans.* 70 inches.

6. A belt transmits 35 horse-power when moving at 3,300 feet per minute. Find the net driving tension. If the coefficient of friction be ·3, and the belt embraces half the circumference of the pulley, find the tensions on the driving and slack sides respectively. What width of belt would you use? *Ans.* 350 lbs.; 575 lbs.; 225 lbs.; 11·5 inches.

7. Explain the construction of cotton and hemp ropes for driving machinery, and give their respective strengths and advantages.

8. Sketch a section of the rim of a rope pulley for a 1-inch rope, marking all the dimensions.

9. What is "Telodynamic Transmission of Power?" How is it applied for short and long distances? Give sections and description of the ropes, and of the single and double grooved pulleys used in this system.

10. Describe the machinery employed in the manufacture of wire ropes, and give a detailed account of the process of constructing a wire rope. With the aid of sketches, describe how the ends of such a rope are secured so that its full strength may be utilised (Sc. & A. Hons. Mach. Cons. Exam., 1895). (See Mr. Smith's paper in the *Proc. Mech. Eng.* 1862, and Unwin's *Machine Design*, part I.)

11. Give a sketch and short description of tramway rope haulage.

LECTURE XVIII.

Velocity-Ratio with Belt and Rope Transmission.—It is shown in our elementary treatise that when two pulleys of diameters, D_1, D_2, are connected by a belt or rope, their angular velocity-ratios are inversely as their diameters—*i.e.*, if N_1 and N_2 be their respective number of revolutions in a given time, then :—

$$N_1 : N_2 = D_2 : D_1 \qquad \text{(I)}$$

This equation is only true on the supposition that there is no slipping between the belt or rope and the pulleys ; and also, that the thickness of the belt or rope is so small in comparison with the diameters of the pulleys, that it may be neglected.

Let δ = Diameter of the rope or thickness of the belt.

Then, the working diameter of the pulleys will be $D_1 + \delta$, and $D_2 + \delta$, respectively. Consequently :—

$$N_1 : N_2 = D_2 + \delta : D_1 + \delta \qquad \text{(II)}$$

EXAMPLE I.—Compare the angular velocities of two pulleys of diameters 24 and 10 inches respectively when the thickness of the belt is $\frac{3}{8}$ inch.

(1) Neglecting the thickness of the belt, we get :—

$$\frac{N_1}{N_2} = \frac{D_2}{D_1} = \frac{24}{10} = \frac{2 \cdot 4}{1}.$$

(2) Taking the thickness of the belt into account, we get :—

$$\frac{N_1}{N_2} = \frac{D_2 + \delta}{D_1 + \delta} = \frac{24 + \frac{3}{8}}{10 + \frac{3}{8}} = \frac{195}{83} = \frac{2 \cdot 35}{1}.$$

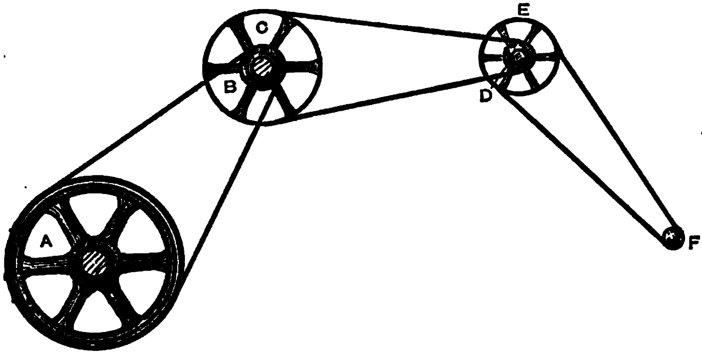
We thus see that by taking the thickness of the belt into account the velocity-ratio is diminished, and in this example in the proportion of 240 to 235, or by 2·1 per cent.

Velocity-Ratio in a Compound System of Belt Gearing.—When there are a number of pulleys, A, B, C, D, E, F, connected together by belting, as illustrated by the above figure, let D_A, D_B, &c., denote their respective diameters, and N_A, N_B, &c., their revolutions per minute.

Then, $\dfrac{N_B}{N_A} = \dfrac{D_A}{D_B}$, and $\dfrac{N_D}{N_C} = \dfrac{D_C}{D_D}$, also $\dfrac{N_F}{N_E} = \dfrac{D_E}{D_F}$.

By multiplying together the corresponding members of these equations, we get:—

$$\frac{N_B}{N_A} \times \frac{N_D}{N_C} \times \frac{N_F}{N_E} = \frac{D_A}{D_B} \times \frac{D_C}{D_D} \times \frac{D_E}{D_F}.$$



VELOCITY-RATIO IN A COMPOUND SYSTEM OF BELT GEARING.

Since the pairs of pulleys, B and C, and D and E, are fixed to their respective shafts, $N_B = N_C$ and $N_D = N_E$. By cancelling these equal values, we get:—

$$\frac{N_F}{N_A} = \frac{D_A \times D_C \times D_E}{D_B \times D_D \times D_F} \quad . \quad . \quad . \quad . \quad . \quad \text{(III)}$$

If we call pulleys A, C, and E the *drivers* and B, D, and F the *followers*, we obtain the following rule:—

$$\frac{\text{Revolutions of last follower}}{\text{Revolutions of first driver}} = \frac{\text{Product of diameters of all the drivers.}}{\text{Product of diameters of all the followers.}}$$

Or, *Speed of first driver* × *product of the diameters of all the drivers.*

= *Speed of last follower* × *product of the diameters of all the followers.*

The radii or circumferences of the pulleys may be substituted for the diameters.

If the thickness of the belts be taken into account, then the pulley diameters must be increased by the thickness of the belts.

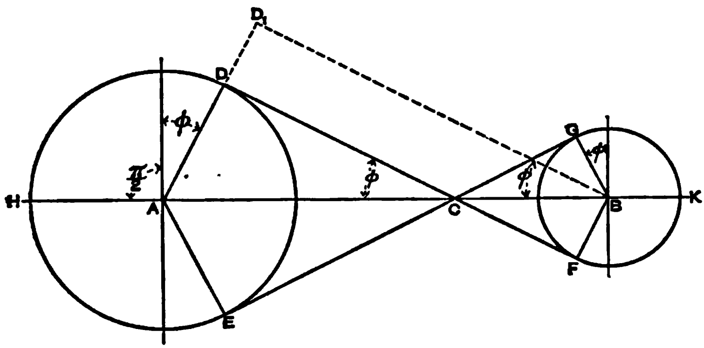
Example II.—An engine is employed to drive a fan by means of pulleys and bands, the first driving band passing over the flywheel of the engine. The train consists of (1) the fly-wheel, A; (2) two pulleys, B and C, on one axis; (3) two other pulleys, D and E, on another axis; and (4) a pulley, F, to the axis of which the fan is attached. The diameters of A, B, C, D, E, F are as 12, 3, 8, 2, 5, 1 respectively, and A makes 20 revolutions per minute. How many revolutions does F make per minute?

Here $D_A = 12$; $D_B = 3$; $D_C = 8$; $D_D = 2$; $D_E = 5$; $D_F = 1$; $N_A = 20$.

Then, $$\frac{N_F}{N_A} = \frac{D_A \times D_C \times D_E}{D_B \times D_D \times D_F} = \frac{12 \times 8 \times 5}{3 \times 2 \times 1} = 80.$$

$$\therefore \quad N_F = N_A \times 80 = 20 \times 80 = \mathbf{1{,}600} \text{ revs. per min.}$$

Length of a Crossed Belt.—An endless belt stretched over two pulleys may be either crossed or open, according as the pulleys are required to rotate in the same or in the opposite direction. We shall now prove by aid of the following figure



Length of a Crossed Belt.

that a driving belt when crossed will serve for any pair of pulleys, so long as the distance between the centres of the pulleys is the same, and the sum of the diameters is constant.

Let R, D = Radius and diameter of pulley, A,
„ r, d = Radius and diameter of pulley, B,
„ a = Distance, A B,
„ L = Length of belt,
„ Σ = Sum of diameters = $D + d$,
„ Δ = Difference of diameters = $D - d$.

Then, $L = 2\,\{\text{arc } HD + DF + \text{arc } FK\}$.

Or, $$L = 2\left\{R\left(\frac{\pi}{2} + \phi\right) + a\cos\phi + r\left(\frac{\pi}{2} + \phi\right)\right\}$$

$$\therefore \quad L = \left(\frac{\pi}{2} + \phi\right)(D + d) + 2a\cos\phi. \quad . \quad . \quad . \quad (IV)$$

If $(D + d)$ has always the same value, then ϕ will also remain constant; and, therefore, L will be a constant length.

From the figure we see that :—

$$\left.\begin{aligned} \sin\phi &= \frac{AD_1}{AB} = \frac{R + r}{a} = \frac{D + d}{2a} = \frac{\Sigma}{2a} \\ \text{But, } \cos\phi &= \sqrt{1 - \sin^2\phi} = \frac{\sqrt{4a^2 - \Sigma^2}}{2a} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (V)$$

$$\therefore \quad L = \left(\frac{\pi}{2} + \phi\right)\Sigma + \sqrt{4a^2 - \Sigma^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (IV_a)$$

This is one equation from which the length of a crossed belt may be calculated.

In using equation (IV_a) it must be observed, that ϕ is expressed in *circular* measure. After obtaining $\sin\phi$ from equation (V), the angle ϕ can be found in circular measure or in degrees by referring to Trigonometrical Tables of Natural Sines. If ϕ be expressed in degrees, let it be denoted by ϕ°, then the circular measure of ϕ is found by multiplying ϕ° by 0·0175.*

Thus, $$\phi = \phi^\circ \times 0{\cdot}0175.$$

EXAMPLE III.—A shaft making 100 revolutions per minute carries a driving pulley $2\frac{1}{2}$ feet in diameter and communicates motion by means of a belt to a parallel shaft at a distance of 6 feet, carrying a pulley 1 foot in diameter. Find the speed of the belt and an expression for its length when crossed. Find also the number of revolutions per minute of the driven shaft, allowing a slip of $1\frac{1}{2}$ per cent. (S. & A. Adv. Exam., 1893).

* How this number is obtained is explained further on in this Lecture.

ANSWER.—(1) *Speed of belt = Circumferential speed of driving pulley,*

$$,, \quad ,, \quad = \pi D N,$$

$$,, \quad ,, \quad = 3{\cdot}1416 \times 2\tfrac{1}{2} \times 100 = \mathbf{785{\cdot}4 \text{ feet per minute.}}$$

(2) The length of a crossed belt has been shown in the text to be:—

$$L = \left(\frac{\pi}{2} + \phi\right)\Sigma + \sqrt{4\,a^2 - \Sigma^2}.$$

From the data, we get:—$\Sigma = D + d = 2{\cdot}5 + 1 = 3{\cdot}5$ feet, $a = 6$ feet, and $\sin\theta = \dfrac{\Sigma}{2\,a} = \dfrac{3{\cdot}5}{12} = {\cdot}2917$.

Referring to a table of Natural Sines, we find the angle whose sine is $= {\cdot}2917$ to be about 17°.

Hence, $\phi = 17 \times 0{\cdot}0175 = {\cdot}2975$ radians.

$$\therefore \qquad L = \left(\frac{3{\cdot}1416}{2} + {\cdot}2975\right) 3{\cdot}5 + \sqrt{4 \times 6^2 - 3{\cdot}5^2}.$$

$$i.e., \qquad L = 6{\cdot}54 + 11{\cdot}48 = \mathbf{18{\cdot}02 \text{ feet.}}*$$

(3) If there were no slip, the speed of the driven pulley would be:—

$$n = N \times \frac{D}{d} = 100 \times \frac{2{\cdot}5}{1} = \mathbf{250 \text{ revolutions per minute.}}$$

* The student should notice that $\sin\phi$ and ϕ agree to the second decimal figure, and this is true for all angles up to about 21°. The smaller the angle the more nearly do $\sin\phi$ and ϕ agree in numerical value. Now, in the examination room, no tables of Natural Sines are allowed, and the student is not expected to calculate ϕ from the value of its sine. But in engineering problems of this kind it is considered sufficiently accurate to take numbers to two or three decimal figures; hence the student may assume that—

$$\phi = \sin\phi = {\cdot}292.$$

$$\text{Then,} \qquad L = \left(\frac{3{\cdot}1416}{2} + {\cdot}292\right) 3{\cdot}5 + \sqrt{4 \times 6^2 - 3{\cdot}5^2},$$

$$i.e., \qquad L = 6{\cdot}52 + 11{\cdot}48 = \mathbf{18 \text{ feet.}}$$

This only differs by about ·1 per cent. from the correct answer. The error introduced when we take $\phi = \sin\phi$ is $\cdot2975 - \cdot292 = \cdot0055$, or ·55 per cent. only.

But since there is $1\frac{1}{2}$ per cent. slip the actual speed is only $98\frac{1}{2}$ per cent. of this.

$$\therefore \quad n_1 = \frac{98 \cdot 5}{100} \times 250 = 246 \cdot 25 \text{ revolutions per minute.}$$

The length of the belt could also be obtained by drawing the figure accurately to scale. The student should, therefore, obtain an answer in this way to the second part of the above question, and compare his results with those which we have just found.

EXAMPLE IV.—A shaft, having a stepped speed-cone with four steps, revolves at a constant speed of 180 revolutions per minute, and is connected, by a crossed belt, to another shaft having a similar stepped cone. The diameter of the largest step of the cone on the driving shaft is 16 inches. The driven shaft is required to run at speeds 480, 300, 160, and 90 revolutions per minute respectively. Determine the diameters of the remaining steps of the two cones.

ANSWER.—Let D_1, D_2, D_3, D_4 denote the diameters of the four steps of the cone on the driving shaft; D_1 being the diameter of the largest step.

Let d_1, d_2, d_3, d_4 denote the diameters of the four steps of the cone on the driven shaft; d_1 being the diameter of the smallest step.

Then, $\quad D_1 + d_1 = D_2 + d_2 = D_3 + d_3 = D_4 + d_4 = \Sigma.$

Let N denote the speed of the driving shaft, and let N_1, N_2, N_3, N_4 denote the four different speeds of the driven shaft, so that

$$N = 180; \; N_1 = 480; \; N_2 = 300; \; N_3 = 160; \; N_4 = 90.$$

We have first to determine d_1, and thereby Σ.

Here, $\quad \dfrac{d_1}{D_1} = \dfrac{N}{N_1}.$

$$\therefore \quad d_1 = \frac{180}{480} \times 16 = 6 \text{ inches.}$$

$$\therefore \quad \Sigma = 16 + 6 = 22 \text{ inches.}$$

For steps D_2, d_2, we get :—

$$\frac{D_2}{d_2} = \frac{N_2}{N}; \quad \therefore \frac{D_2}{D_2 + d_2} = \frac{N_2}{N_2 + N}.$$

Hence, $\quad D_2 = \dfrac{300}{300 + 180} \times 22 = 13 \cdot 75$ inches.

And, $\quad d_2 = 22 - 13 \cdot 75 = 8 \cdot 25$ inches.

Similarly, $D_3 = \frac{N_3}{N_3 + N} \Sigma = \frac{160}{160 + 180} \times 22 = \mathbf{10{\cdot}35}$ **ins.**

And, $d_3 = 22 - 10{\cdot}35 = \mathbf{11{\cdot}65}$ **inches.**

Again, $D_4 = \frac{N_4}{N_4 + N} \Sigma = \frac{90}{90 + 180} \times 22 = \mathbf{7{\cdot}\dot{3}}$ **inches.**

And, $d_4 = 22 - 7{\cdot}\dot{3} = \mathbf{14{\cdot}\dot{6}}$ **inches.**

Hence, we have:—

Driving Cone.	Driven Cone.
$D_1 = 16$ inches.	$d_1 = 6$ inches.
$D_2 = 13{\cdot}75$ „	$d_2 = 8{\cdot}25$ „
$D_3 = 10{\cdot}35$ „	$d_3 = 11{\cdot}65$ „
$D_4 = 7{\cdot}\dot{3}$ „	$d_4 = 14{\cdot}\dot{6}$ „

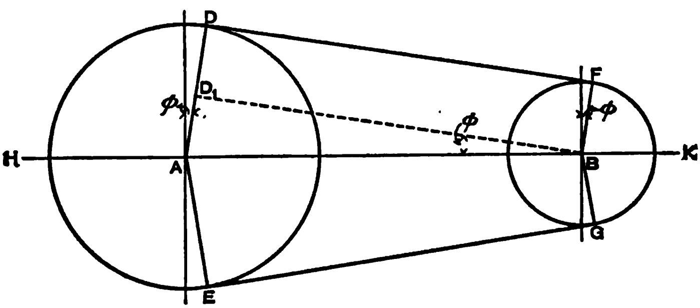
Length of an Open Belt.—By referring to the next figure and proceeding as before, we can determine the length of an open belt connecting any pair of pulleys.

Here, $L = 2 \{ \text{arc } HD + DF + \text{arc } FK \}$.

Or, $L = 2 \left\{ R\left(\frac{\pi}{2} + \varphi\right) + a \cos \varphi + r\left(\frac{\pi}{2} - \varphi\right) \right\}$.

Or, $L = \frac{\pi}{2}(D + d) + \varphi (D - d) + 2a \cos \varphi$.

$\therefore \quad L = \frac{\pi}{2} \Sigma + \varphi \Delta + 2a \cos \varphi. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad$ (VI)



Length of an Open Belt.

Here Σ and Δ cannot both be constant quantities for two or more pairs of pulleys. Hence, if the sum of the diameters of each pair of pulleys be the same, an *open* belt of constant length

cannot be made to work equally tight on the various pairs of pulleys.

We shall now obtain an approximate and more convenient formula for the length of an open belt than that given by equation (VI).

Since, in this case, φ is always very small, we may write :—

$$\varphi = \sin \varphi.$$

$$\left.\begin{aligned} \text{But,} \quad & \sin \varphi = \frac{\mathrm{A\,D_1}}{\mathrm{A\,B}} = \frac{\Delta}{2\,a}. \\ \therefore \quad & \varphi = \frac{\Delta}{2\,a}, \text{ approximately.} \end{aligned}\right\} \quad \ldots\ldots \quad \text{(VII)}$$

$$\therefore \quad \varphi\,\Delta = \frac{\Delta^2}{2\,a}, \quad \text{"}$$

$$\text{Also,} \quad \cos \varphi = \sqrt{1 - \sin^2 \varphi} = \frac{\sqrt{4\,a^2 - \Delta^2}}{2\,a}$$

Substituting these values of $\varphi\,\Delta$ and $\cos \varphi$ in equation (VI),

$$\text{We get,} \quad \mathrm{L} = \frac{\pi}{2}\Sigma + \frac{\Delta^2}{2\,a} + \sqrt{4\,a^2 - \Delta^2}.$$

$$\text{Or,} \quad \mathrm{L} = \frac{\pi}{2}\Sigma + 2\,a\left\{\frac{\Delta^2}{4\,a^2} + \left(1 - \frac{\Delta^2}{4\,a^2}\right)^{\frac{1}{2}}\right\}$$

By the binomial theorem, we can expand the last expression in this equation to any required number of terms.

$$\text{Thus,} \left(1 - \frac{\Delta^2}{4\,a^2}\right)^{\frac{1}{2}} = 1 - \tfrac{1}{2}\left(\frac{\Delta^2}{4\,a^2}\right) + \frac{\frac{1}{2}(\frac{1}{2} - 1)}{1 \cdot 2} \cdot \left(\frac{\Delta^2}{4\,a^2}\right)^2, \text{ \&c.}$$

The third and following terms on the right-hand side are very small, since $\frac{\Delta^2}{4\,a^2}$ is of itself very small. We may, therefore, neglect all terms after the second.

$$\text{Thus,} \left(1 - \frac{\Delta^2}{4\,a^2}\right)^{\frac{1}{2}} = 1 - \frac{\Delta^2}{8\,a^2}, \text{ approximately.}$$

$$\text{Hence,} \quad \mathrm{L} = \frac{\pi}{2}\,\Sigma + 2\,a\left\{\frac{\Delta^2}{4\,a^2} + 1 - \frac{\Delta^2}{8\,a^2}\right\}.$$

$$\textit{i.e.,} \quad \mathrm{L} = \frac{\pi}{2}\,\Sigma + 2\,a\left\{1 + \frac{\Delta^2}{8\,a^2}\right\}. \quad \ldots \quad (\mathrm{VI}_a)$$

This is evidently a more convenient expression for the length of an open belt than equation (VI).

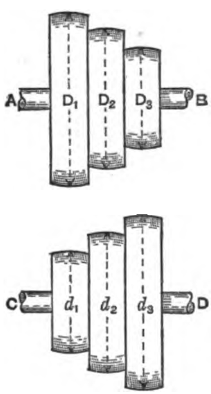
In the case of a crossed belt it has been shown that the belt will work equally tight and well on any pair of pulleys on the same shafts so long as the sum of the diameters of the two pulleys is constant. Thus, if a crossed belt be used for connecting two stepped speed cones, the sum of the diameters of alternate pulleys must be constant. If, however, an open belt be used the sum of the diameters of each pair of working pulleys will not be constant. Consider two pairs of pulleys on the stepped cones.

Let D_1, d_1 = Diameters of first pair.
„ D_2, d_2 = Diameters of second pair.
„ $\Sigma_1 = D_1 + d_1$; $\Sigma_2 = D_2 + d_2$.
„ $\Delta_1 = D_1 - d_1$; $\Delta_2 = D_2 - d_2$.

Then, since the length of the belt is constant, we get :—

$$\frac{\pi}{2}\Sigma_2 + 2a\left\{1 + \frac{\Delta_2^2}{8a^2}\right\} = \frac{\pi}{2}\Sigma_1 + 2a\left\{1 + \frac{\Delta_1^2}{8a^2}\right\}.$$

$$\therefore \qquad \Sigma_2 = \Sigma_1 + \frac{\Delta_1^2 - \Delta_2^2}{2\pi a} \quad . \quad . \quad . \text{(VIII)}$$



STEPPED CONES.

This formula gives us the sum of the diameters of the second pair of pulleys in terms of the sum and difference of the diameters of the first pair. The difference, Δ_2, of the diameters of the second pair of pulleys also enters into this equation, and this must be found before Σ_2 can be obtained.

Let A B be the main shaft running constantly at N revolutions per minute. Let N_1, N_2 be the speeds of the shaft C D when the belt is on the pulleys 1 and 2 respectively.

Now, calculate Δ_2 on the assumption that the belt is crossed. This is not exactly, but very approximately, correct. Hence, neglecting the thickness of the belt, we get :—

$$\frac{D_2}{d_2} = \frac{N_2}{N}; \qquad \therefore \frac{D_2 - d_2}{D_2 + d_2} = \frac{N_2 - N}{N_2 + N}.$$

$$\textit{i.e.,} \qquad \frac{\Delta_2}{\Sigma_2} = \frac{N_2 - N}{N_2 + N}; \quad \therefore \Delta_2 = \frac{N_2 - N}{N_2 + N}\Sigma_2. \quad . \quad . \quad \text{(IX)}$$

But, since we are to *assume* that the belt is crossed in this calculation, we must have $\Sigma_2 = \Sigma_1$.

$$\therefore \qquad \Delta_2 = \frac{N_2 - N}{N_2 + N} \Sigma_1, \text{ approximately.} \quad . \quad . \quad . \quad . \quad (IX_a)$$

Substituting this approximate value for Δ_2 in equation (VIII), we get Σ_2. Having now calculated Σ_2, we can easily find D_2 and d_2. Thus:—

$$\frac{D_2}{d_2} = \frac{N_2}{N}; \therefore \frac{D_2}{D_2 + d_2} = \frac{N_2}{N_2 + N},$$

$$\therefore \qquad D_2 = \frac{N_2}{N_2 + N} \Sigma_2, \text{ and } d_2 = \frac{N}{N_2 + N} \Sigma_2. \quad . \quad . \quad (X)$$

In the very same way, the diameters of all the other pairs of pulleys in the stepped cone can be found.

Hence, the following practical rule for designing a set of stepped speed-cones, worked by an open belt:—

Let N = Constant speed of driving or main shaft, A B.

N_1, N_2, N_3, &c., = Required speeds of driven shaft, C D.

(*a*) Fix on convenient diameters D_1, d_1 to give the required velocity-ratio with any one pair of pulleys. This will give Σ_1 and Δ_1.

(*b*) Next calculate Δ_2, Δ_3, &c., for the other pulleys *on the assumption that the belt is crossed;* or by formula (IX_a).

(*c*) Insert these values successively in equation (VIII), from which Σ_2, Σ_3, &c., can be found.

(*d*) The diameters can then be found from equations (X).

EXAMPLE V.—The centres of two pulleys, 4 and 2 feet in diameter respectively, are 8 feet apart. The pulleys are connected by an open belt; find its length.

ANSWER.—Here $\Sigma = 6$ feet; $\Delta = 2$ feet; $a = 8$ feet.

From equation (VI_a) we get:—

$$L = \frac{\pi}{2} \Sigma + 2a \left\{ 1 + \frac{\Delta^2}{8a^2} \right\}$$

$$,, = \frac{3 \cdot 1416}{2} \times 6 + 2 \times 8 \left\{ 1 + \frac{4}{8 \times 64} \right\} \text{ feet.}$$

$$,, = 9 \cdot 4248 + 16 \cdot 125 = 25 \cdot 55 \text{ feet.}$$

EXAMPLE VI.—If the speed cones in Example IV. are connected by means of an open belt instead of a crossed one, and

the distance between the centres of the two shafts is 8 feet, determine the diameters of the various steps.

Answer.—Using the same notation as in Example IV., we get, as before, $D_1 = 16$ inches; $d_1 = 6$ inches; $a = 8 \times 12 = 96$ inches; $\Sigma_1 = 22$ inches; and $\Delta_1 = 10$ inches.

(1) *To find* D_2, *and* d_2.

First, calculate D_2 and d_2 on the assumption that the belt is crossed.

From Example IV. we get:—

$$D_2 = 13{\cdot}75 \text{ inches, and } d_2 = 8{\cdot}25 \text{ inches.}$$

Hence, as a first approximation, we may write:—

$$\Sigma_2 = 22 \text{ inches, and } \Delta_2 = 5{\cdot}5 \text{ inches.}$$

Next, recalculate Σ_2 from equation (VIII) in the text, viz.:—

$$\Sigma_2 = \Sigma_1 + \frac{\Delta_1^2 - \Delta_2^2}{2\pi a} = 22 + \frac{10^2 - 5{\cdot}5^2}{2 \times \frac{22}{7} \times 96} = 22{\cdot}1155 \text{ inches.}$$

Substituting this new value of Σ_2 in equation (X), we get:—

$$D_2 = \frac{N_2}{N_2 + N}\Sigma_2 = \frac{300}{300 + 180} \times 22{\cdot}1155 = 13{\cdot}822 \text{ inches.}$$

$$d_2 = 22{\cdot}1155 - 13{\cdot}822 = 8{\cdot}293 \text{ inches.}$$

These are the diameters of the second step as obtained for *one correction*. If a closer approximation be required, then we must again recalculate Σ_2 from the results just obtained. Thus:—

$$\Delta_2 = 13{\cdot}822 - 8{\cdot}293 = 5{\cdot}529 \text{ inches.}$$

$$\therefore \Sigma_2 = \Sigma_1 + \frac{\Delta_1^2 - \Delta_2^2}{2\pi a} = 22 + \frac{10^2 - 5{\cdot}529^2}{2 \times \frac{22}{7} \times 96} = 22{\cdot}1148 \text{ inches.}$$

Substituting this new value for Σ_2 in equation (X), we get:—

$$D_2 = \frac{300}{300 + 180} \times 22{\cdot}1148 = 13{\cdot}8217 \text{ inches.}$$

$$\therefore d_2 = 22{\cdot}1148 - 13{\cdot}8217 = 8{\cdot}2931 \text{ inches.}$$

which are the values of D_2 and d_2 *corrected twice.*

It will be at once seen that these last values differ very slightly from those obtained by one correction. Hence, it will be sufficiently accurate to make one correction only.

(2) *To find* D_3 *and* d_3.

As before, calculate D_3 and d_3 on the assumption that the belt is crossed. From Example IV., we get:—

$$D_3 = 10{\cdot}35 \text{ inches, and } d_3 = 11{\cdot}65 \text{ inches.}$$

$$\therefore \quad \Sigma_3 = 22 \text{ inches, and } \Delta_3 = 1{\cdot}3 \text{ inches.}$$

Next recalculate Σ_3 from equation (VIII).

$$\therefore \quad \Sigma_3 = \Sigma_1 + \frac{\Delta_1^2 - \Delta_3^2}{2\pi a} = 22 + \frac{10^2 - 1{\cdot}3^2}{2 \times \frac{22}{7} \times 96} = 22{\cdot}163 \text{ inches.}$$

Substituting this value in equation (X), we get:—

$$D_3 = \frac{N_3}{N_3 + N}\Sigma_3 = \frac{160}{160 + 180} \times 22{\cdot}163 = 10{\cdot}43 \text{ inches.}$$

$$\therefore d_3 = 22{\cdot}163 - 10{\cdot}43 = 11{\cdot}733 \text{ inches.}$$

(3) In a similar way, we get:—

$$D_4 = 7{\cdot}359 \text{ inches.}$$

And, $\quad d_4 = 14{\cdot}718$ inches.

Hence, for an open belt we get the following sizes for the steps of the cones:—

Driving Cone.	Driven Cone.
$D_1 = 16$ inches.	$d_1 = 6$ inches.
$D_2 = 13{\cdot}822$ „	$d_2 = 8{\cdot}293$ „
$D_3 = 10{\cdot}43$ „	$d_3 = 11{\cdot}733$ „
$D_4 = 7{\cdot}359$ „	$d_4 = 14{\cdot}718$ „

Frictional Resistance between a Belt or Rope and its Pulley.— In Lecture VII., we deduced a formula for the ratio of the tensions of the two parts of a belt or rope when stretched round a pulley or post, viz.:—

$$\log_e \frac{T_d}{T_s} = \mu\,\theta.$$

Where T_d = Tension on driving or tight side of belt or rope.

„ T_s = Tension on driven or slack side of belt or rope.

„ μ = Coefficient of friction between the belt or rope and the pulley.

„ θ = Circular measure of angle subtended at the centre of the pulley by the belt or rope.

The following values of μ have been obtained by experiment:—

For leather belts on iron pulleys,	$\mu = 0{\cdot}3$ to $0{\cdot}4$.
For hemp or cotton ropes on iron pulleys,	$\mu = 0{\cdot}35$ (*pulleys dry*). $\mu = 0{\cdot}15$ (*pulleys greased*).
For wire ropes on iron pulleys,	$\mu = 0{\cdot}15$.
For wire ropes on iron pulleys, the grooves being lined with leather or guttapercha,	$\mu = 0{\cdot}25$.

Slipping of a belt generally occurs at the smaller pulley, the angle of contact, θ, being smaller on this pulley than on the larger one. Hence, in calculations relating to the slipping of a belt it is usual to consider the small pulley only. The average length of arc embraced by the belt on the small pulley may be taken at $\frac{4}{10}$ of the whole circumference—*i.e.*, $\theta = {\cdot}4 \times 2\,\pi = 2{\cdot}5133$ radians. If, then, we take $\mu = 0{\cdot}3$, we can easily find the ratio between T_d and T_s in the case of a leather belt.

Thus, $$\log_e \frac{T_d}{T_s} = 0{\cdot}3 \times 2{\cdot}5133.$$

Or, changing this into common logarithms, by multiplying by $0{\cdot}4343$, we get:—

$$\text{Log} \frac{T_d}{T_s} = 0{\cdot}3 \times 2{\cdot}5133 \times 0{\cdot}4343 = {\cdot}327457$$

$$\text{,,} \quad \text{,,} = \log 2{\cdot}125, \text{ nearly.}$$

$$\therefore \qquad T_d = 2\,T_s \text{ approximately.}$$

If the angle of contact between the belt and the pulley be expressed in degrees, then the equation requires to be modified.

Let θ° = angle of contact expressed in degrees. Then, we know that:—

$$\frac{\theta}{\theta^\circ} = \frac{\text{Circular measure of two right angles}}{\text{Number of degrees in two right angles}} = \frac{\pi}{180}.$$

$$\therefore \qquad \theta = \frac{3{\cdot}1416}{180} \times \theta^\circ = 0{\cdot}0175\,\theta^\circ.$$

Substituting this value for θ, we get:—

$$\log_e \frac{T_d}{T_s} = 0{\cdot}0175\,\mu\,\theta^\circ. \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad \text{(XI)}$$

Or, changing into common logarithms, by multiplying by $0{\cdot}4343$, we get:—

$$\log \frac{T_d}{T_s} = 0{\cdot}4343 \times 0{\cdot}0175\,\mu\,\theta^\circ = 0{\cdot}0076\,\mu\,\theta^\circ. \quad . \quad (\text{XI}_a)$$

GREATEST VALUE OF THE RATIO OF TENSIONS ON TIGHT AND SLACK SIDES OF BELTING.*

Angle embraced by Belt.			Ratio of Tensions $= \frac{T_d}{T_s}$.			
In Degrees. $\theta°$.	In Circular Measure. θ.	In Fraction of Circumference. $\theta°/360° = \theta/2\pi$.	$\mu = 0·2$.	$\mu = 0·3$.	$\mu = 0·4$.	$\mu = 0·5$.
30	·524	·083	1·110	1·170	1·233	1·299
45	·785	·125	1·170	1·266	1·369	1·481
60	1·047	·167	1·233	1·369	1·521	1·689
75	1·309	·208	1·299	1·481	1·689	1·924
90	1·571	·250	1·369	1·602	1·874	2·193
105	1·833	·319	1·443	1·733	2·082	2·500
120	2·094	·334	1·521	1·875	2·312	2·851
135	2·356	·375	1·602	2·027	2·565	3·247
150	2·618	·417	1·689	2·194	2·849	3·702
165	2·880	·458	1·778	2·372	3·163	4·219
180	3·142	·500	1·875	2·566	3·514	4·808
195	3·403	·541	1·975	2·776	3·901	5·483
210	3·665	·583	2·082	3·003	4·333	6·252
240	4·188	·666	2·311	3·514	5·340	8·119
270	4·712	·750	2·566	4·112	6·589	10·550
300	5·236	·833	2·849	4·808	8·117	13·700

Frictional Resistance between a Rope and a Grooved Pulley.—In rope transmission the pulleys over which the rope passes are provided with grooves, and we saw in our last Lecture that for hemp or cotton ropes the grooves are so constructed that the ropes get wedged into them and bear on their sides. This, as we have seen, is the method adopted in order to increase the resistance to slipping. In Lecture VII. we found that the ratio between T_d and T_s in this case is given by the equation :—

$$\text{Log}_e \frac{T_d}{T_s} = \mu \operatorname{cosec} \frac{\alpha}{2} \theta.$$

Where μ = Coefficient of friction between rope and sides of groove.
„ α = Angle which sides of groove make with each other.
„ θ = Angle embraced by rope on pulley.

Comparing this equation with that for flat pulleys we see that the logarithm of the ratio of the tensions is increased in this case in the proportion :—cosec $\frac{\alpha}{2}$: 1.

* From Unwin's *Machine Design*, part I., p. 377.

Generally the groove angle, α, is about 45°, and then we get:—

$$\operatorname{cosec} 22\tfrac{1}{2}° = 2\cdot6.$$

Hence,
$$\operatorname{Log}_e \frac{T_d}{T_s} = 2\cdot6\ \mu\ \theta.$$

Or, converting into common logarithms, we get:—

$$\operatorname{Log} \frac{T_d}{T_s} = 0\cdot4343 \times 2\cdot6\ \mu\ \theta = 1\cdot13\ \mu\ \theta.$$

We have already stated that $\mu = 0\cdot15$ to $0\cdot35$ for hemp or cotton ropes, the smaller value being taken when the pulleys are greased. Substituting these values in the last equation, we get:—

$$\operatorname{Log} \frac{T_d}{T_s} = 0\cdot17\ \theta \text{ (pulleys greased).}$$

Also,
$$\operatorname{Log} \frac{T_d}{T_s} = 0\cdot39\ \theta \text{ (pulleys dry).}$$

If for $\mu \operatorname{cosec} \frac{\alpha}{2}$ we write μ_1 the equation becomes:—

$$\log_e \frac{T_d}{T_s} = \mu_1 \theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(XII)}$$

Where μ_1 can be looked upon as the coefficient of friction for grooved pulleys.

Converting into common logarithms, by multiplying by 0·4343, and substituting $\theta°$ for θ, equation (XII) becomes:—

$$\operatorname{Log} \frac{T_d}{T_s} = 0\cdot4343 \times 0\cdot0175\,\mu_1\,\theta° = 0\cdot0076\,\mu_1\,\theta°. \quad . \quad . \quad . \quad (\text{XII}_a)$$

VALUES OF μ_1 CORRESPONDING TO DIFFERENT VALUES OF μ AND α.

Angle of Groove in Degrees α.	Values of μ_1 when				
	$\mu = 0\cdot15$.	$\mu = 0\cdot20$.	$\mu = 0\cdot25$.	$\mu = 0\cdot30$.	$\mu = 0\cdot35$.
30	·58	·77	·97	1·16	1·35
35	·50	·66	·83	1·00	1·16
40	·44	·58	·73	·88	1·02
45	·39	·52	·65	·78	·91

GREATEST VALUE OF THE RATIO OF TENSIONS ON TIGHT AND SLACK SIDES OF A ROPE WORKING IN GROOVED PULLEYS.

Angle Embraced by Rope.			Ratio of Tensions $= \frac{T_d}{T_s}$.					
In Degrees. $\theta°$.	In Circular Measure. θ.	In Fraction of Circumference. $\frac{\theta°}{360°} = \frac{\theta}{2\pi}$.	$\mu_1 = 0·4$.	$\mu_1 = 0·5$.	$\mu_1 = 0·6$.	$\mu_1 = 0·7$.	$\mu_1 = 0·8$.	$\mu_1 = 0·9$.
60	1·047	·167	1·52	1·69	1·87	2·08	2·31	2·57
90	1·571	·250	1·87	2·19	2·57	3·00	3·51	4·11
120	2·094	·334	2·31	2·85	3·51	4·33	5·34	6·59
150	2·618	·417	2·85	3·70	4·81	6·25	8·12	10·55
180	3·142	·500	3·51	4·81	6·59	9·02	12·35	16·90
210	3·665	·583	4·33	6·25	9·02	13·01	18·77	27·08
240	4·188	·666	5·34	8·12	12·35	18·77	28·53	43·38

"Slip" or "Creep" of Belts due to Elasticity.—Although there may be no actual slipping as a whole between a belt and its pulleys, yet the unequal stretching of the two parts of the belt, due to the unequal tensions, T_d and T_s, causes a difference in angular velocity-ratio, which becomes very serious when this requires to be kept uniform

It will be seen in a subsequent Lecture that any elastic material subjected to tension becomes elongated, and that the elongation depends directly on the stretching force. Hence, the belt will be stretched to a greater extent on the driving side than on the slack side.

Let l = Length of belt which would leave either pulley in unit time, if the belt were unstretchable or inelastic.

" l_d = Length of belt running on to driving pulley in unit time.

" l_s = Length of belt running on to driven pulley in unit time.

" e = Elongation produced by a tension of 1 lb. on a length of 1 foot of belting.

Then, $$l_d = l + e\,T_d\,l = (1 + e\,T_d)\,l.$$

Similarly, $$l_s = (1 + e\,T_s)\,l.$$

Since we have supposed that there is no actual slipping between the belt and pulley as a whole, it is clear that the

length of belting coming on to a pulley in unit time will be equal to the length of the arc described by a point on the circumference of the pulley in the same time. In other words, the length of belting coming on to the pulley in unit time, is equal to the circumferential speed of pulley. Hence, for the driving pulley, $\pi D N = l_d$; and for the driven pulley, $\pi d n = l_s$.

$$\therefore \qquad \frac{d\,n}{D\,N} = \frac{l_s}{l_d} = \frac{(1 + e\,T_s)\,l}{(1 + e\,T_d)\,l}.$$

Or,
$$\frac{n}{N} = \left(\frac{1 + e\,T_s}{1 + e\,T_d}\right)\frac{D}{d}. \quad . \quad . \quad . \quad . \quad \text{(XIII)}$$

If we knew the value of e in all cases, and then calculated T_d and T_s by the previous formulæ, we might obtain the actual velocity-ratio for such cases. From experiments by M. Kretz, it appears that:—

$$\frac{1 + e\,T_s}{1 + e\,T_d} = 0{\cdot}975 \textit{ for new, and } = 0{\cdot}978 \textit{ for old, leather belts.}$$

Taking these results, we get:—

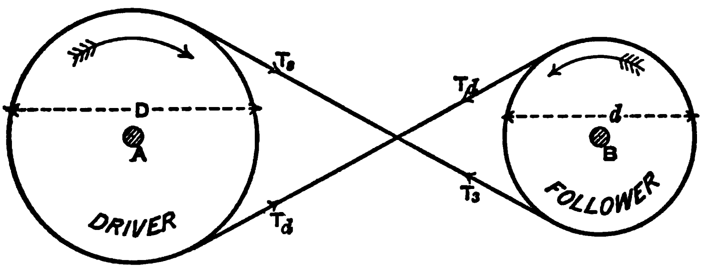
$$\left.\begin{aligned} \frac{n}{N} &= 0{\cdot}975 \times \frac{D}{d} \textit{ for new leather belts.} \\ \text{And,} \quad \frac{n}{N} &= 0{\cdot}978 \times \frac{D}{d} \textit{ for old leather belts.} \end{aligned}\right\} \quad . \quad . \quad \text{(XIII}_a\text{)}$$

It therefore appears that the velocity of the driven pulley is about 2 per cent. less than it would be if the belt were inelastic. This loss of velocity is termed the "slip," or "creep," due to elasticity. Consequently, when motion has to be transmitted through several belts, this loss becomes serious, and renders belt gearing unsuitable for exact velocity-ratios.

Horse Power transmitted by Belt and Rope Gearing.—When one pulley, A, drives another pulley, B, by means of a belt or rope, there is necessarily a difference in tension in the two parts of the belt or rope. First of all, there is a resistance to motion offered by the follower, B. Hence, when the driver, A, is made to rotate, the friction between the belt or rope and the pulleys prevents slipping, with the result that the tension in that part of the belt or rope going on to the driver *increases*, while the tension in the other part *decreases*. This increase of tension in the driving side, and the decrease in the slack side, will go on until the resultant moment of the tensions in the two parts

of the belt or rope acting on the follower overcomes the resisting couple acting on that pulley. When this takes place:—

Let T_d = Tension in *driving* side of belt or rope.
„ T_s = Tension in *slack* side of belt or rope.
„ M = Moment of couple resisting rotation of follower.
„ V = Velocity of belt or rope in feet per minute.
„ D = Diameter of driver in feet.
„ d = Diameter of follower in feet.
„ N = Speed of driver in revolutions per minute.
„ n = Speed of follower in revolutions per minute.
„ H.P. = Horse-power transmitted.



POWER TRANSMITTED BY A BELT OR ROPE.

Then, $$M = (T_d - T_s)\frac{d}{2}.$$

$$\therefore \left.\begin{array}{c}\textit{Work transmitted}\\ \textit{per minute}\end{array}\right\} = M \times 2\pi n = (T_d - T_s) \times \pi d n.$$

$$\left.\begin{array}{lr}\therefore & \text{H.P.} = \dfrac{(T_d - T_s)\pi d n}{33{,}000}\\ \text{But,} & V = \pi D N = \pi d n.\\ \therefore & \text{H.P.} = \dfrac{(T_d - T_s)V}{33{,}000} = \dfrac{PV}{33{,}000}\end{array}\right\} \quad . \quad . \quad \text{(XIV)}$$

Where $P = T_d - T_s$, and is called the driving force or tension.

$$\therefore \qquad P = \frac{33{,}000 \text{ H.P.}}{V}$$

Let T = Initial or average tension; or tension in *both* parts of belt when at rest.

The average tension during motion must be the same as the tension before motion commenced, since the lengthening of the belt on the driving side must be equal to the shortening of the belt on the slack side.

$$\therefore \qquad T = \frac{T_d + T_s}{2}.$$

This must be the tension with which the belt should be initially stretched over the pulleys.

If, for these reasons, we suppose $T_d = 2\,T_s$, we get the following practical rules for belt gearing :—

$$\textit{Driving Tension} = P = \frac{33{,}000 \text{ H.P.}}{V}.$$

$$\textit{Greatest Tension} = T_d = 2\,P.$$

$$\textit{Initial Tension} = T = 1\tfrac{1}{2}\,P.$$

$$\textit{Breadth of Belt} = \beta = \frac{T_d}{f} = \frac{2\,P}{f}.$$

Where, f is the safe working tension per inch of width or breadth β.

The following table gives the safe working tension, f, in lbs. per inch of width for leather belts, when the safe stress in lbs. per square inch cross sectional area is known :—

Working Tension of Leather Belts.

Thickness of Belt in Inches.	Safe working tension, f, in lbs. per inch of width when the safe stress in lbs. per square inch of cross sectional area is :—		
	250	300	350
$\frac{3}{16}$	47	56	66
$\frac{7}{32}$	55	66	77
$\frac{1}{4}$	62	75	87
$\frac{5}{16}$	78	94	109

In the case of hemp or cotton rope gearing, the tension T_d may be taken at 140 lbs. per square inch of *gross sectional area*, or about $\frac{1}{10}$ of the working strength of the rope.

The following table gives the working strength and driving force for hemp or cotton ropes:—

WORKING STRENGTH, &c., FOR DRIVING ROPES.

Diameter of Rope in inches. $= \delta$	Girth of Rope in inches. $= \gamma$	Working strength in lbs.	Driving Force in lbs. $= P$
1	$3\frac{1}{8}$	842	76
$1\frac{5}{16}$	$4\frac{1}{8}$	1,548	140
$1\frac{1}{2}$	$4\frac{3}{4}$	1,940	176
$1\frac{11}{16}$	$5\frac{1}{4}$	2,602	236
$2\frac{1}{16}$	$6\frac{1}{2}$	3,633	330

Taking the greatest stress in a hemp or cotton rope at 140 lbs. per square inch gross sectional area, and assuming that $T_d = 4\, T_s$, so that the driving tension $P = T_d - T_s$ is 105 lbs. per square inch gross sectional area, we get the following table showing the horse-power transmitted by a single rope for given speeds:—*

HORSE-POWER TRANSMITTED BY A SINGLE ROPE AT HIGH SPEEDS.

Diameter of Rope in Inches.	Girth of Rope in Inches.	Horse-power Transmitted by one Rope when the Speed in Feet per Minute is:—						
δ	γ	3,000	3,500	4,000	4,500	5,000	5,500	6,000
1	3·14	7·50	8·75	10·00	11·25	12·50	13·75	15·00
$1\frac{1}{8}$	3·53	9·48	11·07	12·66	14·24	15·82	17·40	18·98
$1\frac{1}{4}$	3·93	11·72	13·67	15·62	17·58	19·53	21·48	23·44
$1\frac{3}{8}$	4·32	14·18	16·54	18·91	21·27	23·63	26·00	28·36
$1\frac{1}{2}$	4·71	16·87	19·69	22·50	25·31	28·12	30·94	33·75
$1\frac{5}{8}$	5·10	19·80	23·11	26·41	29·71	33·01	36·31	39·61
$1\frac{3}{4}$	5·50	22·97	26·80	30·62	34·45	38·28	42·11	45·94
$1\frac{7}{8}$	5·89	26·37	30·76	35·15	39·55	43·95	48·34	52·73
2	6·28	30·00	35·00	40·00	45·00	50·00	55·00	60·00

The following table of particulars regarding power transmitted, &c., by wire ropes, as calculated by Roebling, is given in Unwin's *Machine Design*, Part I., p. 434. The ropes have each 42 wires.

* From Low & Bevis's *Machine Drawing and Design*, p. 161.

Power Transmitted by Wire Ropes.

Diameter of Rope in Inches.	Diameter of Pulley in Feet.	Number of Revolutions of Pulley per Minute.	Velocity of Belt in Feet per Second.	Breaking Strength of Rope in lbs.	Horse-power Transmitted.
$\frac{7}{16}$	5	100	26	4,260	8·6
$\frac{15}{32}$	6	100	31	5,660	13·4
$\frac{1}{2}$	7	100	36	8,200	21·1

Example VII.—The belt pulley of an engine is 6 feet in diameter, and the engine makes 80 revolutions per minute. What width of belt would be required to transmit 30 H.P. from the pulley, supposing the belt to be $\frac{3}{8}$ of an inch in thickness, and the tension on the slack side to be $\frac{4}{10}$ of that on the tight side of the belt? The stress in the belt is not to exceed 300 lbs. per square inch of section. (S. & A. Adv. Exam., 1892.)

Answer.—Here D = 6 ft.; N = 80; H.P. = 30.

Thickness of belt $= \delta = \frac{3}{8}$ inch.

Greatest stress allowed $= f =$ 300 lbs. per sq. inch.

Let β = required width of belt in inches.

Then, $\left.\begin{array}{r}\textit{Total stress on}\\ \textit{driving side of belt}\end{array}\right\} = \mathrm{T}_d = \beta\,\delta f,$

„ „ $= \beta \times \frac{3}{8} \times 300 = 112{\cdot}5\,\beta$ lbs.

Again, $\mathrm{T}_s = \frac{4}{10}\mathrm{T}_d = 45\,\beta$ lbs.

∴ *Driving force* $= \mathrm{P} = \mathrm{T}_d - \mathrm{T}_s = 67{\cdot}5\,\beta$ lbs.

$$\therefore \quad \text{H.P.} = \frac{\mathrm{P} \times \pi \mathrm{D N}}{33{,}000}.$$

$$\text{Or,} \quad 30 = \frac{67{\cdot}5\,\beta \times \frac{22}{7} \times 6 \times 80}{33{,}000},$$

$$\therefore \quad \beta = \frac{33{,}000 \times 30 \times 7}{67{\cdot}5 \times 22 \times 6 \times 80} = 9{\cdot}72 \text{ ins.}$$

Example VIII.—Two pulleys on parallel shafts 10 feet apart, and having diameters of 6 feet and 1 foot respectively, are connected by an open belt 8 inches broad. The larger pulley makes 120 revolutions per minute. Find the greatest power which can be transmitted, when the maximum stress in the belt

has not to exceed 80 lbs. per inch of width, and taking the coefficient of friction between belt and pulleys at 0·4.

ANSWER.—The first thing to be determined here is the ratio of the tensions in the two parts of the belt. This is obtained from the equation:—

$$\text{Log}\,\frac{T_d}{T_s} = \cdot 4343\,\mu\,\theta.$$

Where, θ is the circular measure of the angle subtended at the centre of the *smaller pulley* by the part of the belt in contact with that pulley.

To obtain this angle we must first find the angle, ϕ, used in equations (VI) and (VII) in this Lecture, for finding the length of an open belt. Referring to these equations and the figure at that part of the text, we clearly see that:—

$$\theta = 180^\circ - 2\,\phi^\circ = (\pi - 2\,\phi) \text{ radians.}$$

From equation (VII),

$$\phi = \sin\phi \text{ (approx.)} = \frac{D - d}{2\,a} = \frac{6 - 1}{2 \times 10} = \cdot 25.$$

$$\therefore \quad \theta = 3\cdot 1416 - 2 \times \cdot 25 = 2\cdot 64 \text{ radians, approx.}$$

Hence, $\text{Log}\,\dfrac{T_d}{T_s} = \cdot 4343 \times \cdot 4 \times 2\cdot 64 = \cdot 458621.$

Referring to a table of common logarithms, we find that:—

$$458621 = \text{Log } 2\cdot 87,$$

$$\therefore \quad \frac{T_d}{T_s} = 2\cdot 87.$$

Since, from the question, the greatest stress in the belt must not exceed 80 lbs. per inch of width, and the width is 8 inches, we get:—

$$T_d = 8 \times 80 = 640 \text{ lbs.}$$

$$\therefore \quad T_s = \frac{640}{2\cdot 87} = 223 \text{ lbs.}$$

$$\therefore \quad P = T_d - T_s = 417 \text{ lbs.}$$

And, $V = \pi D N = \dfrac{22}{7} \times 6 \times 120$ ft. per minute.

$$\therefore \quad \text{H.P.} = \frac{P\,V}{33{,}000} = \frac{417 \times \frac{22}{7} \times 6 \times 120}{33{,}000} = 28\cdot 6 \text{ nearly.}$$

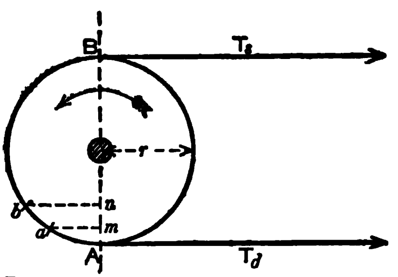
This is the greatest power which can be transmitted. If the power to be transmitted be greater than this, then slipping will take place between the belt and the smaller pulley.

Influence of Centrifugal Tension on the Strength of High-Speed Belts and Ropes.—When belts or ropes are run at high speeds the tensions in the two parts of the belt or rope between the pulleys are greater than that calculated from the horse-power transmitted. This increase of tension is due to the centrifugal force set up in those parts of the belt which are in contact with the pulleys. In addition to this increase in tension the centrifugal action has also the effect of diminishing the normal pressure between the belt and the pulleys, and, therefore, of diminishing the resistance to slipping.

Let w = Weight of belt or rope, in *lbs. per linear foot.*
„ v = Velocity of belt or rope, in *feet per second.*
„ r = Radius of the pulley in *feet.*
„ T_d, T_s = Tensions in driving and slack sides of belt or rope *as calculated from power transmitted.*
„ T^1_d, T^1_s = Tensions in driving and slack sides of belt or rope *corrected for centrifugal action.*

For simplicity, suppose the belt embraces half the circumference of the pulley considered.

Then, *Centrifugal force of belt per linear foot* $= \dfrac{w\,v^2}{g\,r}$ lbs.



BELT EMBRACING HALF CIRCUMFERENCE OF PULLEY.

In the figure, let $a\,b$ represent a part of the belt 1 foot in length. Let $m\,n$ be the projection of $a\,b$ on the diameter A B. It will be shown in a subsequent Lecture, that the horizontal component of the centrifugal force on the part, $a\,b$, of the belt is:—

$$\frac{w\,v^2}{g\,r} \times m\,n.$$

Hence, for the whole arc of contact, A C B, we get:—

$$\left.\begin{array}{l}\textit{Total horizontal component}\\ \textit{of centrifugal force}\end{array}\right\} = \frac{w\,v^2}{g\,r} \times \mathrm{A\,B} = \frac{2\,w\,v^2}{g}\ \text{lbs.}$$

One-half of this centrifugal tension is felt at A, and the other half at B. Therefore, the actual tensions in the two parts of the belt are :—

$$\left.\begin{aligned} T_d^1 &= T_d + \frac{w\,v^2}{g} \\ T_s^1 &= T_s + \frac{w\,v^2}{g} \end{aligned}\right\} \quad \ldots\ldots \quad \text{(XV)}$$

From these equations it will be seen :—

(1) That the diameters of the pulleys have nothing to do with the results, and

(2) That the driving force or tension, $P = T_d^1 - T_s^1 = T_d - T_s$ and is, therefore, the same as before.

If the speed be great, the above value for T_d^1 must be taken when calculating the size of a belt or rope to transmit a given power.

The values of w may be calculated from the following approximate formulæ :—

Let β = Breadth of leather belt in *inches*.

„ δ = Diameter of rope, or thickness of belt in *inches*.

„ γ = Girth of rope in inches.

Then, $w = 0{\cdot}43\,\beta\,\delta$ *lbs.*, *nearly*, for *leather belts*.

$$\left.\begin{aligned} w &= 0{\cdot}281\,\delta^2 \quad ,, \\ w &= 0{\cdot}0285\,\gamma^2 \quad ,, \end{aligned}\right\} \quad ,, \ \textit{dry hemp or cotton ropes.}$$

$$\left.\begin{aligned} w &= 0{\cdot}3376\,\delta^2 \quad ,, \\ w &= 0{\cdot}0342\,\gamma^2 \quad ,, \end{aligned}\right\} \quad ,, \ \textit{wet or tarred hemp or cotton ropes.}$$

$$w = 1{\cdot}341\,\delta^2 \quad ,, \qquad ,, \ \textit{wire ropes.}$$

Example IX.—Determine the horse-power which may be transmitted by a leather belt, 5 inches wide and $\frac{1}{4}$ inch thick, running at a speed of 50 feet per second, the tension in the loose side being $\frac{4}{10}$ of that on the tight side of the belt, and the stress allowed being 275 lbs. per square inch (S. & A. Hons. Mach. Const. Exam., 1886). Again, taking the weight of a cubic foot of the leather at 60 lbs., determine the effects due to centrifugal action.

Answer.—Here, V = 3,000 ft. per minute; $\beta = 5$ inches; $\delta = \frac{1}{4}$ inch; f = 275 lbs. per square inch; $T_s = \frac{4}{10}\,T_d$.

(1) *Neglecting centrifugal action.*

$$P = T_d - T_s = \cdot 6\, T_d.$$

But, $T_d = \beta\, \delta f = 5 \times \frac{1}{4} \times 275 = 343\cdot 75$ lbs.

Hence, $P = \cdot 6 \times 343\cdot 75 = 206\cdot 25$ lbs.

$$\therefore \quad \text{H.P.} = \frac{P\,V}{33{,}000} = \frac{206\cdot 25 \times 3{,}000}{33{,}000} = 18\cdot 75.$$

(2) *Taking centrifugal action into account.*

Let W = Weight of a *cubic* foot of leather in lbs.

„ w = Weight of a *linear* „ „

„ A = Cross sectional area of belt in square inches.

Then, clearly, $w : W = A : 144.$

$$\therefore \qquad w = \frac{W\,A}{144}.$$

Substituting the values given in the question, we get :—

$$w = \frac{60 \times 5 \times \frac{1}{4}}{144} = \cdot 521 \text{ lbs.}$$

Substituting this in equation (XV), in the text, and observing that v = 50 ft. per second, we get :—

$$\left.\begin{array}{c}\textit{Increase of tension due}\\ \textit{to centrifugal force}\end{array}\right\} = \frac{w\,v^2}{g} = \frac{\cdot 521 \times 50 \times 50}{32} = 40\cdot 7 \text{ lbs.}$$

But the maximum tension allowed in the belt is 343·75 lbs., hence :—

Maximum effective tension = 343·75 − 40·7 = 303 lbs. nearly.

$$\therefore \qquad P = \cdot 6 \times 303 = 181\cdot 8 \text{ lbs.}$$

$$\therefore \qquad \text{H.P.} = \frac{P\,V}{33{,}000} = \frac{181\cdot 8 \times 3{,}000}{33{,}000} = 16\cdot 53$$

This example shows that the power is reduced by about 12 per cent. when centrifugal action is taken into account.

Lecture XVIII.—Questions.

1. Find an expression for the length of a crossed belt, and show that the same driving belt will serve for any pair of pulleys, so long as the belt is crossed and the distance apart of their centres and the sum of their diameters remains constant.

2. The centres of two pulleys, 4 and 2½ feet in diameter respectively, are 12 feet apart. Find length of crossed belt required. *Ans.* 35·1 feet.

3. A crossed belt is employed to connect two equal coned drums, having their axes parallel, and their vertices lying in opposite directions. Prove that the belt will be equally tight in all positions when shifted along the cones. Would the same be true if the belt were not crossed?

4. By aid of a *graphical construction* determine the length of a crossed belt required to embrace either of two pairs of pulleys which are mounted on parallel shafts 3 feet 6 inches apart. The smaller pulley on each shaft is 8 inches diameter, and the velocity-ratio at the higher speed is required to be four times that at the lower speed. (S. & A. Adv. Mach. Const. Exam., 1892.) *Ans.* 10·4 feet.

5. The diameters of the pulleys of a stepped speed-cone for a machine are 13½, 11½, 9½, and 7½ inches respectively, and the diameter of the smallest pulley of the stepped driver is 8½ inches. The connection being made by means of a crossed belt, what should be the diameters of the other pulleys of the stepped driver? If the driving shaft makes 120 revolutions per minute, find the revolutions per minute of the machine pulley for all positions of the belt. (S. & A. Adv. Mach. Const. Exam., 1885.) *Ans.* (1) 10½, 12½, and 14½ inches; (2) 75·5, 109·6, 157·9, and 232.

6. A shaft having a stepped speed-cone, with four steps, revolves at a constant speed of 150 revolutions per minute, and is connected by means of a crossed belt to another shaft having a similar stepped cone. The diameter of the largest step of the cone on the driving shaft is 13⅛ inches. The driven shaft is required to run at speeds 250, 200, 120, and 60 revolutions per minute. Determine the diameters of all the remaining steps of the two cones, and the length of belt required, the distance between the two shafts being 8 feet. *Ans.* (1) $D_2 = 12$ inches, $D_3 = 9·333$ inches, $D_4 = 6$ inches; $d_1 = 7·875$ inches, $d_2 = 9$ inches, $d_3 = 11·667$ inches, $d_4 = 15$ inches; (2) 18·94 feet.

7. Find an expression for the length of an open belt. The centres of two pulleys, 5 and 2¾ feet in diameter respectively, are 15 feet apart. Find length of open belt required. *Ans.* 42·26 feet.

8. Two pulleys, whose diameters are 4 feet 8 inches and 2 feet 3 inches respectively, their centres being 10 feet apart, are connected by an open belt, determine, by a graphical construction, the length of belt required. *Ans.* 31 feet.

9. Explain how you would design a set of speed cones to be worked by an open belt, the angular velocity-ratios being given you.

10. A countershaft revolves at a constant speed of 250 revolutions per minute, and carries a stepped speed-cone with four steps, and drives a similar cone on another shaft by means of an open belt. The driven shaft is required to run at speeds 520, 300, 245, and 180 revolutions per minute respectively. Given the diameter of the largest step on the driving cone 22 inches, and the distance between the shafts 8½ feet; find the remaining sizes of the steps on both cones. *Ans.* $D_2 = 17·87$ inches; $D_3 = 16·22$ inches; $D_4 = 13·70$ inches; $d_1 = 10·58$ inches; $d_2 = 14·90$ inches; $d_3 = 16·56$ inches; $d_4 = 19·04$ inches.

11. Deduce a formula for the greatest ratio of the tensions in the two parts of a belt stretched over a pulley when slipping is just about to take place. Two pulleys, whose diameters are $5\frac{3}{4}$ and 2 feet respectively, are 15 feet apart. Find the maximum ratio of tensions in tight and slack sides of belt (1) when the belt is crossed, (2) when the belt is open, given $\mu = 0.3$. *Ans.* (1) 3 : 1; (2) 2·38 : 1.

12. Deduce a formula for the greatest ratio of the tensions in the two parts of a rope stretched over a grooved pulley, the rope being wedged into the grooves slipping being just about to occur. Find the greatest ratio of the tensions in the two parts of a cotton rope running over grooved pulleys, the arc of contact on the smaller pulley being $\frac{5}{12}$ of the whole circumference, angle of groove 40°, coefficient of friction, $\mu = 0.25$. *Ans.* 6·77 : 1.

13. Explain the following paradox in connection with belt gearing:—For every foot of belt length that goes on to the driving pulley, less than a foot comes off and goes on to the driven pulley. Would the statement still be true if we substituted unit weight of belt instead of unit length of belt? If not, why not?

14. Explain how the formula for obtaining the power transmitted by a stretched belt running over pulleys is arrived at. What horse-power may be transmitted by a belt 10 inches wide, and $\frac{1}{4}$ inch thick, running at a speed of 42 feet per second; the tension on the slack side of the belt being 0·4 of that on the driving side? The stress allowed is 300 lbs. per square inch of belt section. *Ans.* 68·72 H.P.

15. A belt is required to transmit 4 horse-power from a shaft running at 120 revolutions to one at 160 revolutions per minute. Find the stresses in the belt, the small pulley being 2 feet in diameter, and the ratio of the tensions on the belt being as 7 is to 4. Find also the width of belt that would be required in the above case, if the stress is taken at 100 lbs. per inch of width. *Ans.* 306·25 lbs.; 175 lbs.; 3·06 inches.

16. A pulley, 3 feet 6 inches in diameter, and making 150 revolutions per minute, drives by means of a belt a machine which absorbs 7 horse-power. What must be the width of the belt so that its greatest tension shall be 70 lbs. per inch of width, it being assumed that the tension in the driving side is twice that in the slack side? Take $\pi = 3\frac{1}{7}$. (S. & A. Exam., 1891.) *Ans.* 4 inches.

17. In the modern system of transmitting power through long distances by a slender wire rope moving at a high velocity, the following example occurs:—The pulley which drives the rope is 15 feet in diameter, making 100 revolutions per minute, and the energy to be transmitted is measured by 250 horse-power. Find the tension of the wire rope, which in this case is $\frac{3}{4}$ inch in diameter. *Ans.* 1,750 lbs., or 3,960 lbs. per square inch nearly.

18. A leather belt is required to transmit 2 H.P. from a shaft running at 80 revolutions per minute to a shaft running at 160. Find the stresses in the belt, assuming that the smaller pulley is 12 inches in diameter, and that the ratio of the tensions in the tight and slack sides of the belt is $2\frac{1}{4}$: 1. Hence, find the width of belt, taking the working stress at 100 lbs. per inch of width. (S. & A. Hons. Mach. Const. Exam., 1882.) *Ans.* 236·3 lbs.; 105 lbs.; 2·36 inches.

19. Suppose the friction of two pulleys is such that the ratio of the tensions in the tight and slack sides of the belt is 1·75. Also, suppose the greatest safe working tension to be 120 lbs. per inch width of belt. Find the width of a belt to transmit 10 horse-power, the circumferential speed of the pulleys being 20 feet per second. (S. & A. Hons. Mach. Const. Exam., 1883.) *Ans.* 5·34 inches.

20. Assuming that the arc embraced by a belt on the smaller of two

pulleys over which it runs is $\frac{4}{10}$ of the circumference, and that μ is taken $= 0{\cdot}3$, prove the following simple approximate rule for the breadth of a leather belt:—$\beta = \frac{208 \text{ H.P.}}{V \delta}$ inches; where H.P. is the horse-power transmitted, V the velocity of belt in feet per minute, and δ the thickness of belt in inches. Greatest working stress, 300 lbs. per square inch.

21. It is required to transmit 10 H.P. from a pulley 5 feet in diameter, and making 200 revolutions per minute, to one 18 inches in diameter, by means of an open belt, the centres of the pulleys being 12 feet apart. Taking coefficient of friction between belt and pulley at 0·35; find (1) angle of contact on smaller pulley; (2) the speed of smaller pulley; and (3) the width of single belt $\frac{3}{16}$ inch thick which will be necessary. *Ans.* (1) 163°; (2) 52·36 feet per second; (3) 3 inches.

22. It is required to transmit 16 H.P. from a pulley 20 inches in diameter by means of a belt which embraces only $\frac{3}{8}$ of the circumference of the pulley. Find the tensions in the two parts of the belt when slipping is just prevented, and the width of belt required, the thickness of the belt being $\frac{3}{8}$ inch, speed of pulley 120 revolutions per minute, coefficient of friction $\mu = 0{\cdot}35$. *Ans.* 2174·5 lbs.; 1333·75 lbs.; 19·35 inches.

23. What circumstances affect the action of a belt when the speed is high? (S. & A. Hons. Mach. Const. Exam., 1882.)

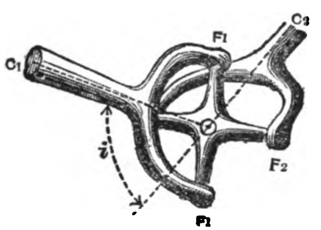
24. Find an expression for the increase in the tensions in the tight and slack sides of a belt, taking centrifugal action into account. In question 14 make the necessary corrections for centrifugal action, being given that the weight of a cubic foot of leather weighs 60 lbs. *Ans.* 63·45 H.P.

25. In question 17 make the necessary corrections for centrifugal action, given that the weight of a linear foot of wire rope $\frac{3}{4}$ inch in diameter is 3 lbs. *Ans.* 2,330 lbs.

LECTURE XIX.

In this Lecture we shall examine a few more of the many devices for transmitting circular motion and for converting it into rectilinear motion, or *vice versâ*, together with other miscellaneous mechanisms.

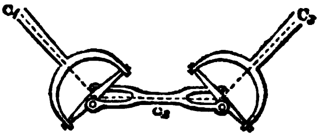
Hooke's Coupling or Universal Joint.—This is a contrivance sometimes used for connecting two intersecting shafts. Each of the shafts ends in a fork, F_1, F_2, which embraces two arms of the crosspiece, O. The four arms of this cross are of equal length. As C_1 rotates, F_1 and F_2 describe circles in planes perpendicular to their respective axes. Since these planes are inclined to each other the angular velocity of C_2 at any instant is different from that of C_1, but the mean angular velocities are equal to one another, because at one instant C_2 goes faster than C_1, and at another slower. This joint will not work when the two shafts are inclined at 90°, or any smaller angle, to each other.



Hooke's Joint.

Double Hooke's Joint.—The variable velocity ratio obtained with a Hooke's joint may be obviated by the use of two joints instead of one. The forks are connected by an intermediate link, C_2, which must be carried on corresponding arms of the two crosses, as shown in the next figure. If the intermediate shaft be equally inclined to the other two shafts, the irregularities caused in the motion by its transmission through the first coupling are exactly neutralised by the equal and opposite ones caused by the second joint. The first and third shafts, therefore, revolve with the same

velocity at every instant. The double joint works equally well whether the two extreme axes are inclined as shown in the figure, or are parallel to each other but not in line.



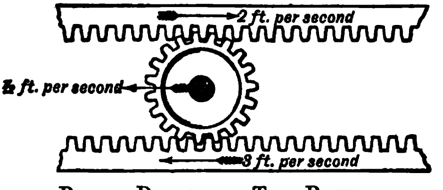
DOUBLE HOOKE'S JOINT.

Both the single and double Hooke's joint are, as a rule, used only for light work, such as for astronomical instruments.*

Aggregate Motion.—The motion of a piece of machinery is not always a simple one, but is very often the resultant or *aggregate* of several independent motions impressed upon it simultaneously.

Thus, the motion of a screw working in a fixed nut is the aggregate of the circular motion of the cylinder and the axial motion caused by the thread. These two together, give a helical motion to any point on the screw. Weston's differential pulley block, which the student has already studied, forms a very good example of aggregate motion. Here, the actual motion of the load is the resultant of two opposite and nearly equal motions imparted by the two parts of the chain supporting it.

In some printing machines, the following arrangement is adopted in order to double the horizontal motion obtained from a crank and connecting-rod. The end of the connecting-rod carries a pinion which runs between two racks. One of these racks moves horizontally between guides while the other is fixed. The motion of the movable rack is composed of that of the connecting-rod end plus that due to the rotation of the pinion. Sometimes, as in Example II., two wheels of different sizes are keyed together on the connecting-rod end, one gearing with a fixed rack and the other with a movable one. In this way the travel of the rack may be made greater, or less, than the diameter of the crank-pin circle in any desired proportion. In all these cases, where the several impressed motions are in parallel directions, the resultant is simply their algebraic sum.



PINION DRIVEN BY TWO RACKS.

EXAMPLE I.—A toothed wheel runs in gear with two parallel racks, one above and the other below it, the wheel being free to run between the racks. If the upper rack

* A recent application of this device is to be met with in the steering gear for the Admiralty torpedo-boat destroyers.

has a velocity of 2 feet per second in one direction, and the lower rack a velocity of 3 feet per second in the opposite direction, what is the velocity of the centre of the wheel? (S. & A. Exam., 1887.)

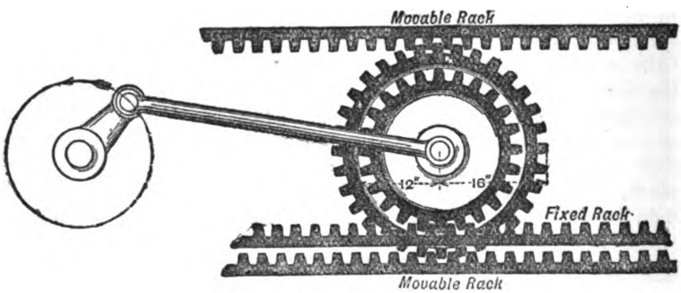
ANSWER.—When a wheel rolls along a road with a velocity, v, it is clear, that the point on the rim which is at any instant touching the ground is for the moment at rest, while the point on the rim vertically over the centre will be moving with a velocity twice that of the centre of the wheel; that is, its velocity will be $2v$.

Now, suppose the lower rack fixed. Then, from what has been said, it is evident that the wheel will run between the racks with a velocity equal to half the velocity of the upper rack. That is, the wheel will be moving to the right with a velocity of 1 foot per second.

In the same way, if we suppose the upper rack fixed the wheel will move to the left with a velocity of $1\frac{1}{2}$ feet per second.

When both racks move, as shown, then the wheel will have a velocity equal to half the difference of their velocities; that is, the velocity of the wheel will be $= 1\frac{1}{2} - 1 = \frac{1}{2}$ foot per second, and in the same direction as the lower rack.

EXAMPLE II.—A crank, 12 inches long, is attached by a connecting-rod to the axis of a spur wheel 24 inches in diameter, which runs upon a *fixed horizontal rack*. On the axis of the spur



RACK DRIVEN BY A CRANK AND MOVABLE PINION.

wheel, and locked to it, is a second spur wheel, 32 inches in diameter, which gears with a *free horizontal rack sliding in guides*. Find the travel of the rack in inches for each revolution of the crank. (S. and A. Exam., 1890.)

ANSWER.—There are two answers to this question, according as we consider the two racks (the fixed and movable racks) to be on the same, or on the opposite, sides of the axis of the spur wheels. In either case, the motion of the movable rack is made up of two

motions—one due to the motion of the axis of the wheels, and the other due to the circular motion of the wheels about their common axis. Suppose the crank to make one-half turn from the inside dead point to the outside dead point in the direction shown by the arrow. Then, evidently, the axis of the wheels will be moved towards the left by an amount equal to twice the length of the crank $= 2 \times 12 = 24$ inches. Also, any tooth on the smaller wheel will turn through an arc of the circumference equal to 24 inches. Any tooth on the larger wheel will turn through an arc of its pitch circle equal to $\frac{16}{12} \times 24 = 32$ inches. Hence, the exact motion of the movable rack consists of a motion of 24 inches along with the axis of the wheels, and another of 32 inches due to the turning of the wheels about their common axis.

First, suppose the racks to be on opposite sides of the axis. Then, from an inspection of the figure, it is clear that the motion of the movable rack will be the sum of these two separate motions.

$$\therefore \left.\begin{array}{c}\text{Motion of movable rack per}\\ \text{half turn of crank}\end{array}\right\} = 24 + 32 = 56 \text{ inches.}$$

During the other semi-revolution, the rack moves back the same distance—*i.e.*, it moves 112 inches in all.

Secondly, when the racks are on the same side of the axis, it is equally clear that the motion will be equal to the difference of the two separate motions.

$$\therefore \left.\begin{array}{c}\text{Motion of movable rack per}\\ \text{half turn of crank}\end{array}\right\} = 24 - 32 = -8 \text{ inches.}$$

This shows that the rack moves in the opposite direction to the axis. The whole motion in this case per revolution of the crank is 8 inches each way or 16 inches in all. In this latter case it is evident that the axis of the wheels must be guided horizontally in order to keep them in gear with the racks.

Epicyclic Trains of Wheels.—We sometimes find that the axes of some of the wheels in a train are not fixed, but rotate around another axis. Such trains are called **Epicyclic Trains.** The movable axes are fixed to an arm, called the **Train Arm,** which rotates about that axis. Epicyclic trains are used in those machines where some motion is required which it would be difficult, or inconvenient, to obtain with an ordinary train. For instance, they are used in the "Cordelier," or rope making machine, in order to twist the fibres of the strands in one direction while the strands themselves are being twisted together

in the opposite direction. If this were not done the fibres would be getting untwisted while the strands were being twisted, and a useless rope would result. By putting a little extra twist on the fibres, the rope will be hard and firm and will not tend to untwist.

The student has already studied an application of the epicyclic train in Lecture VIII.—viz., the Rotatory Dynamometer. In this case the train is one of bevel and not of spur wheels as in the other examples we will consider here.

The motion of any wheel in an epicyclic train is an aggregate motion; for, the wheel has a certain angular velocity due to its rotating about its own axis, and another caused by the rotation of that axis along with the train arm. In this case also, the resultant motion is the algebraic sum of the several parts.

Let, N_D = Number of revolutions of driver in unit time relative to some fixed point.

„ N_F = Number of revolutions of follower in the same time relative to the same fixed point.

„ N_A = Number of revolutions of the arm in the same time relative to the same fixed point.

„ e = Value of the train = $\left\{\dfrac{\text{Number of revolutions of follower in a given time relative to the arm.}}{\text{Number of revolutions of driver in the same time relative to the arm.}}\right.$

Care must be taken to give e its proper sign; for, e is negative if the driver and follower rotate in opposite directions relative to the arm, and positive if in the same direction.

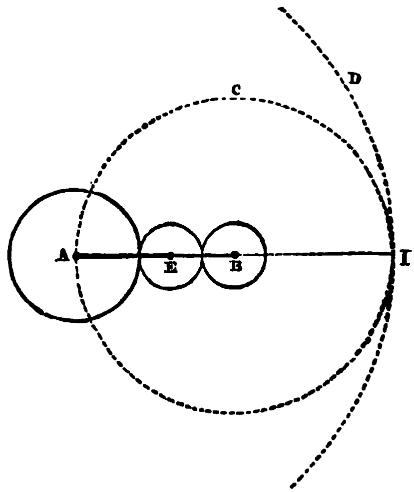
Now, since the driver rotates N_D times in unit time and the train arm N_A times, their relative motion will be $(N_D - N_A)$ turns. Similarly the number of revolutions of the follower relative to the arm will be $(N_F - N_A)$.

$$\therefore \qquad e = \frac{N_F - N_A}{N_D - N_A}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (I)$$

It should be noted that if the driver and follower rotate in opposite directions, one of them must be considered the positive, and the other the negative, direction.

Epicyclic Train for Drawing Ellipses.—The figure shows the wheel work of an instrument for tracing ellipses by means of rolling circles. Suppose the circle A C I to roll inside a circular

arc, D I, of twice its radius. Then, as was proved in Lecture XIII. in connection with the hypocycloid, the point, I, will move along the straight line I A (A being the centre of the arc, D I), and it is manifest that B (the centre of A C I) will describe a circle round A. Any other point in B I, or B I produced, must, therefore, trace out an ellipse. It is inconvenient in practice to roll the circle, A C I, inside D I, and the same result may be obtained by aid of an epicyclic train. Let A be a wheel *fixed* at the centre of the arc, D I, and B another, of half its size, concentric with the rolling circle, A C I. These are connected by the train arm, A B, which rotates about A, and an idle wheel, E. B I is a tracing arm, rigidly fixed to the wheel, B. Let the train arm, A B, now rotate. A, the driver, does not rotate; therefore, $N_D = 0$, and $e = \frac{A}{B} = 2$.



EPICYCLIC TRAIN FOR TRACING ELLIPSES.

Then, from equation (I) we get:—

$$e = \frac{N_F - N_A}{N_D - N_A}, \quad \therefore 2 = \frac{N_F - N_A}{0 - N_A}. \quad \therefore N_F = -N_A.$$

Or, the tracing arm, B I, and train arm, A B, rotate in opposite directions with equal velocities. This is obviously the same as if the tracing arm were a radius of the imaginary rolling circle, A C I, rolling inside the arc, D I. We can thus see that if we put a tracing point on B I, it will trace out an ellipse.

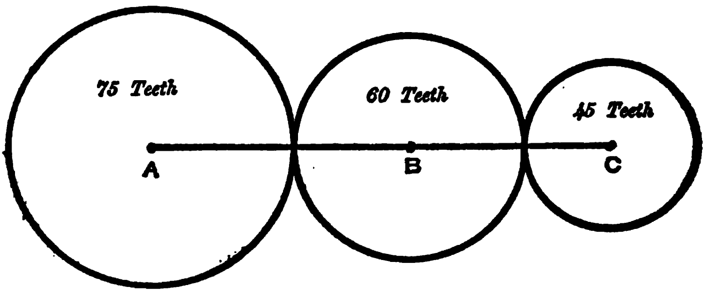
EXAMPLE III.—An epicyclic train consists of three wheels, A, B, C, taken in order, and in gear with each other. The first wheel, A, has 75 teeth, B has 60 teeth, C has 45 teeth; also, the driver, A, rotates three times in a minute, and the arm rotates four

times in a minute, and in the opposite direction. How many rotations do B and C make respectively per minute?

ANSWER.—The arrangement is shown diagrammatically in the above figure.

(1) *To find the motion of* B.—Here, using the same notation as in the text, we have:—

$$N_D = 3\,;\ N_A = -4\,;\ \text{and}\ e_1 = -\frac{75}{60} = -\frac{5}{4}.$$



EPICYCLIC TRAIN OF WHEELS.

From equation (I) we get:—

$$e_1 = \frac{N_{F1} - N_A}{N_D - N_A}, \qquad \therefore\ -\frac{5}{4} = \frac{N_{F1} + 4}{3 + 4},$$

$$\therefore \qquad N_{F1} = -\frac{35}{4} - 4 = -12\tfrac{3}{4}.$$

That is, the follower, B, rotates $12\frac{3}{4}$ times per minute, and in the opposite direction to the driver, A.

(2) *To find the motion of* C.—In this case, $e_2 = \frac{75}{60} \times \frac{60}{45} = \frac{5}{3}$.

As before, $$e_2 = \frac{N_{F2} - N_A}{N_D - N_A}, \qquad \therefore\ \frac{5}{3} = \frac{N_{F2} + 4}{3 + 4},$$

$$\therefore \qquad N_{F2} = \frac{35}{3} - 4 = 7\tfrac{2}{3}.$$

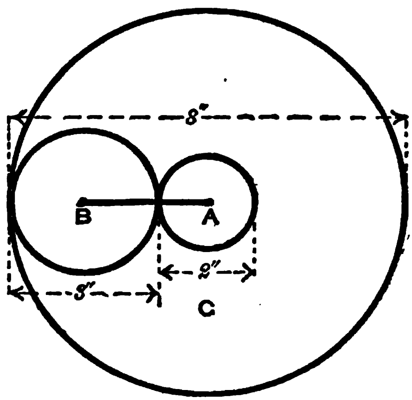
Whence, C turns $7\frac{2}{3}$ times per minute, in the same direction as A.

These results may be arrived at otherwise as follows:—Suppose the wheels and arm rigidly connected, so as to move as one piece; then, let the arm turn for one minute, so as to receive its four negative turns; each of the wheels will then also receive four

negative turns. Now, suppose the arm to be fixed, and let the wheel, A, receive seven positive turns, so as to cancel the four negative turns already given, and leave the nett motion of three positive turns, as required by the question.

The effect on B will be to give it $7 \times \frac{75}{60} = 8\frac{3}{4}$ negative turns, and on C to give it $7 \times \frac{75}{60} \times \frac{60}{45} = 11\frac{2}{3}$ positive turns; hence, the total motion of B in one minute will be $-4 - 8\frac{3}{4} = 12\frac{3}{4}$ negative turns, and of C, $-4 + 11\frac{2}{3} = 7\frac{2}{3}$ positive turns.

EXAMPLE IV.—What is an epicyclic train of wheels? Two spur wheels, A and B, whose diameters are 2 and 3 respectively, are in gear with an annular wheel, C, whose diameter is 8. The wheels A and C have a common axis, but B is carried by an arm centred on the axis of A. If A make five revolutions while C makes one revolution, both in the same direction, find the angle described by the arm during this time. (S. & A. Exam., 1888.)



EPICYCLIC TRAIN OF WHEELS.

ANSWER.—An epicyclic train of wheels is one in which the axes of the wheels are not fixed in space, but are attached to a rotating frame or bar, in such a manner that the whole train of wheels can derive motion from the rotation of the bar.

Using the same letters as before, and calling A the driver and C the follower, we have :—$N_D = 5$; $N_F = 1$; and $e = -\frac{3}{3} \times \frac{3}{8} = -\frac{1}{4}$.

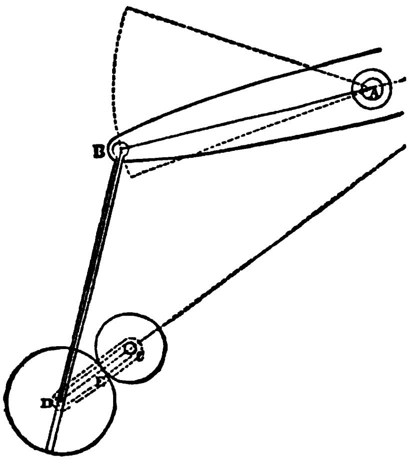
Then, from equation (I), $e = \dfrac{N_F - N_A}{N_D - N_A}$.

$$\therefore \qquad -\frac{1}{4} = \frac{1 - N_A}{5 - N_A}.$$

$$\therefore \qquad N_A = 1\frac{4}{5} \text{ turns.}$$

That is, the train arm has made $1\frac{4}{5}$ turns in the same direction as A and C.

Sun and Planet Wheels.—This device was invented by Watt to convert the oscillatory motion of the beam in his engines into the circular motion of the flywheel. As will be seen from the first figure, it consists of a wheel, D, rigidly fixed to the connecting rod, D B, and kept in gear with another wheel, C, by the link, D E C. The wheel, C, is keyed to the flywheel shaft. As the beam oscillates up and down, the connecting-rod pulls D up one side of C, and pushes it down the other. It thereby causes C to rotate, and with it the flywheel and shaft. The student will easily see that the wheels, D and C, form an epicyclic train, of which the link, D E C, is the train arm. We may, therefore, apply the formula already given for epicyclic trains, to find how often the flywheel revolves for each up and down movement of the beam. Doing this, and assuming D and C to be of the same size, we get:—



SUN AND PLANET WHEELS.

N_D = Number of revolutions of the driver, D, for each up and down movement = 0.

N_F = Number of revolutions of the follower, C, for each up and down movement.

N_A = Number of revolutions of the arm, D E C, for each up and down movement = 1.

$e = -1.$

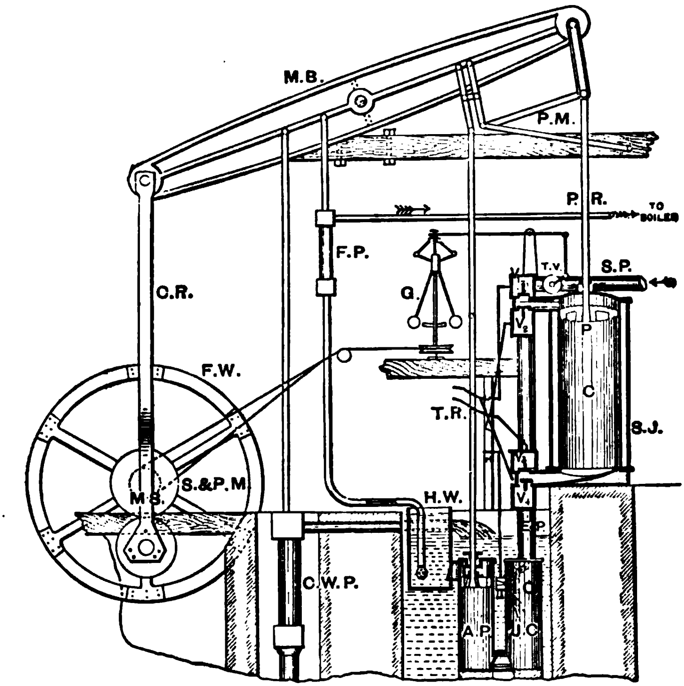
From equation (I), $e = \dfrac{N_F - N_A}{N_D - N_A}$; *i.e.*, $-1 = \dfrac{N_F - 1}{0 - 1}$.

Or, $N_F = 2.$

From this we see, when the wheels are equal, that the flywheel goes round twice while the connecting-rod goes once up and down. If the wheel D be twice as large as the wheel C, we would find, in the same way, that C went round three times during this period. The following figure shows these sun and planet wheels as applied to Watt's double-acting steam engine.*

* See Lecture II. of the Author's *Text-Book on Steam and Steam Engines.*

Sun and Planet Cycle Gear.—The illustration on next page (which has been kindly supplied by the makers of this gear, the "Elliptic" Cycle Syndicate of Grantham) shows a recent interesting application of the sun and planet wheels. Both wheels are elliptic in this case, and, therefore, give a variable velocity-ratio. By means of this gear, the pedals, which travel in an elliptical path, are caused to move at a uniform speed. The velocity-ratio



WATT'S DOUBLE-ACTING STEAM ENGINE, SHOWING SUN AND PLANET MOTION (S. & P.M.) AND PARALLEL MOTION (P.M.).

between the cranks and driving wheel has a double variation at each revolution of the cranks, which move more quickly at the top and bottom positions. The quicker movement caused by the gearing is counteracted by the quick vertical movement of the pedals, due to, and governed by, the position at which the crank-pin bearing is attached to the pedal bar. The result is a regular and uniform movement of the pedals in an elliptical path; the train value being 2 : 1.

It is claimed by the inventor, that a bicycle having an elliptical pedal path, with a uniform movement, has a great advantage over a circular pedal path, as the pressure can be applied continuously, whilst in the circular path nearly one-quarter of the travel at top and bottom is horizontal, and, therefore, not in an effective direction.

The shape of the planet wheel governs the double variation, and, consequently, must always be an ellipse, but the sun wheel may be made eccentric, and with half the number of teeth of the elliptical wheel, the value of the train then being 3 : 1.

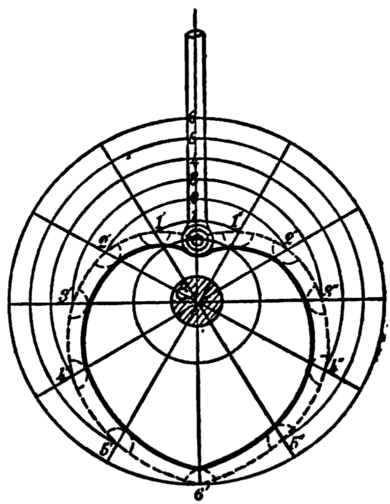
HARRISON'S ELLIPTIC CYCLE GEAR.

Cams.—Cams are usually of the form of discs or cylinders. They rotate about an axis, and give a reciprocating motion to some point in a rod by means of the form of their periphery or surface, or by grooves in their surface.

The cam generally revolves uniformly round its axis, whilst the reciprocating motion may be of any nature, depending on the shape of the cam, and may be in a plane inclined at any angle to the axis of rotation. In the following examples, uniformity of rotation is assumed in the case of the cam, and the motion of the reciprocating piece takes place in a plane perpendicular to the axis.

Heart Wheel or Heart-shaped Cam.—Suppose that it is required to give a uniform reciprocating motion to a bar moving vertically between guides, and in a line passing through, C, the centre of motion of the cam plate.

Let the sliding bar be at its lowest position, as shown, and when in its highest position let its extremity be at the point 6. The distance thus moved is called the *travel* and will be passed over during one-half revolution of the cam. The required curved outline may be obtained in the following manner:—With centre, C, describe circles passing through the extreme positions of the end of the rod. Divide the travel into, say, six equal parts at the points 1, 2, 3, &c. Divide the semi-circumference into the same number of equal parts by radial lines C 1′, C 2′, &c. Then with centre, C, draw the concentric arcs 1, 1′; 2, 2′; &c., intersecting these radii in the points 1′, 2′, 3′, &c. The dotted line drawn through these points will represent the required curve.



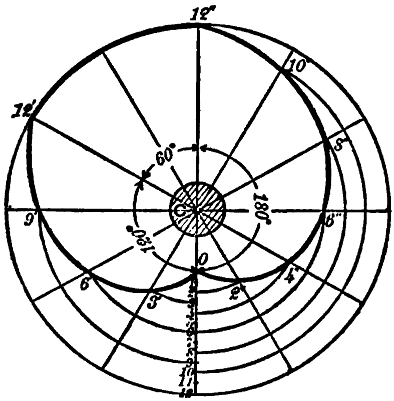
HEART-SHAPED CAM.

If the end of the sliding bar rests on this curve it is clear, that for equal angles turned through by the cam, the bar will move outwards through equal distances, and consequently, will have uniform linear motion imparted to it. The return motion will evidently be obtained by the similar and equal curve 1″, 2″, 3″, &c., on the opposite side of the cam.

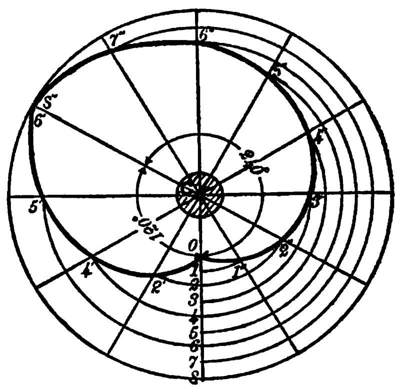
A cam so formed would impart the required motion to a *point*. If the end of the sliding bar be provided with a roller in order to diminish the friction, then the shape of the cam must be altered so that the centre of the roller shall move over the outline of the cam as traced above. To accomplish this we must draw a curve inside the original one by describing small arcs with centres on the original curve as at 1′, 2′, 3′, &c., and with a radius equal to that of the roller, and then by drawing a smooth curve touching these arcs, as shown by the heavy line in the figure.

Cam for Intermittent Motion.—Sometimes the motion imparted by a cam is intermittent. For instance, a common form of lever punching machine is fitted with a cam which gives the punch an upward movement, then a period of rest, and finally a downward movement during each revolution. As an example of this, let us set out a cam to impart vertical motion to a bar, so that the latter shall be raised uniformly during the first half revolution, remain at rest during the next one-sixth, and descend uniformly during the remainder of the revolution.

As before, suppose the reciprocation to be in a line passing through, C, the centre of motion of the cam plate. Then, with centre, C, draw circles passing through the extreme positions of the end of the bar. Divide the circumference into three parts corresponding to the periods of one-half, one-sixth, and one-third



CAM GIVING AN INTERVAL OF REST.



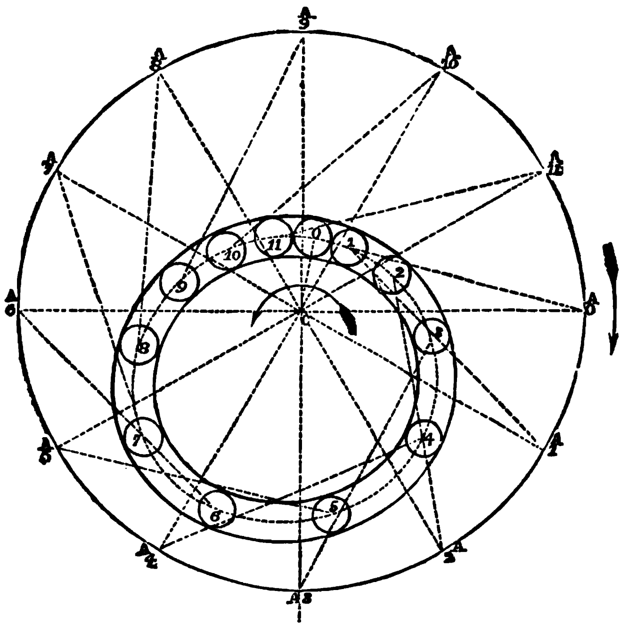
CAM GIVING A QUICK RETURN.

revolution, by drawing radial lines making angles of 180°, 60°, and 120°. Since the motion is to be uniform, divide the travel into a convenient number of equal parts, say twelve; and the circumference into the same number of equal parts by radial lines. Draw the concentric arcs 2, 2″; 4, 4″; &c., and 3, 3′; 6, 6′; &c., as shown. The curves through the points so determined will give the required motions. The interval of rest will evidently be given by the circular portion from 12″ to 12′. The complete outline is represented by the heavy line in the diagram.

Quick Return Cam.—The student will readily understand from the right-hand figure, that if two-thirds of a revolution be occupied in raising the motion bar and the remainder in lowering the same, the return stroke will be performed in half the time of

forward stroke. The curves of this cam are found in the same way as in the previous examples.

Cam with Groove on Face.—When the reciprocating bar has to be pulled as well as pushed by the cam, it is evident that the cams already considered would not drive it, but leave it at its extreme position. In such a case the periphery of the cam plate is not used, but a groove is cut in its face, as shown by the accompanying figure. The end of the rod carries a pin which works in this groove. The rod, therefore, gets pushed out by the inner face of



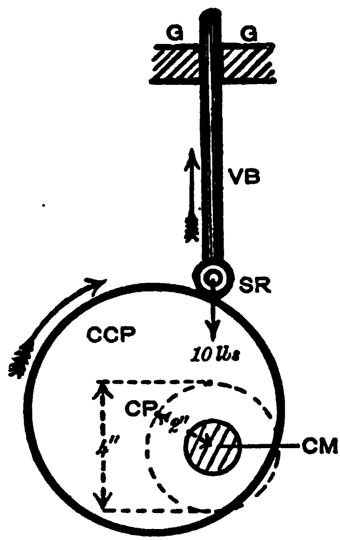
Cam with Groove on Face.

the slot and pulled in by the outer face. The central dotted curve is obtained in precisely the same way as before, and the two full curves are drawn parallel to it at a distance on each side of it equal to the radius of the pin.

Cylindrical Grooved Cam.—This differs from the above in that its rim is cylindrical and long. A groove is cut around its cylindrical surface, but it is not made circular. Parts of it are spiral, and so act on a pin like a screw. This gives a motion to the bar parallel to the axis about which the cam rotates.

Example V.—A vertical bar, moving in guides, is driven by a circular cam plate having a centre of motion in the centre line of the bar. The distance from the centre of motion to the centre of the plate is 2 inches, and the bar exerts a pressure of 10 lbs. when rising, but falls by its own weight. Find the work done in 100 revolutions of the plate.

Answer.—Since the distance between the roller, S R, and the centre of the plate, C P, remains constant as the plate revolves,



Circular Cam Plate.

Index to Parts.

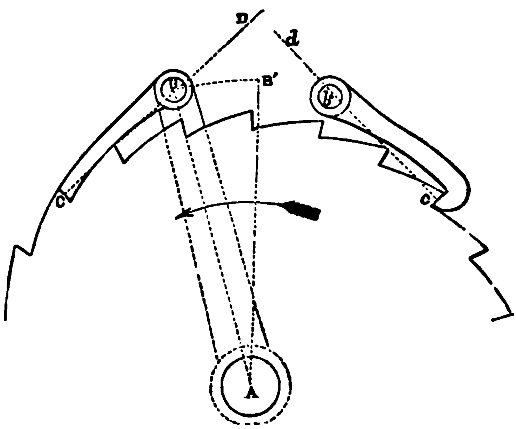
G	represents	Guides.
V B	,,	Vertical bar.
S R	,,	Slipping roller.
C C P	,,	Circular cam plate.
C P	,,	Centre of plate.
C M	,,	Centre of motion.

it is evident that the bar will move as if it were actuated by a crank of length equal to the distance between C M and C P, and a connecting-rod of length equal to the radius of the plate. Hence the stroke of the bar will be 4 inches, or $\frac{1}{3}$ foot—*i.e.*, twice the length of the equivalent crank. Neglecting friction, the work done in raising the bar by one revolution of the plate, will be :—

$$10 \times \frac{1}{3} \text{ (ft.-lbs).}$$

$\therefore$ *Work done in* 100 *revolutions* $= 100 \times 10 \times \frac{1}{3} =$ **333·$\dot{3}$ ft.-lbs.**

Pawl and Ratchet Wheel.—A toothed wheel which is acted upon by a vibrating piece, termed a *click* or *pawl*, is called a *ratchet wheel*. Ratchet wheels are made in many different forms, and are used for a variety of purposes. For instance, clocks and watches are usually provided with ratchet wheels to allow the spring or weight to be wound up, without disturbing the rest of the works, and they are used to drive the feeding arrangements of many machines. When, as in the latter case, the click or pawl drives the ratchet wheel, it is carried on a vibrating arm. In the first

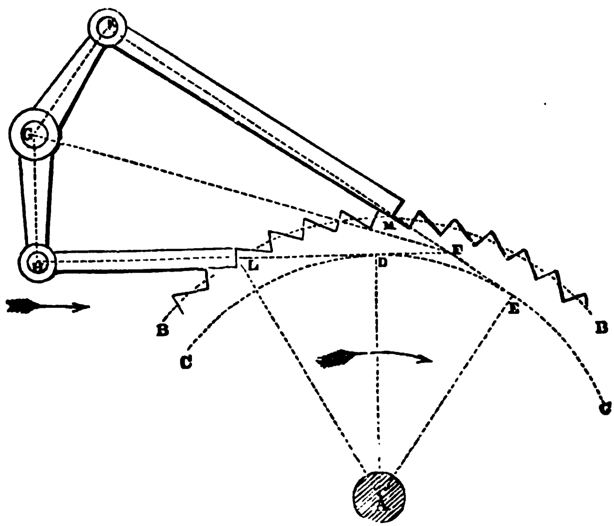


PAWL AND RATCHET.

figure, A B is the vibrating bar which drives the ratchet wheel, by means of the click, B C, and teeth, C c, when moving in the direction shown by the arrow. When A B moves back to A B′, the click slides over the top of the next tooth and drops behind it. It is then ready to drive the wheel through the space of another tooth when A B again moves forward. While the pawl is moving back from B to B′, the wheel is prevented from moving with it by another *pawl* or *detent*, *b c*. In this case, the vibrating bar is on the same axis as the ratchet wheel; but this is not always so, as will be seen from the next example. The reactions between the teeth and the pawl have to keep them in contact with each other. The resultant pressure of the teeth on the pawl must therefore be such, that its moment tends to turn the pawl towards A, the centre of the ratchet wheel. This condition evidently is satisfied if C D, the direction of the resultant pressure at C, passes between A and the axis, B, about which the pawl

turns. Similarly, the moment of the resultant pressure on the detent must tend to turn it towards A, but its direction, *d c* (not *c d*), must lie outside A *b*, because this detent ends in a hook. Both pawls might have been like B C, which acts by pushing, or both hooks, which act by pulling, like *b c*. The pawls are pressed against the ratchet by their own weight, or by springs, according to circumstances. When a ratchet wheel is used only to prevent the récoil of the axis on which it is fixed, the vibrating arm is, of course, not required, and only the detent is used.

La Garousse's Double-Acting Pawl.—This is a pawl which advances the ratchet wheel at each stroke. As will be seen from

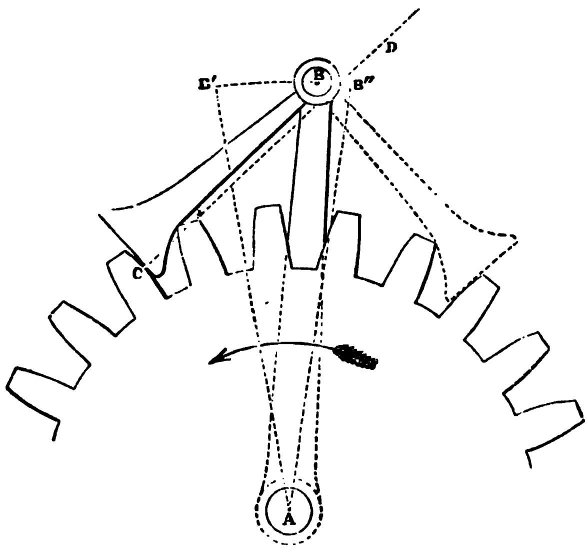


DOUBLE-ACTING PAWL.

the diagram, it is composed of two clicks, K M and H L, carried by an arm, K G H, which vibrates about its centre, G. While the arm is turning in the direction of the straight arrow, H L advances the ratchet by the space of half a tooth while K M retires another half, and, therefore, drops behind the next tooth. During the return of K G H, K M drives the ratchet while H L moves back. It will thus be seen that no detent is required, and that the motion of the ratchet is nearly continuous. The pawls may be hooks, when, of course, the teeth will be modified to suit. The positions of the clicks and arm may be found in the following manner:—Draw any convenient radius, A L, of the pitch circle, B B, and from it set out

the angle, L A D, equal to the desired mean obliquity of the clicks. Draw L D perpendicular to A D, and describe the circle C C with radius A D; the directions of the clicks at mid-stroke will be tangents to this circle. Make angle D A E equal to an *odd* number of times half the pitch angle of the teeth, and draw E M, the tangent, at E. Let this intersect the tangent L D at F, and the pitch circle in M. Draw F G, bisecting angle M F L, and take G, any convenient point in it, for the centre of the rocking shaft. Lastly, make G H and G K perpendicular to H L and K M respectively. Then, K G H is the position of the vibrating arm, and H L and K M the lengths of the two clicks, and their positions at mid-stroke. The *effective* stroke of the clicks is half the pitch of the teeth, and the *total* stroke as much greater as may be necessary to ensure their clearing the teeth.

Reversible Pawl.—The next figure shows a form of click used in the feed motion of shaping and other machines. The ratchet

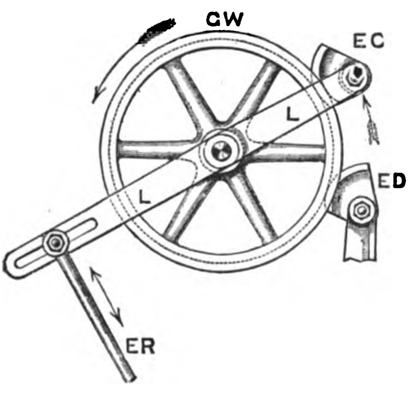


REVERSIBLE CLICK.

wheel is here an ordinary toothed wheel, and the click, B C, is so shaped as to be able to drive it either way. When the click is in the position shown in full lines, it drives the ratchet wheel in the direction of the arrow. When the wheel is required to rotate

the other way, the click is lifted over to the dotted position; and, if it be desired to stop the feed motion without stopping the machine, the click is put in an upright position. A portion of the pin at B, which turns with the click, is triangular in section. A spring presses on this part and so keeps the click in any one of its three positions. The ratchet wheel is keyed to A, the axis of the screw which moves the slide carrying the cutter, and the friction between this screw and its nut is sufficient, without any detent, to prevent the ratchet from moving back. The vibrating arm, A B, which carries the click is driven by a small eccentric or crank. The pawl may, of course, be made to move the rachet more than one tooth at a time by adjusting the angle through which A B vibrates.

Masked Ratchet.—In numbering machines it is often necessary to print the same number twice, as in cheques and their counterfoils. The ratchet which shifts the type wheels must, therefore, be moved at every alternate back-stroke of the printing machine. This may be accomplished by putting a second ratchet, running free on the shaft, alongside the driving one and making the pawl broad enough to move both. The second ratchet has the same number of teeth as the other, but its teeth are made alternately deep and shallow. It is also a little larger than the driving ratchet, so that the pawl passes over the top of the teeth of the latter, without moving it, when in a shallow tooth. Next stroke the pawl drops into a deep tooth. This allows it to catch the



INDEX TO PARTS.

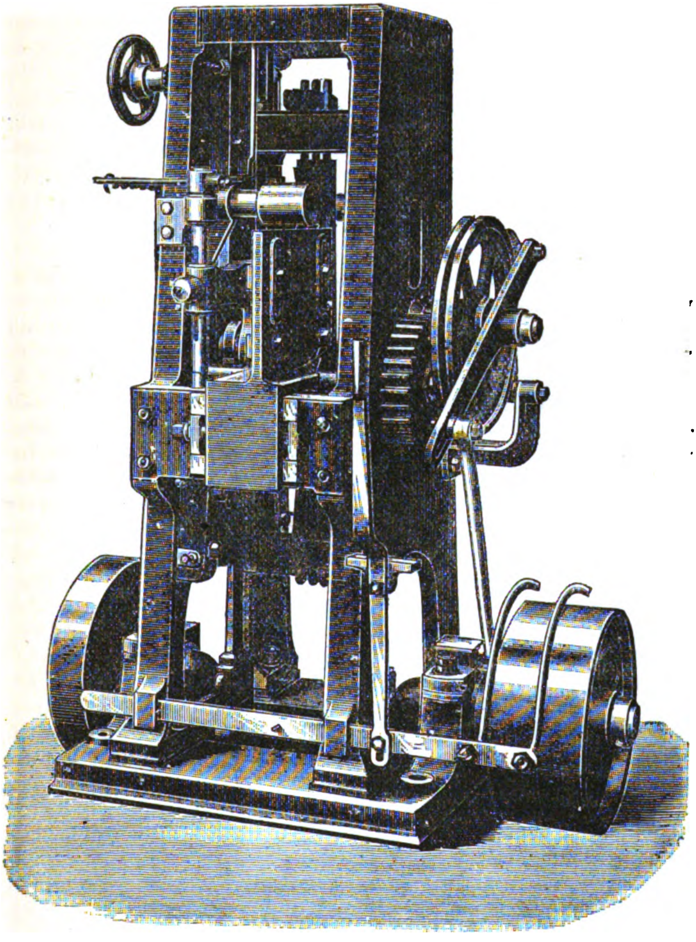
G W for Grooved wheel.
E C ,, Eccentric cam.
L ,, Lever.
E D ,, Eccentric detent.
E R ,, Eccentric rod.

WORSSAM'S SILENT FEED.

teeth of the main ratchet and so shift the type wheel. This arrangement is called a *masked ratchet*.

Silent Feed.—A ratchet wheel is always more or less noisy in

action, and the wear caused by the sudden drop of the pawl is considerable. To avoid this, a friction catch is sometimes substituted for the pawl and a grooved wheel for the toothed one.



VERTICAL SAWING MACHINE, BY JOHN M'DOWAL & SONS OF JOHNSTONE, SHOWING SILENT FEED.

The pawl and ratchet then becomes a *silent feed*. The action of this arrangement will be easily understood by a reference to

the figure.* E C is an eccentric cam which is tapered at its edge to fit the groove in the grooved wheel, G W. It can turn on a pin carried by the lever, L. When E C moves as shown by the arrow, the friction causes it to turn about its axis, and, since the axis is not concentric with the circular part of its rim, it gets wedged in the groove. Hence, for the rest of the stroke, the lever carries G W round with it. At the beginning of the return stroke, E C turns in the opposite direction, and so gets released from the groove. A detent, E D, precisely similar to E C, but carried on a fixed arm, prevents the wheel from moving backwards. The lever, L L, is worked by an eccentric, and the length of its stroke may be adjusted by altering the position of the end of the eccentric rod, E R, in the slot. The second illustration shows a sawing machine, with this feed motion at the right-hand side.

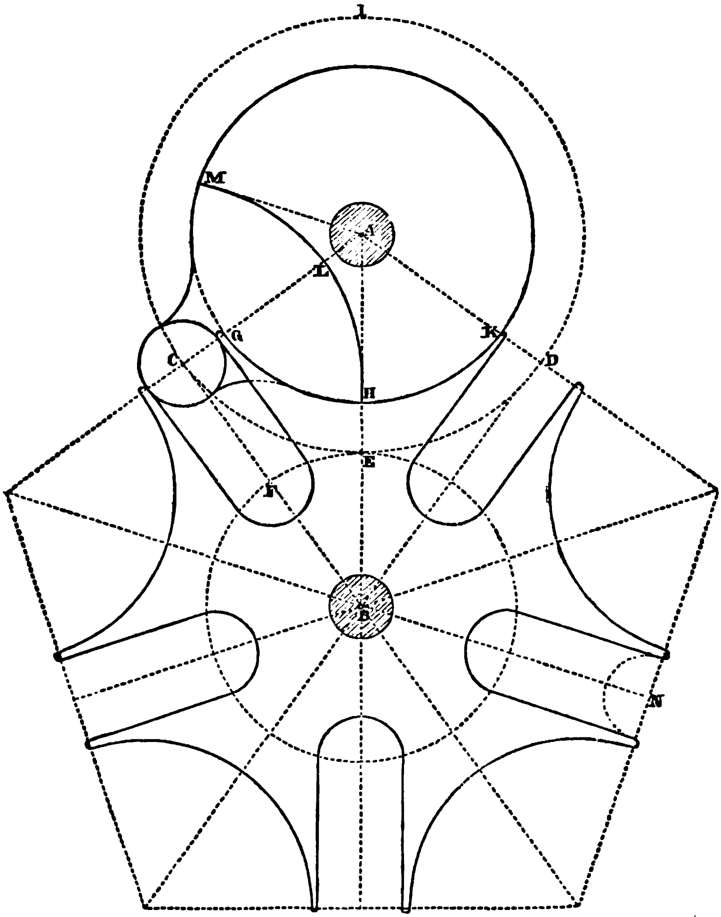
Counting Wheels.—In counting machines, the wheel carrying the figures for the tens must turn through one-tenth of a revolution while the units wheel shifts from 9 to 0, and it must remain at rest at other times. The same is true of the wheels for the hundreds and tens, and so on. The most obvious way to do this is to put ten teeth on the follower on the tens shaft, and only one on the driver on the units shaft. Every time the units wheel passes a certain point it will, therefore, shift the tens wheel by one tooth. The teeth on the follower are usually pins, and a roller is pressed between them by a spring. This roller serves to bring the wheel to its exact position, and to lock it there.

Another device is shown in the accompanying figure, which avoids the shock that always takes place in the first arrangement. Here, A is the shaft whose revolutions require to be counted, and B the centre of the counting wheel. The wheel fixed to the shaft, A, carries a pin, C, which moves the counting wheel by gearing with the sides of the slots. While the pin is in a slot, the horns, G, K, &c., pass through the part, M G H L, of the driving wheel, which is cut for the purpose. After the pin has left the slot, the curved part, G H K, bears on the convex arc, H K M, and so locks the counting wheel in the position shown in the figure. The figure shows only five slots, but there may be ten, or any other required number. When used for counting, there would, of course, be ten.

The following construction may be used for finding the proportions of the various parts:—Join A B, and set out angles C B A, A B D, &c., each equal to $\frac{360^\circ}{2n}$, or $\frac{180^\circ}{n}$, where n is the number of

* We have to thank Messrs John M'Dowal & Sons, of Johnstone, the makers of this machine, for these two figures.

slots required. Draw A C and A D perpendicular to B C and B D respectively, and complete the regular polygon, of which C A and A D are the halves of two adjacent sides. In the figure, this is a



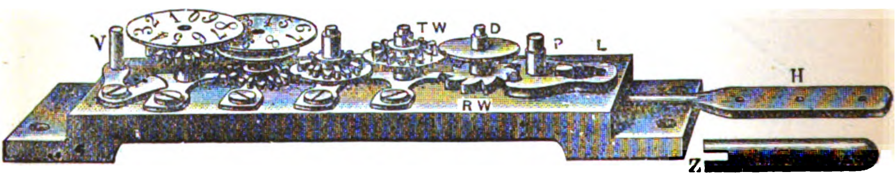
Counting Wheel.

regular pentagon. Then each alternate line radiating from B, such as B C or B D, is the centre line of a slot, and the others, as B A, the centre lines of the circular arcs, G H K, &c. With centre

A and radius A C, describe the circle C E D, to represent the path of the centre of the pin, and another circle, with B as centre, to touch it. Then, at the points where this latter circle cuts the centre lines of the slots, make semicircles of the same radius as the pin, C. The sides of the slots may then be drawn parallel to their respective centre lines to touch these semicircles. The arcs, G H K, are drawn with radius equal to A H, and their centres at the angles of the polygon, so as to be concentric with H K M when in gear with it. The arc, M L H, has its centre in A C, and its radius equal to B G or B K.

Geneva Stop.—This is a modification of the above, used to prevent watches being overwound, and such like purposes. This is effected by filling up one of the slots, as shown by the dotted circle at N. The pin on A is arrested when N reaches C or D. The same thing would result from filling up one of the hollows like G H K. It is obvious that the shaft, A, can make one complete turn for every slot, except the stopping one, and a complete turn all but the angle, G A K, for that one.

Counting Machines.—The accompanying illustration shows a very good form of counting-machine (with three of its dials removed)



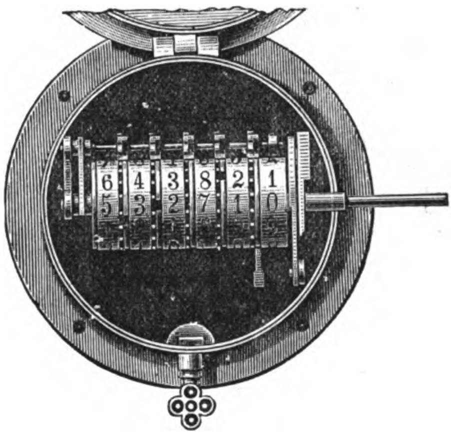
THE "UNIVERSAL COUNTER.

which works on a modification of the first of the above-mentioned methods.* The driving mechanism consists essentially of a short lever, L, which can oscillate about the pin, P, and which drives the ratchet wheel, R W, by two projections on its end. Below this lever, L, there is a circular plate from which a pin projects into the hole in L. This pin, not being concentric with the circular plate, will cause L to oscillate when the plate rotates. The rod, Z, is attached to the back of this plate when the instrument is used to count revolutions, and the lever, H, to its edge, as shown in the figure, when used to count oscillations. The ratchet wheel spindle carries the first or units dial, and also a disc, D, having a pin projecting downwards from its lower edge. The next spindle, to which the tens dial is attached, has a toothed

* This and the following figure were kindly supplied by the makers of these instruments, Messrs. Schäffer & Budenberg of Glasgow, &c.

wheel, T W, with twenty teeth. These teeth are alternately broad and narrow, and the wheel is locked by the disc, D, gearing between two consecutive broad teeth. At the proper time, the pin on D comes into gear with a narrow tooth, and, at the same time, a notch on the edge of the disc allows one broad tooth to pass round. The tens spindle, therefore, makes one-tenth of a revolution. The same arrangement is adopted for the other dials. The case of the instrument has windows which allow only one figure on each dial to be seen at a time. In order that the dials may be easily set to zero, their spindles are each mounted on separate levers, which are locked in their places by the bent lever to which V is attached. Pulling V to the left frees these levers, and permits them to be so turned as to put the toothed wheels out of gear with their respective discs, when the dials may be set to zero.

Another counter by the same makers is shown in the second

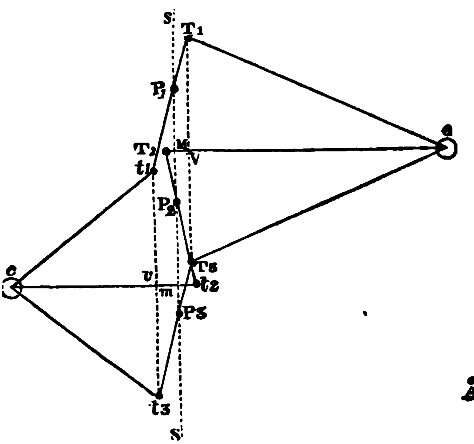


Harding Counter.

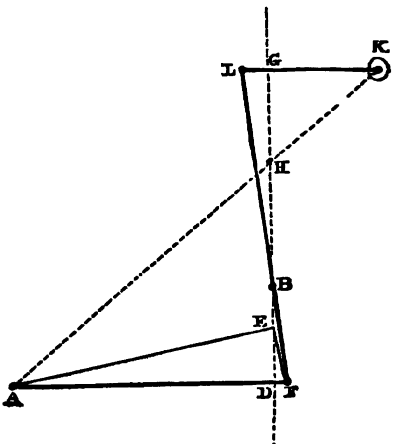
figure. In this instrument the dials are cylindrical and all run loose on one shaft. To the right-hand side of each (except the first) is fixed a number of pins, and to the left (except on the last) two only. Above the dials, between each pair, a set of little toothed wheels is mounted loosely on a secondary spindle. These toothed wheels gear with the pins on the dials, and every alternate tooth is also broad enough to gear with the side of the right-hand dial, which locks them in the same way as the disc in the "Universal" counter. The two pins on the left of a dial come into

gear at the proper time, and a slot on the side of the dial allows the little pinion to turn two teeth. This is just sufficient to cause the next dial to turn through one-tenth of a revolution. The same thing is true of each of the other dials. This instrument is driven in a similar way to the previous one.

Watt's Parallel Motion.—Referring to the illustration of Watt's double-acting engine, previously given in this Lecture, the student will notice that the beam and piston-rod are connected by a set of links. This system of links has been called *Watt's Parallel Motion.* The first part—viz., that for guiding a point in a straight line—is usually termed a "parallel motion," although this term properly belongs to the portion which makes certain other points travel in paths parallel to that of this guided point. The next figure will



WATT'S APPROXIMATE STRAIGHT-LINE MOTION.



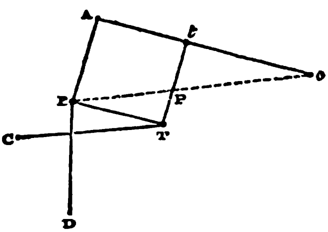
CONSTRUCTION FOR LENGTHS OF LINKS.

serve to show the principle on which an approximate rectilinear motion is obtained. Part of the beam of the engine is shown in three different positions, C T_1, C T_2, and C T_3. The point, T, in it is connected by the link, T *t*, to the end of a lever or radius rod, *c t*, pivotted at *c*. In their mid positions, C T_2, *c* t_2, these two levers are usually parallel to each other, and perpendicular to the line P_1 P_2 P_3. The point, T, describes an arc of a circle round C, and *t* round *c*. As these arcs curve in opposite directions, we should expect some intermediate point on the link, T *t*, to curve in neither direction, but to describe an approximate straight line.

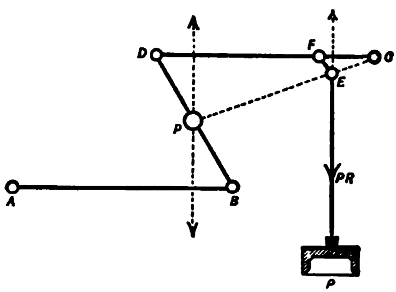
This point, P, may be found by making $\frac{\mathrm{P}\,t}{\mathrm{P\,T}} = \frac{\mathrm{C\,T}}{c\,t}$. The actual path of P is like the figure 8, and the parts which cross are very nearly exact straight lines for a short distance on either side of the crossing.

Prof. Rankine gives the following construction for the lengths of the links in his *Machinery and Millwork* :—Let A be the centre of the beam, G D the centre line of the piston-rod's motion, and B the mid position of its end. Draw A D perpendicular to G D. Make D E equal to one-fourth of the stroke, and join A E. Draw E F perpendicular to A E, and meeting A D in F. A F is the length of the beam. If G be the point where the radius rod cuts G D, draw G K at right angles to G D, and make D H equal to G B. Join A to H, and F to B, and produce A H and F B to meet G K in K and L. Then, F L is the connecting link, K L is the radius rod, and B is the point on the link, F L, to which the piston-rod must be attached.

Parallel Motion.—We will now consider the parallel motion proper. In the accompanying figure A B T t is a parallelogram, and c is a point in A t produced. In the meantime we will



Parallel Motion.



Parallel Motion for Richard's Indicator.

leave the links C T and B D out of account and consider the parallelogram only. Join B c and we have two similar triangles, B A c and P t c.

$$\therefore \quad \frac{\mathrm{P}\,t}{\mathrm{B\,A}} = \frac{t\,c}{\mathrm{A}\,c}. \qquad \therefore \mathrm{P}\,t = \mathrm{B\,A}\,\frac{t\,c}{\mathrm{A}\,c} = \text{a constant,}$$

That is, in every position of the parallelogram the point, P, remains in one fixed position in the link, T t. Moreover, the ratio

$\frac{Bc}{Pc}$ is constant, and, therefore, whatever path P traces out, B will trace out a similar one. This is the principle of the *pantograph*, which is used for enlarging or reducing drawings. Now, we have just seen how we may make P move in an approximate straight line by the link, C T. B will, therefore, also move in an approximate straight line. We might have guided B instead of P with a radius rod, but this would have necessitated longer and heavier links and would have occupied more space.

In applying this motion to his engine, Watt made A t c the beam, and attached the piston-rod to B and the air pump-rod to P. The lengths A c, t c were, therefore, proportional to the strokes of the piston and pump bucket respectively. Sometimes a third link was added so as to get a second parallelogram and a second point moving parallel to P, and this was used to drive the feed-pump.

The right-hand figure shows the parallel motion of Richard's steam engine indicator.* The student will at once see that it is a modification of Watt's parallel motion. In this case the piston-rod, P R, is attached by the link, E F, to the bar, C D, between D and the centre, C. The motion of p, to which the pencil is attached, is, therefore, a *magnified* copy of the piston's motion.*

* See Lecture XVI. of the author's *Elementary Manual of Steam and the Steam Engine* for a description of this indicator.

Lecture XIX.—Questions.

1. Describe Hooke's joint for connecting two axes whose directions meet in a point. Investigate a method of setting out in a diagram the angles described by the axes in the same given time. (Hons. S. and A. Exam., 1895.)

2. Sketch and describe the double Hooke's joint, and explain why it is used in certain cases in preference to the single joint.

3. The rods of a double-barrelled pump are attached to a double-cranked shaft, at the end of which is a wheel with 30 teeth. The wheel gears with a pinion of 8 teeth driven by a winch handle. Find the number of single strokes performed by each pump rod while the winch handle makes 15 revolutions, and sketch the arrangement. *Ans.* 8.

4. In printing machines the table is sometimes made to reciprocate by running a pinion between two racks, whereof one is fixed and the other is attached to the table. The pinion may be actuated by a crank and connecting-rod, but in that case the reciprocation of the table is not uniform, how may a uniform reciprocation be obtained? (Hons. S. and A. Exam., 1893.)

5. What is an epicyclic train, and where are such trains chiefly employed? Investigate a formula for ascertaining the relative velocities of the first and last wheels of such a train. In an epicyclic train, where the first wheel has 20 teeth and is fixed, the second and third wheels are on one axis and have 30 and 40 teeth respectively, and the last wheel has 50 teeth, find the number of rotations of the last wheel for 30 rotations of the arm. In which direction does the last wheel rotate relatively to the arm? (S. and A. Exam., 1893.) *Ans.* 14; in the same direction.

6. In a rope-making machine, the reels containing the strands are carried round in a circular path, but no twisting or untwisting of the strands occurs during the operation. Sketch and describe the epicyclic train, or other device, by which you would accomplish this. If a little extra twist be required to be put on the strands, how may this be done? Explain your answer fully. (S. and A. Exam., 1891.)

7. Prove the formula which gives the velocity of rotation of the last wheel of an epicyclic train in terms of the velocities of the first wheel and the arm, and arrange an epicyclic train in which the last wheel and the arm shall rotate with equal velocities in opposite directions. (S. and A. Exam., 1889.)

8. A train of three spur wheels is carried by a revolving arm, the first is a dead wheel of 60 teeth, the second has 30 teeth, and the third has 45 teeth. Prove the formula for determining the number of revolutions of the second and third wheels for each revolution of the arm, and ascertain the actual numbers in this example. (S. and A. Exam., 1890.) *Ans.* $3; -\frac{1}{3}$.

9. Investigate the kinematic properties of an epicyclic train formed by a combination of three equal bevel wheels in gear. Describe, with sketches, the manner in which this combination has been applied in Houldsworth's differential motion. Mention other useful applications of the combination, pointing out the special results obtained. (Hons. S. and A. Exam., 1895.)

10. An epicyclic train supported on a frame consists of (1) a spur wheel, A, having 40 teeth; (2) a disc, B, having the same axis as A, and carrying at equal intervals three pinions of 16 teeth, each of which gears with A; (3) an annular wheel, C, of 72 teeth coaxial with A and B, and gearing with the three pinions. If A be made a dead wheel and C be the driver, find the velocity ratio of B to C, both as regards magnitude and direction, proving

any formula which you employ. If B be locked to the frame, and A be the driver, find the same as regards C and A. How would you alter the gearing on B, so that C and A may rotate in the same direction while the disc B remains locked to the frame? How may the driving gear of a bicycle be arranged so that the vehicle may travel more slowly up-hill, while the pedal axis runs at the same rate as on level ground? (Hons. S. and A. Exam., 1894.) *Ans.* $\frac{9}{14}$; $-\frac{9}{5}$.

11. Explain the manner in which Watt used the so-called Sun and Planet Wheels as a substitute for a crank and connecting-rod, and account for the result which he obtained. (S. and A. Exam., 1892.)

12. What are elliptical wheels, and for what purpose are they used? What peculiar property of the ellipse has to be taken into account in designing them, and how are they arranged in practice? Give a sketch. (Adv. S. and A. Exam., 1892.) How are these wheels applied to the driving of cycles?

13. Sketch a cam for giving a bar a uniform reciprocating motion, and explain how you find the form of its periphery.

14. Set out a form of cam which, when acting on a bar by uniform rotation, will cause the backward and forward motion of the bar to have an interval of rest between each. Describe some other method of obtaining an intermittent motion of this kind. (Adv. S. & A. Exam., 1888.)

15. Describe, by the aid of the necessary sketches, how the circular motion of the driving pulley is converted into the reciprocating motion of the punch in an ordinary machine for punching holes in metal plates. Calculate the approximate maximum pressure in pounds at the end of a punch in cutting a hole 1 inch in diameter through a steel plate $\frac{5}{8}$ inch thick, the resistance of the plate to shearing being taken as 50,000 lbs. per square inch of section. (Adv. S. & A. Exam., 1894). *Ans.* 98,175 lbs.

16. Sketch and describe what form of cam you would use when it is required to drive the bar both ways, (1) at right angles to the axis of the cam, and (2) parallel to it.

17. Sketch a pawl and ratchet wheel as used for preventing the recoil of the gear.

18. Sketch and describe some form of pawl which will drive a ratchet wheel during both the forward and backward strokes.

19. Sketch a ratchet feed motion, such as is suitable for a planing machine, and explain the manner in which the amount of feed is regulated. (S. & A. Adv. Exam., 1892.)

20. It is sometimes useful to advance a ratchet wheel at every *alternate* forward stroke of the driver, instead of at every stroke, as is commonly the case; describe and sketch a mechanical contrivance which will give such a movement.

21. Describe, with the necessary sketches, some form of silent feed arrangement commonly used instead of a ratchet wheel, for advancing the timber in sawing machines. Explain the principle of the friction grip upon which such a contrivance depends. Within what limit as to deviation of the line of pressure from the common normal is a friction grip possible, and why? (Adv. S. & A. Exam., 1893.)

22. Sketch and describe a vertical sawmill, showing how the silent feed is applied.

23. Sketch and describe an arrangement for counting the number of strokes or revolutions of an engine.

24. Explain the principle of Watt's approximate straight line motion, commonly called a parallel motion. By what combination of linkwork is an exact straight line motion obtained? Prove the geometrical proposition upon which the result depends. (S. & A. Hons. Exam., 1893.)

SCIENCE AND ART DEPARTMENT'S SECOND STAGE OR ADVANCED EXAMINATION IN APPLIED MECHANICS, 1896.

You may not attempt more than six questions.

21. Compare the force expended in pile driving by a ram or monkey of 1 ton falling 20 feet, with that of a weight of 2 tons falling 10 feet. If one blow of the former moves the pile 9 inches, what is the average resistance that is opposed to its motion? *

22. Distinguish between a spur wheel, a bevel wheel, a worm wheel, and a rack. What is the velocity ratio of two wheels? If a bar of cast iron 1 inch square and 1 inch long when secured at one end, breaks transversely with a load of 6,000 lbs. suspended at the free end; what would be the safe working pressure, employing a factor of 10, between the two teeth which are in contact in a pair of spur wheels, whose width of tooth is 6 inches, the depth of the tooth, measured perpendicularly from the point to the root, being 2 inches, and the thickness at the root of the tooth $1\frac{1}{2}$ inches?

23. A king-post truss, whose height is one-fourth of its span, is loaded at the joints with vertical loads of 15, 30, and 45 units respectively. Determine the nature and amount of the stresses in each member of the frame. *

24. What is a cam? For what purposes in mechanism are cams generally used? Sketch and describe the construction and actual form of a cam in use in any machine with which you are acquainted. Sketch a cam which would give a slow forward and quick return motion to a reciprocating piece, with an interval of rest between the two motions.

25. Describe and show by the necessary sketches the construction of a fly-press for punching holes in iron plates. In such a press the two balls weigh 30 lbs. each, and are placed at a radius of 30 inches from the axis of the screw, the screw itself being of 1 inch pitch. What diameter of hole could be punched by such a press in a wrought-iron plate of $\frac{1}{2}$ inch in thickness; the shearing strength of the metal being 22·5 tons per square inch? (Consider that the balls are revolving at the rate of 60 revolutions per minute when the punch comes into contact with the metal, and that the resistance of the plate is overcome in the first sixteenth of an inch of the thickness of the plate.) *

26. Sketch in section, and describe the construction of a differential pulley block working with an endless chain. Why with such a system of blocks does the weight remain suspended after the pull has been taken off the chain? Indicate clearly on your sketch the position of the chain on the pulleys and the snatch block, showing which side of the fall of the chain should be pulled in order to raise and lower the load respectively. If in a Weston's pulley block only 40 per cent. of the energy expended is utilised in lifting the load, what would require to be the diameter of the smaller part of the compound pulley when the largest diameter is 8 inches, in order that a pull of 50 lbs. on the chain may raise a load of 550 lbs.?

27. In a vernier calliper, the bar of the instrument is divided into inches, and each inch is sub-divided into 40 equal divisions. On the sliding jaw of

* For these questions see Vol. II. of this book.

the instrument is carried a vernier whose length is equal to 24 of the small divisions on the bar of the calliper (the vernier therefore measures $\frac{24}{40}$ inch in length), and the vernier scale is divided into 25 equal divisions. When the sliding jaw is brought into close contact with the fixed limb of the calliper, the zero line on the vernier then coincides with the zero line on the bar; what would then be the distance between the first line from zero on the vernier and the first line of the scale on the bar of the calliper? Sketch and describe the construction of the instrument and the method of taking outside measurements with it. What would be the exact position of the vernier on the bar of the instrument when the two jaws of the callipers are separated by a distance of 0·782 inch?

28. A contractor's portable hand crane has a vertical post A B, to which the jib A C is inclined 45°, and the tension rod B C makes with A B an angle A B C of 120°. The back-stay from the head of the post B to the extremity D of the horizontal strut A D is inclined at an angle of 45° to A D. Find the weight of the counterbalance required at D to balance a load of 10 tons suspended from the end C of the jib. Determine also the nature and amount of the stress on the jib A C, and in the rods B C and B D? (The tension in the chain may be neglected.) *

29. A flywheel weighing 5 tons has a mean radius of gyration of 10 feet. The wheel is carried on a shaft of 12 inches diameter and is running at 65 revolutions per minute; how many revolutions will the wheel make before stopping if the coefficient of friction of the shaft in its bearing is 0·065? (Other resistances may be neglected.) *

30. Describe, with the aid of sketches, the construction and use of the quadrant for carrying the change wheels of the screw-cutting lathe, and explain the manner in which change wheels are employed in cutting screw threads of different pitches in the same lathe. What are the sources of inaccuracy in the screw threads so produced? What number of wheels would you employ, how would they be arranged, and how many teeth would each wheel have in order to cut upon a bolt a left-handed screw thread of $\frac{3}{8}$ inch pitch, on a lathe whose leading screw is right handed and has two threads to the inch? In which direction would the saddle of the lathe travel when cutting a left-hand screw thread in the lathe above named?

31. What is the use of an intensifier or intensifying accumulator in the working of hydraulic machinery? Sketch such an apparatus, and explain fully its principle and construction: give also one example of the application of the intensifier to hydraulic machinery.

32. What are the differences in the methods of working of a milling machine and of a planing machine, as arranged for tooling flat surfaces? What are the advantages of milling over planing? Sketch in front and end elevation the cutter or mill for tooling a flat surface, and give any details you can as to the best form for the teeth, and say why for cutting metals the cutting speeds of milling tools can be made greater than those of ordinary planing tools.

SCIENCE AND ART DEPARTMENT'S HONOURS EXAMINATION, 1896.

You may not attempt more than six questions.

41. Compare the physical qualities of cast iron and wrought iron, and of these with mild steel such as is used for boiler construction; also compare them with the steel used for turning tools. Give a numerical statement of the relative powers of these four varieties of iron to resist tensile, compressive, and torsional stresses respectively. What are the fundamental differences in chemical composition between cast iron, wrought iron, and mild steel?

42. Show clearly why, under ordinary conditions, a worm wheel should not be employed to drive a worm, and state also under what conditions such a method of driving becomes possible. In large horizontal boring machines, the boring bar that carries the boring head is slowly revolved by a large worm wheel, which is itself driven by a worm rotated either by suitable pulleys and belting from the main driving shaft of the shop, or by a small engine coupled direct on to the worm shaft. Sketch the boring bar, with the boring head, as also the driving gear, and show how the boring head is traversed along the bar. Why is worm gearing used for driving these heavy machines?

43. Show that the frictional resistance between a belt and a flat pulley may be represented by the formula—

$$\log\left(\frac{T}{t}\right) = 0{\cdot}4343\ \mu\theta$$

when T and t represent the tensions respectively on the two sides of the belt, of which T is greater than t; μ is the coefficient of friction between the belt and the rim of the pulley, and θ is the circular measure of the angle subtended at the centre of the pulley by the part of the belt which is in contact with the pulley. If $\mu = 0{\cdot}1$, what would be the greatest load that could be supported by the rope or chain which passes around the drum, 12 inches in diameter, of a treble purchase crab or winch, which is fitted with a strap friction brake worked by a lever, to the long arm of which a pressure of 60 lbs. is applied? The diameter of the brake pulley is 30 inches, and the brake handle is 3 feet in length from its fulcrum; one end of the brake strap is immovable, being attached to the pin forming the fulcrum of the brake handle, while the other end of the strap or belt is attached to the shorter arm, 3 inches in length, of the brake lever. The angle subtended by the strap at the centre of the brake pulley measures $\frac{3\pi}{2}$. The gearing of the crab is as follows:—On the shaft which carries the brake wheel is a pinion of 15 teeth, and this gears into a wheel of 50 teeth on the second shaft; a pinion of 20 teeth on this latter shaft gears into a wheel with 60 teeth carried upon the drum or barrel shaft. Sketch the crab and show the construction of the brake. Given $\cdot 20466 = \log 1{\cdot}602$.

44. What are the advantages of forging large masses of steel by hydraulic pressure over the same operation performed by the steam

hammer? Show clearly, with the assistance of the necessary sketches, the method employed in hydraulic forging presses for bringing the ram or pressing surface rapidly back from the work after each application of the pressure.

45. Prove an algebraic formula to show that, with a continuous load of uniform intensity passing over a beam A B, such as when a long train passes over a bridge from A to B, the maximum shearing stress at any point K of the beam occurs when the part A K is fully loaded while the part K B is entirely unloaded, and that the magnitude of the stress is proportional to the square of the distance of K from the point A. A train of 1 ton per foot run, and upwards of 100 feet in length, passes over a bridge of 100 feet span; what would be the maximum shearing stresses at distances of 25 and 50 feet respectively from one end of the bridge? Show how to determine graphically the shearing stresses in the beam.*

46. Describe and show, with the necessary sketches, the driving arrangement of the Whitworth double-geared slotting machine. Show and describe clearly how the upward or return stroke of the tool is made more quickly than the downward or cutting stroke. Show also how the length of the stroke is varied; how the height to which the ram can be lifted is adjusted to suit the varying depths of work on the table; and, lastly, indicate how the back gear is thrown in and out of gear.

47. Investigate an expression in terms of p, p_1, and q, which will give the resultant tensile stress p_1 per square inch of section, in a material which is subjected at the same time to a direct tensile stress of p lbs. per square inch, and to a shearing stress of q lbs. per square inch. A bar of iron is at the same time under a direct tensile stress of 5,000 lbs. per square inch and to a shearing stress of 3,500 lbs. per square inch. What would be the resultant equivalent tensile stress in the material?*

48. Distinguish between an absorption and a transmission dynamometer. Describe the action and sketch the construction of an "Epicyclic train form of Dynamometer," and obtain an expression for calculating with the aid of such an apparatus the horse-power being transmitted by a shaft. The power of a portable engine is tested by passing a strap over the flywheel, which is 54 inches in diameter; one end of the strap is fixed, while a weight is suspended from the other end. With such an arrangement, what would be the horse-power transmitted by the engine when running at 160 revolutions per minute, if the suspended weight is 300 lbs. and the tension on the fixed end is found by a spring balance to be 195 lbs.?

49. Describe and show, with the aid of necessary sketches, the construction of the "Pulsometer." Describe how it works, and indicate the contrivances introduced to promote the steady flow of water and to prevent sudden shocks upon the apparatus. Is the pulsometer an economical arrangement for raising water? Give reasons for your answer. What, if any, are its advantages over the ordinary piston pump?

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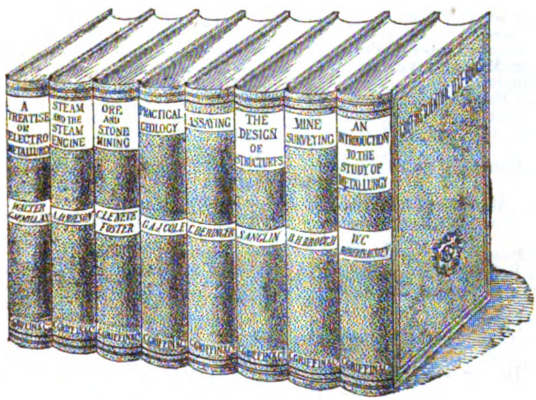

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